# 2018 AMC 10A

Time limit: 75 minutes

Typeset by: LIVE, by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc10/2018A

1. What is the value of

$$\left(\left(\left(2+1\right)^{-1}+1\right)^{-1}+1\right)^{-1}+1?$$

A $\dfrac{5}{8}$

B $\dfrac{11}{7}$

C $\dfrac{8}{5}$

D $\dfrac{18}{11}$

E $\dfrac{15}{8}$

**2.** Liliane has $50\%$ more soda than Jacqueline, and Alice has $25\%$ more soda than Jacqueline. What is the relationship between the amounts of soda that Liliane and Alice have?

A   Liliane has $20\%$ more soda than Alice.

B   Liliane has $25\%$ more soda than Alice.

C   Liliane has $45\%$ more soda than Alice.

D   Liliane has $75\%$ more soda than Alice.

E   Liliane has $100\%$ more soda than Alice.

**3.** A unit of blood expires after

$$10! = 10 \cdot 9 \cdot 8 \cdots 1$$

seconds. Yasin donates a unit of blood at noon of January 1. On what day does his unit of blood expire?

A   January 2

B   January 12

C   January 22

D   February 11

E   February 12

**4.** How many ways can a student schedule $3$ mathematics courses — algebra, geometry, and number theory — in a $6$-period day if no two mathematics courses can be taken in consecutive periods?

(What courses the student takes during the other $3$ periods is of no concern here.)

A    3

B    6

C    12

D    18

E    24

**5.** Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least $6$ miles away," Bob replied, "We are at most $5$ miles away." Charlie then remarked, "Actually the nearest town is at most $4$ miles away."

It turned out that none of the three statements were true. Let $d$ be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of $d$?

A    $(0, 4)$

B    $(4, 5)$

C    $(4, 6)$

D    $(5, 6)$

E    $(5, \infty)$

6. Sangho uploaded a video to a website where viewers can vote that they like or dislike a video. Each video begins with a score of $0$, and the score increases by $1$ for each like vote and decreases by $1$ for each dislike vote.

At one point Sangho saw that his video had a score of $90$, and that $65\%$ of the votes cast on his video were like votes. How many votes had been cast on Sangho's video at that point?

A   200

B   300

C   400

D   500

E   600

7. For how many (not necessarily positive) integer values of $n$ is the following value an integer?

$$4000 \cdot \left(\frac{2}{5}\right)^n$$

A   3

B   4

C   6

D   8

E   9

**8.** Joe has a collection of $23$ coins, consisting of 5-cent coins, 10-cent coins, and 25-cent coins. He has $3$ more 10-cent coins than 5-cent coins, and the total value of his collection is $320$ cents. How many more 25-cent coins does Joe have than 5-cent coins?

A   0

B   1

C   2

D   3

E   4

**9.** All of the triangles in the diagram below are similar to isosceles triangle $ABC$, in which $AB = AC$. Each of the $7$ smallest triangles has area $1$, and $\triangle ABC$ has area $40$. What is the area of trapezoid $DBCE$?
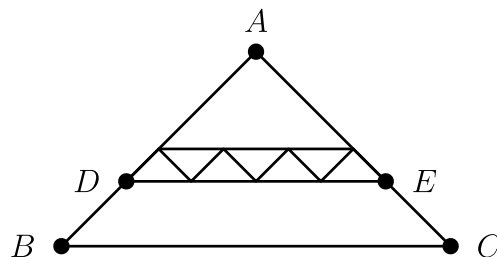


A   16

B   18

C   20

D   22

E   24

**10.** Suppose that real number $x$ satisfies

$$\sqrt{49 - x^2} - \sqrt{25 - x^2} = 3.$$

What is the value of

$$\sqrt{49 - x^2} + \sqrt{25 - x^2}?$$

A   8

B   $\sqrt{33} + 8$

C   9

D   $2\sqrt{10} + 4$

E   12

**11.** When $7$ fair standard $6$-sided dice are thrown, the probability that the sum of the numbers on the top faces is $10$ can be written as

$$\frac{n}{6^7},$$

where $n$ is a positive integer. What is $n$?

A   42

B   49

C   56

D   63

E   84

**12.** How many ordered pairs of real numbers $(x, y)$ satisfy the following system of equations?

$$\begin{cases} x + 3y & = 3 \\ \big||x| - |y|\big| & = 1 \end{cases}$$
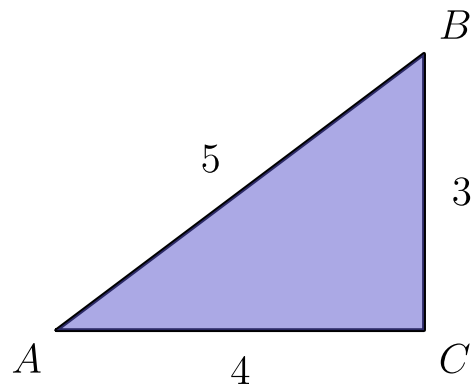
A   1

B   2

C   3

D   4

E   8

**13.** A paper triangle with sides of lengths $3, 4$, and $5$ inches, as shown, is folded so that point $A$ falls on point $B$. What is the length in inches of the crease?



