2013 AMC 8 Solutions

Typeset by: LIVE, by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc8/2013/solutions

1. Danica wants to arrange her model cars in rows with exactly 6 cars in each row. She now has 23 model cars. What is the smallest number of additional cars she must buy in order to be able to arrange all her cars this way?

A

B

C

D

E

Solution(s):

In order for Danica to arrange her model cars in rows of exactly six, the number of model cars she has must be a multiple of six. The smallest multiple of greater than is so she must buy more car to attain this amount.

Thus, **A** is the correct answer.

2. A sign at the fish market says,

" off, today only: half-pound packages for just per package."

What is the regular price for a full pound of fish, in dollars?

A

B

C

D

E

Solution(s):

During the sale, the price for half a pound is and therefore, the price for a full pound is

The sale price is off the regular price, and so the regular price is twice the sale price. Therefore, the regular price for a full pound of fish is

Thus, **D** is the correct answer.

3. What is the value of

A

B

C

D

E

Solution(s):

Within the parentheses, we can group each pair of numbers as follows:

Each pair of grouped terms has a sum of and as there are such pairs, we have a total value of within the parentheses.

Therefore, multiplying this value by yields

Thus, **E** is the correct answer.

4. Eight friends ate at a restaurant and agreed to share the bill equally. Because Judi forgot her money, each of her seven friends paid an extra to cover her portion of the total bill. What was the total bill?

A

B

C

D

E

Solution(s):

Since Judi's portion was fully covered by everyone's contributions, the portion of the bill she was responsible for was equal to

As everyone paid the same amount, we can conclude that the total bill is

Thus, **C** is the correct answer.

5. Hammie is in the grade and weighs pounds. His quadruplet sisters are tiny babies and weigh and pounds. Which is greater, the average (mean) weight of these five children or the median weight, and by how many pounds?

A

B

C

D

E

Solution(s):

The median weight is the middle value, which we can find as:

Therefore, the median is pounds.

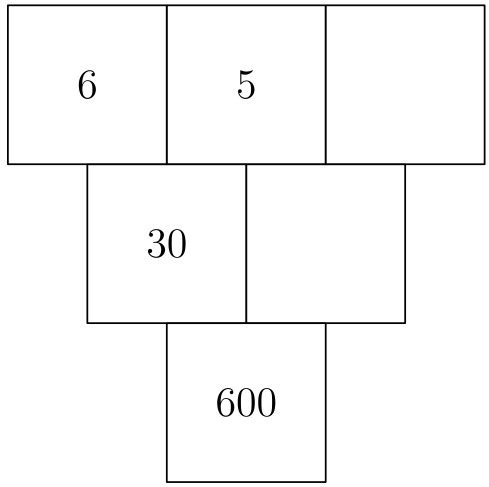
The average (mean) can be calculated to be:

This is greater than the median, specifically, the mean is pounds greater than the median.

Thus, **E** is the correct answer.

6. The number in each box below is the product of the numbers in the two boxes that touch it in the row above. For example,

What is the missing number in the top row?



A

B

C

D

E

Solution(s):

The product of the numbers in the second row is so the missing number in the middle right box is

Now we know that the product of and the missing number in the first row is Therefore, the missing number is

Thus, **C** is the correct answer.

7. Trey and his mom stopped at a railroad crossing to let a train pass. As the train began to pass, Trey counted 6 cars in the first 10 seconds. It took the train 2 minutes and 45 seconds to clear the crossing at a constant speed. Which of the following was the most likely number of cars in the train?

A

B

C

D

E

Solution(s):

Converting minutes and seconds into seconds, we get seconds. As we know that there were cars that passed in seconds, we can set up a proportion to see how many cars passed in seconds:

Cross multiplying, we get

This means that approximately cars passed.

Thus, the correct answer is **C**.

8. A fair coin is tossed 3 times. What is the probability of at least two consecutive heads?

A —

B —

C —

D —

E —

Solution(s):

There are only possible outcomes that result in at least two consecutive heads:

-
-
-

Since there are two possible outcomes for every coin flip, and there are coin flips, there are total outcomes.

Therefore, the probability of any of these outcomes is —

Thus, **C** is the correct answer.

