2001 AMC 12

Time limit: 75 minutes

Typeset by: LIVE by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc12/2001

1. The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

A $2S+3$

B $3S+2$

C $3S+6$

D $2S+6$

E $2S+12$

2. Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n. For example, $P(23)=6$ and $S(23)=5$. Suppose N is a two-digit number such that $N=P(N)+S(N)$. What is the units digit of N?

A 2

B 3

C 6

D 8

E 9

3. The state income tax where Kristin lives is levied at the rate of $p\%$ of the first $\$28000$ of annual income plus $(p+2)\%$ of any amount above $\$28000.$ Kristin noticed that the state income tax she paid amounted to $(p+0.25)\%$ of her annual income. What was her annual income?

- A $\$28000$
- B $\$32000$
- C $\$35000$
- D $\$42000$
- E $\$56000$

4. The mean of three numbers is 10 more than the least of the numbers and 15 less than the greatest. The median of the three numbers is $5.$ What is their sum?

- A 5
- B 20
- C 25
- D 30
- E 36

5. What is the product of all positive odd integers less than $10{,}000$?

A $\frac{10000!}{(5000!)^2}$

B $\frac{10000!}{2^{5000}}$

C $\frac{9999!}{2^{5000}}$

D $\frac{10000!}{2^{5000} \cdot 5000!}$

E $\frac{5000!}{2^{5000}}$

6. A telephone number has the form $ABC - DEF - GHIJ$, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, D, E, and F are consecutive even digits; G, H, I, and J are consecutive odd digits; and $A + B + C = 9$. Find A.

A 4

B 5

C 6

D 7

E 8

7. A charity sells 140 benefit tickets for a total of $\$2001.$ Some tickets sell for full price (a whole dollar amount), and the rest sell for half price. How much money is raised by the full-price tickets?

A $\$782$

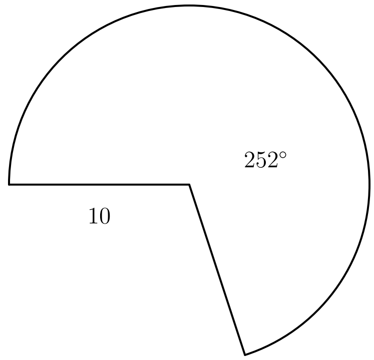
B $\$986$

C $\$1158$

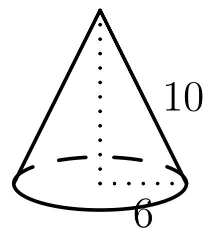
D $\$1219$

E $\$1449$

8. Which of the cones below can be formed from a 252° sector of a circle of radius 10 by aligning the two straight sides?



A



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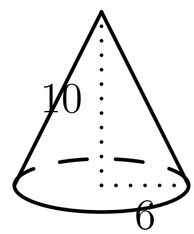

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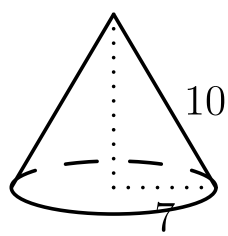

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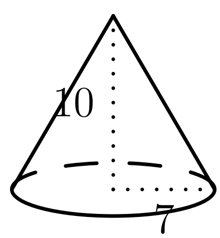

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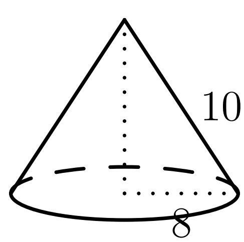


9. Let f be a function satisfying $f(xy) = \frac{f(x)}{y}$ for all positive real numbers x and y. If $f(500) = 3$, what is the value of $f(600)$?