| A | $1 + \dfrac{1}{2}\sqrt{2}$ |
| B | $\sqrt{3}$ |
| C | $\dfrac{7}{4}$ |
| D | $\dfrac{15}{8}$ |
| E | $2$ |

**14.** What is the greatest integer less than or equal to

$$\frac{3^{100} + 2^{100}}{3^{96} + 2^{96}}?$$

| A | $80$ |
| B | $81$ |
| C | $96$ |
| D | $97$ |
| E | $625$ |

**15.** Two circles of radius $5$ are externally tangent to each other and are internally tangent to a circle of radius $13$ at points $A$ and $B$, as shown in the diagram. The distance $AB$ can be written in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. What is $m + n$?
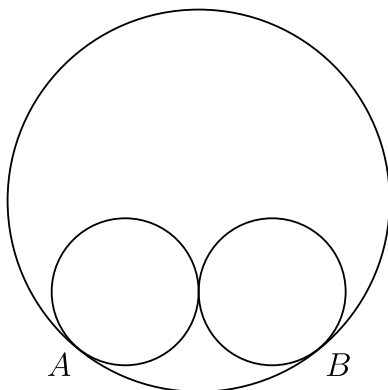


A  21

B  29

C  58

D  69

E  93

**16.** Right triangle $ABC$ has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex $B$ to a point on hypotenuse $\overline{AC}$?

A  5

B  8

C  12

D  13

E  15

**17.** Let $S$ be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if $a$ and $b$ are elements of $S$ with $a < b$, then $b$ is not a multiple of $a$. What is the least possible value of an element in $S$?

A  2

B  3

C  4

D  5

E  7

**18.** How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5$$

$$+a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2$$

$$+a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

A  512

B  729

C  1094

D  3281

E  59,048

**19.** A number $m$ is randomly selected from the set

$$\{11, 13, 15, 17, 19\},$$

and a number $n$ is randomly selected from

$$\{1999, 2000, 2001, \ldots, 2018\}.$$

What is the probability that $m^n$ has a units digit of $1$?

A $\dfrac{1}{5}$

B $\dfrac{1}{4}$

C $\dfrac{3}{10}$

D $\dfrac{7}{20}$

E $\dfrac{2}{5}$

**20.** A scanning code consists of a $7 \times 7$ grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of $49$ squares.

A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of $90°$ counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides.

What is the total number of possible symmetric scanning codes?

A    $510$

B    $1022$

C    $8190$

D    $8192$

E    $65,534$

**21.** Which of the following describes the set of values of $a$ for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real $xy$-plane intersect at exactly $3$ points?

A    $a = \dfrac{1}{4}$

B    $\dfrac{1}{4} < a < \dfrac{1}{2}$

C    $a > \dfrac{1}{4}$

D    $a = \dfrac{1}{2}$

E    $a > \dfrac{1}{2}$

**22.** Let $a, b, c,$ and $d$ be positive integers such that

$$\gcd(a, b) = 24,$$

$$\gcd(b, c) = 36,$$

$$\gcd(c, d) = 54,$$

and

$$70 < \gcd(d, a < 100.$$

Which of the following must be a divisor of $a$?

A    5

B    7

C    11

D    13

E    17

**23.** Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths $3$ and $4$ units. In the corner where those sides meet at a right angle, he leaves a small unplanted square $S$ so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from $S$ to the hypotenuse is $2$ units. What fraction of the field is planted?
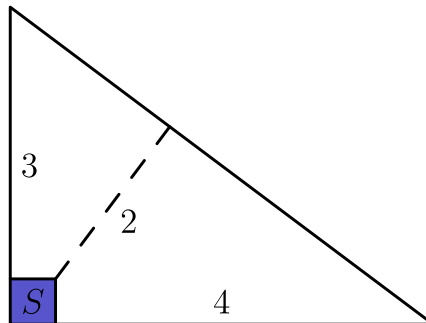


- A  $\dfrac{25}{27}$

- B  $\dfrac{26}{27}$

- C  $\dfrac{73}{75}$

- D  $\dfrac{145}{147}$

- E  $\dfrac{74}{75}$

**24.** Triangle $ABC$ with $AB = 50$ and $AC = 10$ has area 120. Let $D$ be the midpoint of $\overline{AB}$, and let $E$ be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at $F$ and $G$, respectively. What is the area of quadrilateral $FDBG$?

A    60

B    65

C    70

D    75

E    80

**25.** For a positive integer $n$ and nonzero digits $a$, $b$, and $c$, let $A_n$ be the $n$-digit integer each of whose digits is equal to $a$; let $B_n$ be the $n$-digit integer each of whose digits is equal to $b$; and let $C_n$ be the $2n$-digit (not $n$-digit) integer each of whose digits is equal to $c$. What is the greatest possible value of $a + b + c$ for which there are at least two values of $n$ such that

$$C_n - B_n = A_n^2?$$

A    12

B    14

C    16

D    18

E    20

Solutions: https://live.poshenloh.com/past-contests/amc10/2018A/solutions