9. The Incredible Hulk can double the distance he jumps with each succeeding jump. If his first jump is meter, the second jump is meters, the third jump is meters, and so on, then on which jump will he first be able to jump more than kilometer (meters)?

A

B

C

D

E

Solution(s):

If The Incredible Hulk's jumping distance doubles with every successive jump, on the th jump, he will be able to jump meters. As such, we want to find the smallest such that

Calculating powers of we see that:

Therefore, the first time the Hulk jumps meters is on meaning the th jump.

Thus, **C** is the correct answer.

10. What is the ratio of the least common multiple of and to the greatest common factor of and

A

B

C

D

E

Solution(s):

We can get the prime factorization for each number to find these values. We get that:

and

As such, the common prime factors are and which shows that the greatest common factor is

Similarly, taking the largest powers of the two numbers for each prime factor, we get that the least common multiple is

The ratio of these two numbers is

Thus, **C** is the correct answer.

11. Ted's grandfather used his treadmill on days this week. He went miles each day.

On Monday he jogged at a speed of miles per hour. He walked at the rate of miles per hour on Wednesday and at miles per hour on Friday.

If Grandfather had always walked at miles per hour, he would have spent less time on the treadmill. How many minutes less?

A

B

C

D

E

Solution(s):

Grandfather spent — hours jogging on Monday. He spent — hours walking on Wednesday and — — hours on Friday. Converting these times to minutes, we get

—

—

—

for Monday, Wednesday, and Friday respectively. Therefore, Grandfather totaled minutes of exercise throughout the week.

If he walked at a pace of miles per hour each day, he would have spent — — hours each day walking. This equals — minutes every day. If he did this for days, he would have totaled minutes of exercise.

Therefore, Grandfather would have walked for less minutes.

Thus, **D** is the correct answer.

12. At the 2013 Winnebago County Fair a vendor is offering a "fair special" on sandals. If you buy one pair of sandals at the regular price of you get a second pair at a discount, and a third pair at half the regular price.

Javier took advantage of the "fair special" to buy three pairs of sandals. What percentage of the regular price did he save?

A

B

C

D

E

Solution(s):

Javier saved dollars on the second pair. He also saved dollars on the third pair. This shows that he saved a total of dollars.

As

we can conclude that Javier saved compared to the regular price.

Thus, **B** is the correct answer.

13. When Clara totaled her scores, she inadvertently reversed the units digit and the tens digit of one score. By which of the following might her incorrect sum have differed from the correct one?

A

B

C

D

E

Solution(s):

If we express the last two digits of the score as then switching the digits results in

Subtracting these gives us, which means that the difference of the incorrect and correct sums must be divisible by The only answer choice that satisfies this is **A**.

Thus, **A** is the correct answer.

14. Abe holds green and red jelly bean in his hand. Bob holds green, yellow, and red jelly beans in his hand. Each randomly picks a jelly bean to show the other. What is the probability that the colors match?

A —

B —

C —

D —

E —

Solution(s):

There are two possible cases where the colors match: they both pick green jelly beans, or they both pick red jelly beans. There is only way for them to both green jelly beans, and there are ways for them to both choose red jelly beans.

There are a total of ways for Abe and Bob to randomly pick a jelly bean, and as such, the probability the colors match is —

Thus, **C** is the correct answer.

15. If we have

What is the product of and

A

B

C

D

E

Solution(s):

Simplifying the three equations, we get

By inspection, we get that and Their product is

Thus, **B** is the correct answer.

16. A number of students from Fibonacci Middle School are taking part in a community service project. The ratio of -graders to -graders is and the ratio of -graders to -graders is What is the smallest number of students that could be participating in the project?

A

B

C

D

E

Solution(s):

The number of -graders must be a multiple of and which means that it is at least Using this number, we get that the number of -graders is —

Similarly, the number of -graders is —

The total number of students is therefore

Thus, **E** is the correct answer.