A 1

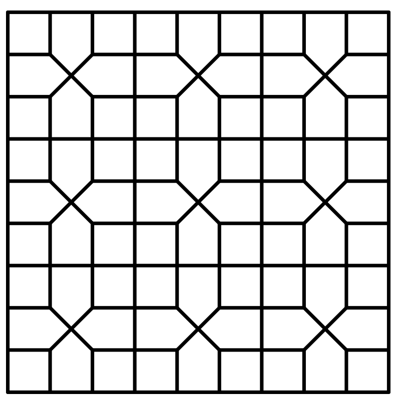
B 2

C $\frac{5}{2}$

D 3

E $\frac{18}{5}$

10. The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to



A 50

B 52

C 54

D 56

E 58

11. A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

A $\frac{3}{10}$

B $\frac{2}{5}$

C $\frac{1}{2}$

D $\frac{3}{5}$

E $\frac{7}{10}$

12. How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

A 768

B 801

C 934

D 1067

E 1167

13. The parabola with equation $y = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y = dx^2 + ex + f$. Which of the following equals $a + b + c + d + e + f$?

A $2b$

B $2c$

C $2a + 2b$

D $2h$

E $2k$

14. Given the nine-sided regular polygon $A_1A_2A_3A_4A_5A_6A_7A_8A_9$, how many distinct equilateral triangles in the plane of the polygon have at least two vertices in the set $\{A_1, A_2, \ldots, A_9\}$?

A 30

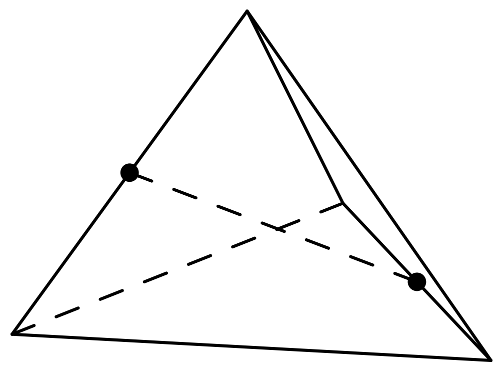
B 36

C 63

D 66

E 72

15. An insect lives on the surface of a regular tetrahedron with edges of length 1. It wishes to travel on the surface of the tetrahedron from the midpoint of one edge to the midpoint of the opposite edge. What is the length of the shortest such trip? (Note: Two edges of a tetrahedron are *opposite* if they have no common endpoint.)



A $\frac{1}{2}\sqrt{3}$

B 1

C $\sqrt{2}$

D $\frac{3}{2}$

E 2

16. A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

A $8!$

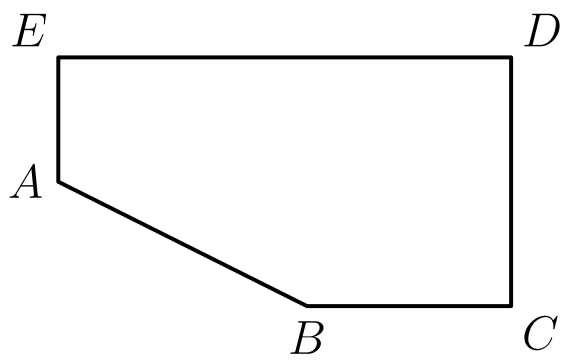
B $2^8\,8!$

C $(8!)^2$

D $\frac{16!}{2^8}$

E $16!$

17. A point P is selected at random from the interior of the pentagon with vertices $A = (0, 2), B = (4, 0), C = (2\pi + 1, 0), D = (2\pi + 1, 4)$, and $E = (0, 4)$. What is the probability that $\angle APB$ is obtuse?



A $\frac{1}{5}$

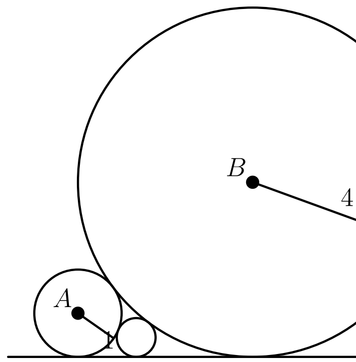
B $\frac{1}{4}$

C $\frac{5}{16}$

D $\frac{3}{8}$

E $\frac{1}{2}$

18. A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first two and to one of their common external tangents as shown. The radius of the third circle is



A $\frac{1}{3}$

B $\frac{2}{5}$

C $\frac{5}{12}$

D $\frac{4}{9}$

E $\frac{1}{2}$

19. The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its zeros, the product of its zeros, and the sum of its coefficients are all equal. If the y-intercept of the graph of $y = P(x)$ is $2,$ what is b?

