2025 AIME II

Time limit: 180 minutes

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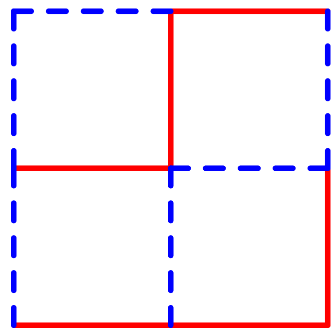
https://live.poshenloh.com/past-contests/aime/2025II

1. Six points $A, B, C, D, E,$ and F lie in a straight line in that order. Suppose that G is a point not on the line and that $AC = 26, BD = 22, CE = 31, DF = 33, AF = 73,$ $CG = 40,$ and $DG = 30.$ Find the area of $\triangle BGE.$

2. Find the sum of all positive integers n such that $n + 2$ divides the product $3(n + 3)(n^2 + 9).$

3. Four unit squares form a 2×2 grid. Each of the 12 unit line segments forming the sides of the squares is colored either red or blue in such a way that each unit square has 2 red sides and 2 blue sides. One example is shown below (red is solid, blue is dashed). Find the number of such colorings.

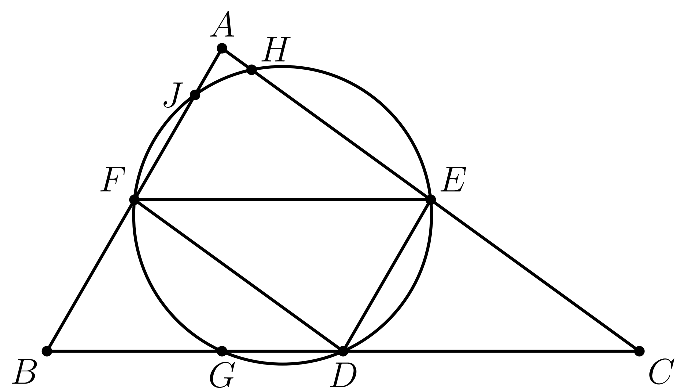


4. The product

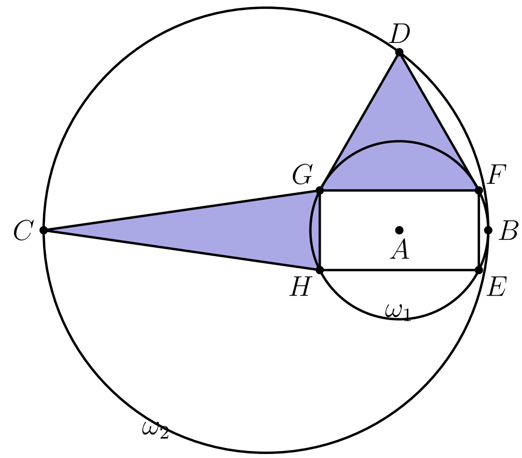
$$\prod_{k=4}^{63} \frac{\log_k(5^{k^2-1})}{\log_{k+1}(5^{k^2-4})} = \frac{\log_4(5^{15})}{\log_5(5^{12})} \cdot \frac{\log_5(5^{24})}{\log_6(5^{21})} \cdot \frac{\log_6(5^{35})}{\log_7(5^{32})} \cdots \frac{\log_{63}(5^{3968})}{\log_{64}(5^{3965})}$$

is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

5. Suppose $\triangle ABC$ has angles $\angle BAC = 84^\circ, \angle ABC = 60^\circ$, and $\angle ACB = 36^\circ$. Let D, E, and F be the midpoints of sides $\overline{BC}, \overline{AC}$, and $\overline{AB}$, respectively. The circumcircle of $\triangle DEF$ intersects $\overline{BD}, \overline{AE}$, and $\overline{AF}$ at points G, H, and J, respectively. The points G, D, E, H, J, and F divide the circumcircle of $\triangle DEF$ into six minor arcs, as shown. Find $\overarc{DE} + 2 \cdot \overarc{HJ} + 3 \cdot \overarc{FG}$, where the arcs are measured in degrees.