17. The sum of six consecutive positive integers is What is the largest of these six integers?

A

B

C

D

E

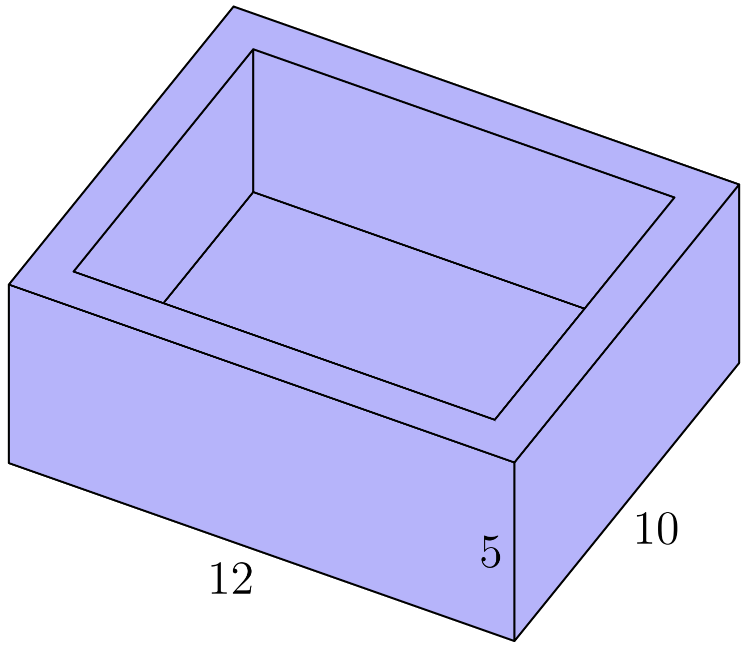
Solution(s):

Let's define the smallest of these integers to be and as such, we want to find We know that:

As such, which implies that and therefore, we have

Thus, **B** is the correct answer.

18. Isabella uses one-foot cubical blocks to build a rectangular fort that is feet long, feet wide, and feet high. The floor and the four walls are all one foot thick. How many blocks does the fort contain?



A

B

C

D

E

Solution(s):

The volume of the fort is equal to the volume enclosed by the outside faces minus the volume inside the walls.

The volume enclosed by the outside faces is cubic feet.

The dimensions of the inside's walls are decreased by and the floor by which makes it This gives the inner section a volume of cubic feet.

Therefore, the volume of the fort is cubic feet.

Thus, **B** is the correct answer.

19. Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, "I didn't get the lowest score in our class," and Bridget adds, "I didn't get the highest score." What is the ranking of the three girls from highest to lowest?

- A Hannah, Cassie, Bridget
- B Hannah, Bridget, Cassie
- C Cassie, Bridget, Hannah
- D Cassie, Hannah, Bridget
- E Bridget, Cassie, Hannah

Solution(s):

Since Cassie deduced she didn't get the lowest score, she must have got a higher score than Hannah. Similarly, Bridget's statement reveals that she must have got a lower score than Hannah.

Therefore, the ranking from highest to lowest is Cassie, Hannah, and then Bridget.

Thus, **D** is the correct answer.

20. A rectangle is inscribed in a semicircle with the longer side on the diameter. What is the area of the semicircle?

A

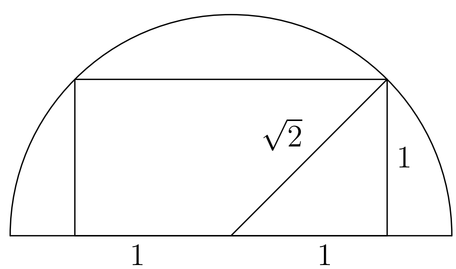
B

C

D

E

Solution(s):



Using the Pythagorean theorem, we get that the radius of the semicircle is
This means that the area of the semicircle is

Thus, **C** is the correct answer.

21. Samantha lives blocks west and block south of the southwest corner of City Park.

Her school is blocks east and blocks north of the northeast corner of City Park. On school days she bikes on streets to the southwest corner of City Park, then takes a diagonal path through the park to the northeast corner, and then bikes on streets to school.

If her route is as short as possible, how many different routes can she take?

