2014 AMC 10B

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc10/2014B

1. Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

A 33

B 35

C 37

D 39

E 41

2. What is $\dfrac{2^3+2^3}{2^{-3}+2^{-3}}$?

A 16

B 24

C 32

D 48

E 64

3. Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles how long was Randy's trip?

A 30

B $\frac{400}{11}$

C $\frac{75}{2}$

D 40

E $\frac{300}{7}$

4. Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

A $\frac{3}{2}$

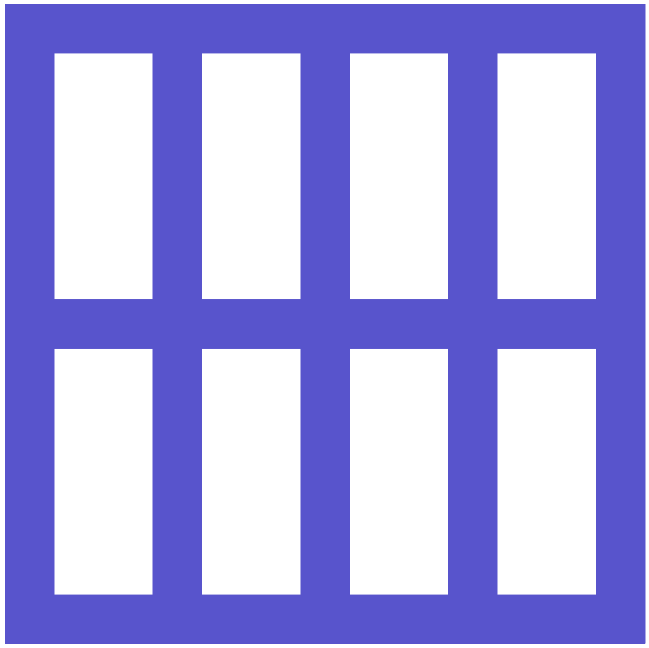
B $\frac{5}{3}$

C $\frac{7}{4}$

D 2

E $\frac{13}{4}$

5. Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5 : 2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?



A 26

B 28

C 30

D 32

E 34

6. Orvin went to the store with just enough money to buy 30 balloons. When he arrived, he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

A 33

B 34

C 36

D 38

E 39

7. Suppose $A > B > 0$ and A is $x\%$ greater than B. What is x?

A $100\left(\frac{A-B}{B}\right)$

B $100\left(\frac{A+B}{B}\right)$

C $100\left(\frac{A+B}{A}\right)$

D $100\left(\frac{A-B}{A}\right)$

E $100\left(\frac{A}{B}\right)$

8. A truck travels $\dfrac{b}{6}$ feet every t seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?

A $\dfrac{b}{1080t}$

B $\dfrac{30t}{b}$

C $\dfrac{30b}{t}$

D $\dfrac{10t}{b}$

E $\dfrac{10b}{t}$

9. For real numbers w and z,

$$\frac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}}=2014.$$

What is $\frac{w+z}{w-z}$?

A -2014

B $\dfrac{-1}{2014}$

C $\dfrac{1}{2014}$

D 1

E 2014

10. In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{r} ABBCB \\ +\ BCADA \\ \hline DBDDD \end{array}$$

A 2

B 4

C 7

D 8

E 9

11. For the consumer, a single discount of $n\%$ is more advantageous than any of the following discounts:

(1) Two successive 15% discounts.

(2) Three successive 10% discounts.

(3) A 25% discount followed by a 5% discount.

What is the smallest possible positive integer value of n?