A -11

B -10

C -9

D 1

E 5

20. Points $A = (3, 9), B = (1, 1), C = (5, 3),$ and $D = (a, b)$ lie in the first quadrant and are the vertices of quadrilateral $ABCD$. The quadrilateral formed by joining the midpoints of $\overline{AB}, \overline{BC}, \overline{CD},$ and $\overline{DA}$ is a square. What is the sum of the coordinates of point D?

A 7

B 9

C 10

D 12

E 16

21. Four positive integers $a, b, c,$ and d have a product of $8!$ and satisfy

$$ab + a + b = 524,$$

$$bc + b + c = 146,$$

$$cd + c + d = 104.$$

What is $a - d$?

A 4

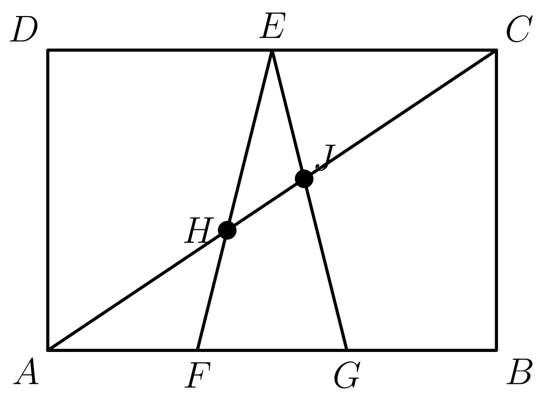
B 6

C 8

D 10

E 12

22. In rectangle $ABCD$, points F and G lie on $\overline{AB}$ so that $AF = FG = GB$ and E is the midpoint of $\overline{DC}$. Also, $\overline{AC}$ intersects $\overline{EF}$ at H and $\overline{EG}$ at J. The area of rectangle $ABCD$ is 70. Find the area of triangle EHJ.



A $\frac{5}{2}$

B $\frac{35}{12}$

C 3

D $\frac{7}{2}$

E $\frac{35}{8}$

23. A polynomial of degree four with leading coefficient 1 and integer coefficients has two real zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

A $\dfrac{1+i\sqrt{11}}{2}$

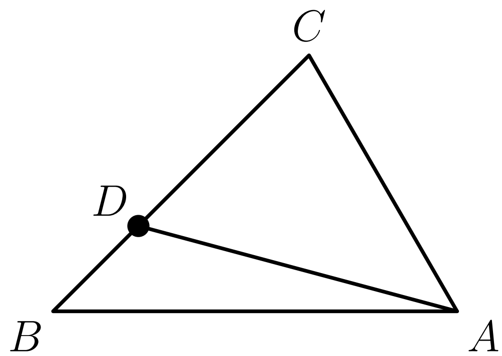
B $\dfrac{1+i}{2}$

C $\dfrac{1}{2}+i$

D $1+\dfrac{i}{2}$

E $\dfrac{1+i\sqrt{13}}{2}$

24. In triangle $ABC, \angle ABC = 45^\circ$. Point D is on $\overline{BC}$ so that $2 \cdot BD = CD$ and $\angle DAB = 15^\circ$. Find $\angle ACB$.



A 54°

B 60°

C 72°

D 75°

E 90°

25. Consider sequences of positive real numbers of the form $x, 2000, y, \ldots,$ in which every term after the first is 1 less than the product of its two immediate neighbors. For how many different values of x does the term 2001 appear somewhere in the sequence?

A 1

B 2

C 3

D 4

E more than 4

Solutions: https://live.poshenloh.com/past-contests/amc12/2001/solutions