6. Circle ω_1 with radius 6 centered at point A is internally tangent at point B to circle ω_2 with radius $15.$ Points C and D lie on ω_2 such that $\overline{BC}$ is a diameter of ω_2 and $\overline{BC} \perp \overline{AD}.$ The rectangle $EFGH$ is inscribed in ω_1 such that $\overline{EF} \perp \overline{BC}$, C is closer to $\overline{GH}$ than to $\overline{EF}$, and D is closer to $\overline{FG}$ than to $\overline{EH}$, as shown. Triangles $\triangle DGF$ and $\triangle CHG$ have equal areas. The area of rectangle $EFGH$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n.$



7. Let A be the set of positive integer divisors of $2025.$ Let B be a randomly selected subset of $A.$ The probability that B is a nonempty set with the property that the least common multiple of its elements is 2025 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n.$

8. From an unlimited supply of 1-cent coins, 10-cent coins, and 25-cent coins, Silas wants to find a collection of coins that has a total value of N cents, where N is a positive integer. He uses the so-called *greedy algorithm*, successively choosing the coin of greatest value that does not cause the value of his collection to exceed N. For example, to get 42 cents, Silas will choose a 25-cent coin, then a 10-cent coin, then 7 1-cent coins. However, this collection of 9 coins uses more coins than necessary to get a total of 42 cents; indeed, choosing 4 10-cent coins and 2 1-cent coins achieves the same total value with only 6 coins.

In general, the greedy algorithm succeeds for a given N if no other collection of 1-cent, 10-cent, and 25-cent coins gives a total value of N cents using strictly fewer coins than the collection given by the greedy algorithm. Find the number of values of N between 1 and 1000 inclusive for which the greedy algorithm succeeds.

9. There are n values of x in the interval $0 < x < 2\pi$ where $f(x) = \sin(7\pi \cdot \sin(5x)) = 0$. For t of these n values of x, the graph of $y = f(x)$ is tangent to the x-axis. Find $n + t$.

10. Sixteen chairs are arranged in a row. Eight people each select a chair in which to sit so that no person sits next to two other people. Let N be the number of subsets of 16 chairs that could be selected. Find the remainder when N is divided by 1000.

11. Let S be the set of vertices of a regular 24-gon. Find the number of ways to draw 12 segments of equal lengths so that each vertex in S is an endpoint of exactly one of the 12 segments.

12. Let $A_1A_2\dots A_{11}$ be an 11-sided non-convex simple polygon with the following properties:

- For every integer $2 \le i \le 10,$ the area of $\triangle A_iA_1A_{i+1}$ is $1.$
- For every integer $2 \le i \le 10, \cos(\angle A_iA_1A_{i+1}) = \frac{12}{13}.$
- The perimeter of the 11-gon $A_1A_2\dots A_{11}$ is equal to $20.$

Then $A_1A_2 + A_1A_{11}$ can be expressed as $\frac{m\sqrt{n}-p}{q}$ where $m, n, p,$ and q are positive integers, n is not divisible by the square of any prime, and no prime divides all of $m, p,$ and $q.$ Find $m+n+p+q.$

13. Let the sequence of rationals $x_1, x_2, \dots$ be defined such that $x_1 = \frac{25}{11}$ and

$$x_{k+1} = \frac{1}{3}\left(x_k + \frac{1}{x_k} - 1\right)$$

for all $k \ge 1.$ Then x_{2025} can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and $n.$ Find the remainder when $m+n$ is divided by $1000.$

14. Let $\triangle ABC$ be a right triangle with $\angle A = 90^\circ$ and $BC = 38$. There exist points K and L inside the triangle such that

$$AK = AL = BK = CL = KL = 14.$$

The area of the quadrilateral $BKLC$ can be expressed as $n\sqrt{3}$ for some positive integer n. Find n.

15. There are exactly three positive real numbers k such that the function

$$f(x) = \frac{(x-18)(x-72)(x-98)(x-k)}{x}$$

defined over the positive real numbers achieves its minimum value at exactly two positive real numbers x. Find the sum of these three values of k.

Solutions: https://live.poshenloh.com/past-contests/aime/2025II/solutions