A 27

B 28

C 29

D 31

E 33

12. The largest divisor of $2,014,000,000$ is itself. What is its fifth-largest divisor?

A $125,875,000$

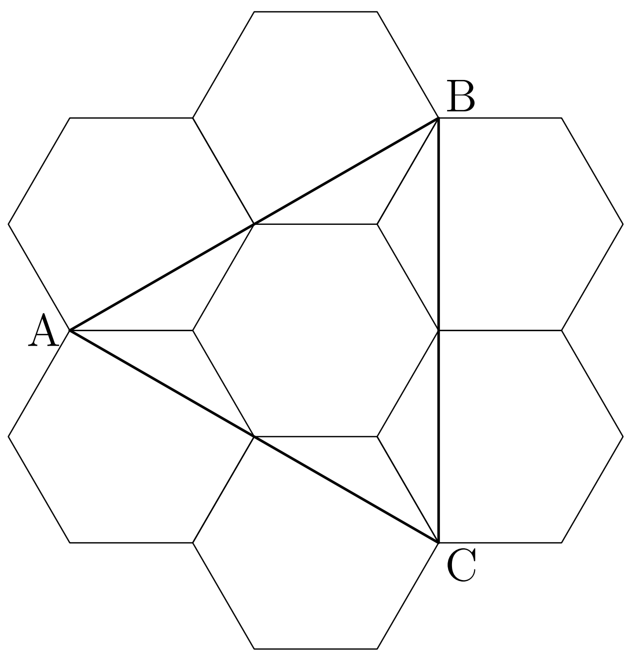
B $201,400,000$

C $251,750,000$

D $402,800,000$

E $503,500,000$

13. Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?



A $2\sqrt{3}$

B $3\sqrt{3}$

C $1+3\sqrt{2}$

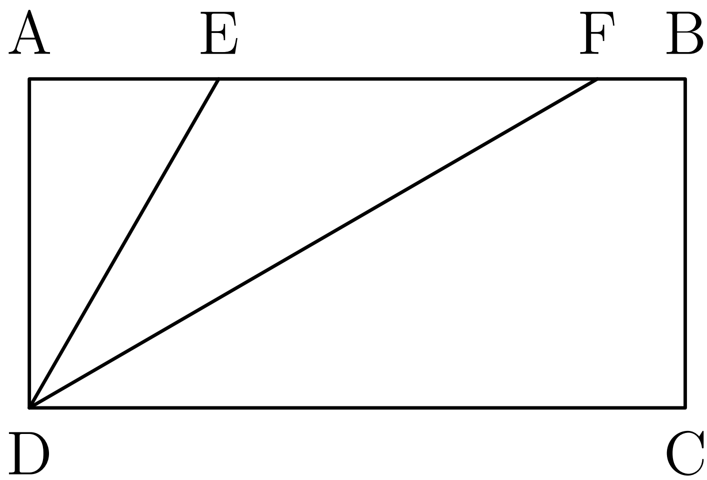
D $2+2\sqrt{3}$

E $3+2\sqrt{3}$

14. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a+b+c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2+b^2+c^2$?

- A 26
- B 27
- C 36
- D 37
- E 41

15. In rectangle $ABCD$, $DC = 2 \cdot CB$ and points E and F lie on $\overline{AB}$ so that $\overline{ED}$ and $\overline{FD}$ trisect $\angle ADC$ as shown. What is the ratio of the area of $\triangle DEF$ to the area of rectangle $ABCD$?



A $\frac{\sqrt{3}}{6}$

B $\frac{\sqrt{6}}{8}$

C $\frac{3\sqrt{3}}{16}$

D $\frac{1}{3}$

E $\frac{\sqrt{2}}{4}$

16. Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

A $\frac{1}{36}$

B $\frac{7}{72}$

C $\frac{1}{9}$

D $\frac{5}{36}$

E $\frac{1}{6}$

17. What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?

A 2^{1002}

B 2^{1003}

C 2^{1004}

D 2^{1005}

E 2^{1006}

18. A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

A 24

B 30

C 31

D 33

E 35

19. Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

A $\frac{1}{6}$

B $\frac{1}{4}$

C $\frac{2-\sqrt{2}}{2}$

D $\frac{1}{3}$

E $\frac{1}{2}$

20. For how many integers x is the number $x^4 - 51x^2 + 50$ negative?

A 8

B 10

C 12

D 14

E 16

21. Trapezoid $ABCD$ has parallel sides $\overline{AB}$ of length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles A and B are acute. What is the length of the shorter diagonal of $ABCD$?

