2012 AMC 10A

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc10/2012A

1. Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

A 10

B 15

C 20

D 25

E 30

2. A square with side length 8 is cut in half, creating two congruent rectangles. What are the dimensions of one of these rectangles?

A 2 by 4

B 2 by 6

C 2 by 8

D 4 by 4

E 4 by 8

3. A bug crawls along a number line, starting at -2. It crawls to -6, then turns around and crawls to 5. How many units does the bug crawl altogether?

A 9

B 11

C 13

D 14

E 15

4. Let $\angle ABC = 24^\circ$ and $\angle ABD = 20^\circ$. What is the smallest possible degree measure for $\angle CBD$?

A 0

B 2

C 4

D 6

E 12

5. Last year 100 adult cats, half of whom were female, were brought into the Smallville Animal Shelter. Half of the adult female cats were accompanied by a litter of kittens. The average number of kittens per litter was 4. What was the total number of cats and kittens received by the shelter last year?

A 150

B 200

C 250

D 300

E 400

6. The product of two positive numbers is 9. The reciprocal of one of these numbers is 4 times the reciprocal of the other number. What is the sum of the two numbers?

A $\frac{10}{3}$

B $\frac{20}{3}$

C 7

D $\frac{15}{2}$

E 8

7. In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

A $\frac{2}{5}$

B $\frac{3}{7}$

C $\frac{4}{7}$

D $\frac{3}{5}$

E $\frac{4}{5}$

8. The sums of three whole numbers taken in pairs are $12, 17,$ and $19.$ What is the middle number?

A 4

B 5

C 6

D 7

E 8

9. A pair of six-sided dice are labeled so that one die has only even numbers (two each of $2, 4$, and 6), and the other die has only odd numbers (two of each $1, 3$, and 5). The pair of dice is rolled. What is the probability that the sum of the numbers on the tops of the two dice is 7?

A $\frac{1}{6}$

B $\frac{1}{5}$

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

10. Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?

A 5

B 6

C 8

D 10

E 12

11. Externally tangent circles with centers at points A and B have radii of lengths 5 and 3, respectively. A line externally tangent to both circles intersects ray AB at point C. What is BC?

A 4

B 4.8

C 10.2

D 12

E 14.4

12. A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February $7, 2012$, a Tuesday. On what day of the week was Dickens born?

A Friday

B Saturday

C Sunday

D Monday

E Tuesday

13. An *iterative average* of the numbers $1, 2, 3, 4,$ and 5 is computed the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

A $\frac{31}{16}$

B 2

C $\frac{17}{8}$

D 3

E $\frac{65}{16}$

14. Chubby makes nonstandard checkerboards that have 31 squares on each side. The checkerboards have a black square in every corner and alternate red and black squares along every row and column. How many black squares are there on such a checkerboard?

A 480

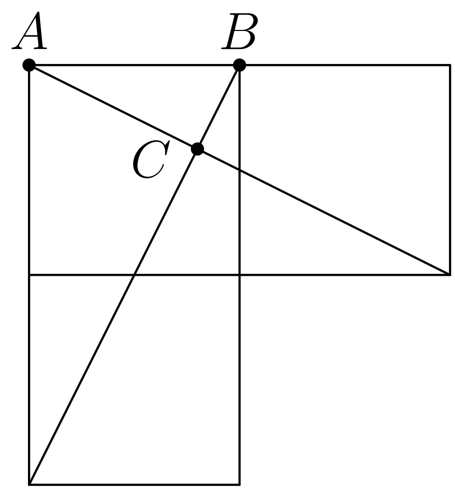
B 481

C 482

D 483

E 484

15. Three unit squares and two line segments connecting two pairs of vertices are shown. What is the area of $\triangle ABC$?



A $\frac{1}{6}$

B $\frac{1}{5}$

C $\frac{2}{9}$

D $\frac{1}{3}$

E $\frac{\sqrt{2}}{4}$

16. Three runners start running simultaneously from the same point on a 500-meter circular track. They each run clockwise around the course maintaining constant speeds of $4.4, 4.8,$ and 5.0 meters per second. The runners stop once they are all together again somewhere on the circular course. How many seconds do the runners run?

A $1,000$

B $1,250$

C $2,500$

D $5,000$

E $10,000$

17. Let a and b be relatively prime positive integers with $a > b > 0$ and

$$\frac{a^3 - b^3}{(a-b)^3} = \frac{73}{3}.$$

What is $a - b$?

A 1

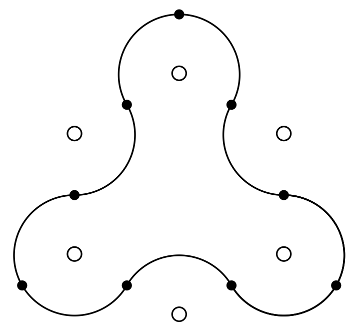
B 2

C 3

D 4

E 5

18. The closed curve in the figure is made up of 9 congruent circular arcs each of length $\dfrac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?



A $2\pi + 6$

B $2\pi + 4\sqrt{3}$

C $3\pi + 4$

D $2\pi + 3\sqrt{3} + 2$

E $\pi + 6\sqrt{3}$

19. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00$ AM, and all three always take the same amount of time to eat lunch.

On Monday the three of them painted 50% of a house, quitting at $4:00$ PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12$ PM. On Wednesday Paula worked by herself and finished the house by working until $7:12$ P.M.

How long, in minutes, was each day's lunch break?

A 30

B 36

C 42

D 48

E 60

20. A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random.

The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability the grid is now entirely black?

A $\frac{49}{512}$

B $\frac{7}{64}$

C $\frac{121}{1024}$

D $\frac{81}{512}$

E $\frac{9}{32}$

21. Let points

$$A = (0,0,0),$$
$$B = (1,0,0),$$
$$C = (0,2,0),$$
$$D = (0,0,3).$$

Points E, F, G, and H are midpoints of line segments $\overline{BD}$, $\overline{AB}$, $\overline{AC}$, and $\overline{DC}$ respectively. What is the area of $EFGH$?

A $\sqrt{2}$

B $\frac{2\sqrt{5}}{3}$

C $\frac{3\sqrt{5}}{4}$

D $\sqrt{3}$

E $\frac{2\sqrt{7}}{3}$

22. The sum of the first m positive odd integers is 212 more than the sum of the first n positive even integers. What is the sum of all possible values of n?

A 255

B 256

C 257

D 258

E 259

23. Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

A 60

B 170

C 290

D 320

E 660

24. Let a, b, and c be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011$$

and

$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc$$
$$= -1997.$$

What is a?

A 249

B 250

C 251

D 252

E 253

25. Real numbers x, y, and z are chosen independently and at random from the interval $[0, n]$ for some positive integer n. The probability that no two of x, y, and z are within 1 unit of each other is greater than $\frac{1}{2}$. What is the smallest possible value of n?

A 7

B 8

C 9

D 10

E 11

Solutions: https://live.poshenloh.com/past-contests/amc10/2012A/solutions