# 2026 AIME II

Time limit: 180 minutes

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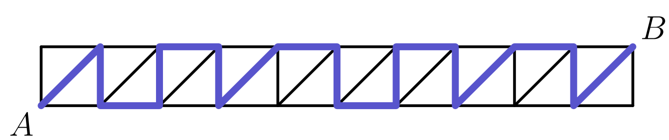
**1.** Find the sum of the $10$th terms of all arithmetic sequences of integers that have first term equal to $4$ and include both $24$ and $34$ as terms.

**2.** The figure below shows a grid of $10$ squares in a row. Each square has a diagonal connecting its lower left vertex to its upper right vertex. A bug moves along the line segments from vertex to vertex, never traversing the same segment twice and never moving from right to left along a horizontal or diagonal segment. Let $N$ be the number of paths the bug can take from the lower left corner $(A)$ to the upper right corner $(B)$. One such path from $A$ to $B$ is shown by the thick line segments in the figure. Find $\sqrt{N}$.



**3.** Let $ABCDE$ be a nonconvex pentagon with internal angles $\angle A = \angle E = 90^\circ$ and $\angle B = \angle D = 45^\circ$. Suppose that $DE < AB$, $AE = 20$, $BC = 14\sqrt{2}$, and points $B$, $C$, and $D$ lie on the same side of line $AE$. Suppose further that $AB$ is an integer with $AB < 2026$ and the area of pentagon $ABCDE$ is an integer multiple of $16$. Find the number of possible values of $AB$.

**4.** For each positive integer $n$ let $f(n)$ be the value of the base-ten numeral $n$ viewed in base $b$, where $b$ is the least integer greater than the greatest digit in $n$. For example, if $n = 72$, then $b = 8$, and $72$ as a numeral in base $8$ equals $7 \cdot 8 + 2 = 58$; therefore $f(72) = 58$. Find the number of positive integers $n$ less than $1000$ such that $f(n) = n$.

**5.** An urn contains $n$ marbles. Each marble is either red or blue, and there are at least $7$ marbles of each color. When $7$ marbles are drawn randomly from the urn without replacement, the probability that exactly $4$ of them are red equals the probability that exactly $5$ of them are red. Find the sum of the five least values of $n$ for which this is possible.

**6.** Find the sum of all real numbers $r$ such that there is at least one point where the circle with radius $r$ centered at $(4, 39)$ is tangent to the parabola with equation $2y = x^2 - 8x + 12.$

**7.** A standard fair six-sided die is rolled repeatedly. Each time the die reads $1$ or $2,$ Alice gets a coin; each time it reads $3$ or $4,$ Bob gets a coin; and each time it reads $5$ or $6,$ Carol gets a coin. The probability that Alice and Bob each receive at least two coins before Carol receives any coins can be written as $\frac{m}{n},$ where $m$ and $n$ are relatively prime positive integers. Find $100m + n.$

**8.** Isosceles triangle $\triangle ABC$ has $AB = BC.$ Let $I$ be the incenter of $\triangle ABC.$ The perimeters of $\triangle ABC$ and $\triangle AIC$ are in the ratio $125 : 6,$ and all the sides of both triangles have integer lengths. Find the minimum possible value of $AB.$

**9.** Let $S$ denote the value of the infinite sum

$$\frac{1}{9} + \frac{1}{99} + \frac{1}{999} + \frac{1}{9999} + \cdots$$

Find the remainder when the greatest integer less than or equal to $10^{100} S$ is divided by $1000.$

**10.** Let $\triangle ABC$ be a triangle with $D$ on $\overline{BC}$ such that $\overline{AD}$ bisects $\angle BAC$. Let $\omega$ be the circle that passes through $A$ and is tangent to segment $\overline{BC}$ at $D$. Let $E \neq A$ and $F \neq A$ be the intersections of $\omega$ with segments $\overline{AB}$ and $\overline{AC}$, respectively. Suppose that $AB = 200$, $AC = 225$, and all of $AE$, $AF$, $BD$, and $CD$ are positive integers. Find the greatest possible value of $BC$.

**11.** Find the greatest integer $n$ such that the cubic polynomial

$$x^3 - \frac{n}{6}x^2 + (n-11)x - 400$$

has roots $\alpha^2$, $\beta^2$, and $\gamma^2$, where $\alpha$, $\beta$, and $\gamma$ are complex numbers, and there are exactly seven different possible values for $\alpha + \beta + \gamma$.

**12.** Consider a tetrahedron with two isosceles triangle faces with side lengths $5\sqrt{10}$, $5\sqrt{10}$, and $10$ and two isosceles triangle faces with side lengths $5\sqrt{10}, 5\sqrt{10}$, and $18$. The four vertices of the tetrahedron lie on a sphere with center $S$, and the four faces of the tetrahedron are tangent to a sphere with center $R$. The distance $RS$ can be written as $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

**13.** Call finite sets of integers $S$ and $T$ *cousins* if

- $S$ and $T$ have the same number of elements,
- $S$ and $T$ are disjoint, and
- the elements of $S$ can be paired with the elements of $T$ so that the elements in each pair differ by exactly $1$.

For example, $\{1, 2, 5\}$ and $\{0, 3, 4\}$ are cousins. Suppose that the set $S$ has exactly $4040$ cousins. Find the least number of elements the set $S$ can have.

**14.** For integers $a$ and $b$, let $a \circ b = a - b$ if $a$ is odd and $b$ is even, and $a \circ b = a + b$ otherwise. Find the number of sequences $a_1, a_2, a_3, \ldots, a_n$ of positive integers such that

$$a_1 + a_2 + a_3 + \cdots + a_n = 12 \quad \text{and} \quad a_1 \circ a_2 \circ a_3 \circ \cdots \circ a_n = 0,$$

where the operations are performed from left to right; that is, $a_1 \circ a_2 \circ a_3$ means $(a_1 \circ a_2) \circ a_3$.

**15.** Find the number of ordered 7-tuples $(a_1, a_2, a_3, \ldots, a_7)$ having the following properties:

- $a_k \in \{1, 2, 3\}$ for all $k$.
- $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$ is a multiple of $3$.
- $a_1a_2a_4 + a_2a_3a_5 + a_3a_4a_6 + a_4a_5a_7 + a_5a_6a_1 + a_6a_7a_2 + a_7a_1a_3$ is a multiple of $3$.

Solutions: https://live.poshenloh.com/past-contests/aime/2026II/solutions