A $10\sqrt{6}$

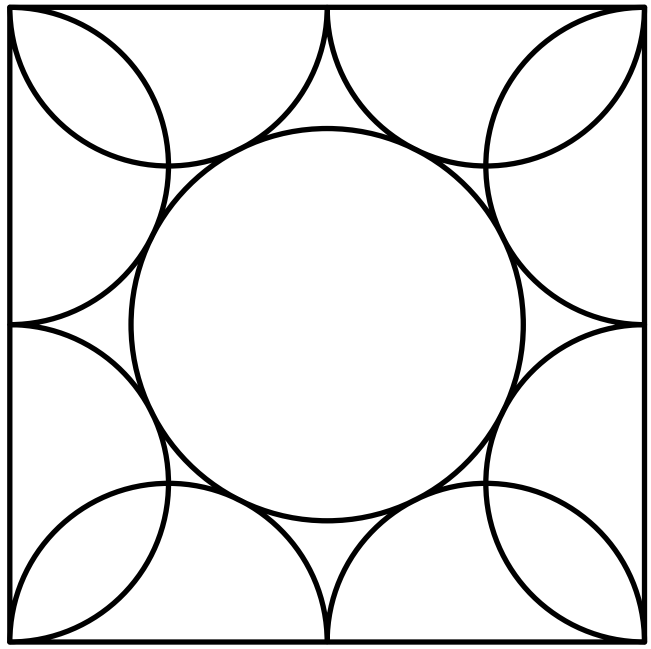
B 25

C $8\sqrt{10}$

D $18\sqrt{2}$

E 26

22. Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?



A $\dfrac{1+\sqrt{2}}{4}$

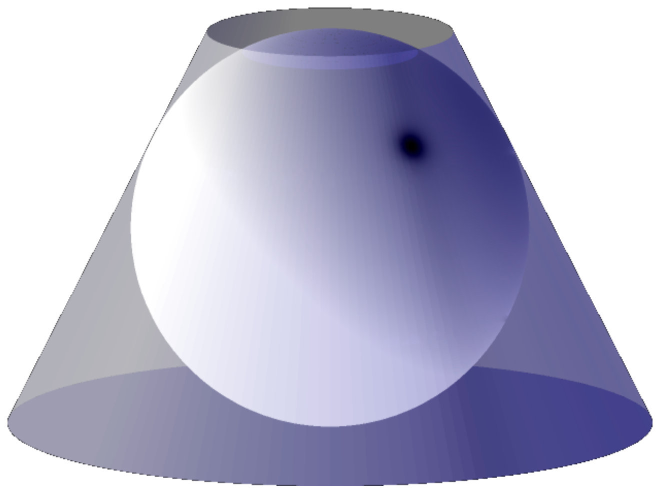
B $\dfrac{\sqrt{5}-1}{2}$

C $\dfrac{\sqrt{3}+1}{4}$

D $\dfrac{2\sqrt{3}}{5}$

E $\dfrac{\sqrt{5}}{3}$

23. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



A $\frac{3}{2}$

B $\frac{1+\sqrt{5}}{2}$

C $\sqrt{3}$

D 2

E $\frac{3+\sqrt{5}}{2}$

24. The numbers $1, 2, 3, 4, 5$ are to be arranged in a circle. An arrangement is bad if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

A 1

B 2

C 3

D 4

E $5.$

25. In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps.

If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape without being eaten by the snake?

A $\frac{32}{79}$

B $\frac{161}{384}$

C $\frac{63}{146}$

D $\frac{7}{16}$

E $\frac{1}{2}$

Solutions: https://live.poshenloh.com/past-contests/amc10/2014B/solutions