A

B

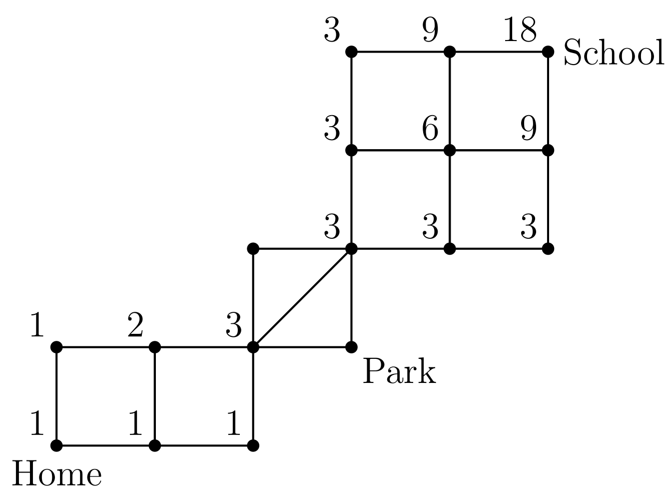
C

D

E

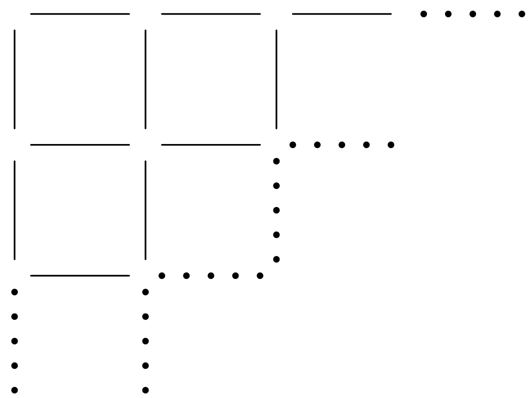
Solution(s):

We can count the number of ways at each intermediate point using the fact that we can only go North and East. The following diagram illustrates this.



Thus, **E** is the correct answer.

22. Toothpicks are used to make a grid that is toothpicks long and toothpicks wide. How many toothpicks are used altogether?



A

B

C

D

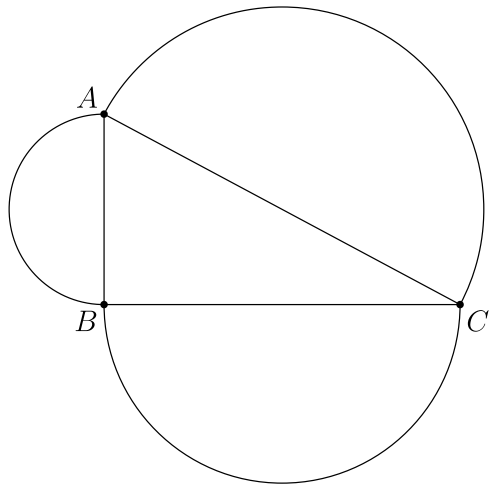
E

Solution(s):

A grid formatted like this has columns of toothpicks and rows of toothpicks. This means that there are toothpicks in total.

Thus, **E** is the correct answer.

23. Angle of is a right angle. The sides of are the diameters of semicircles as shown. The area of the semicircle on equals and the arc of the semicircle on has length What is the radius of the semicircle on



A

B

C

D

E

Solution(s):

We know that if have the area of a semicircle, we can multiply that by to find the area of its completed circle, which we then that to calculate the radius of the semicircle.

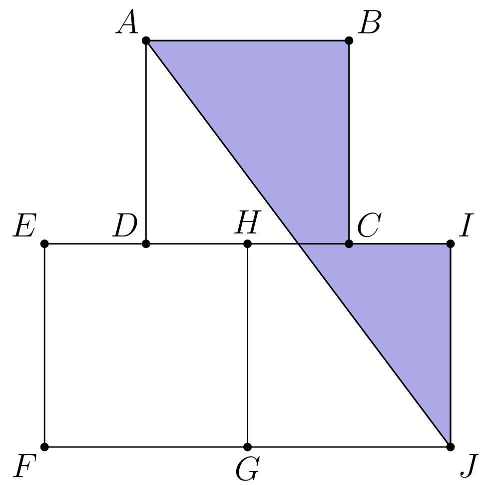
Considering the semicircle on we get that the area of the circle is which shows that its radius would be This means that Similarly, we calculate

We now have two sides of the right triangle. We can use the Pythagorean theorem to calculate

Therefore, the desired radius is

Thus, **B** is the correct answer.

24. Squares and are equal in area. Points and are the midpoints of sides and respectively. What is the ratio of the area of the shaded pentagon to the sum of the areas of the three squares?



A —

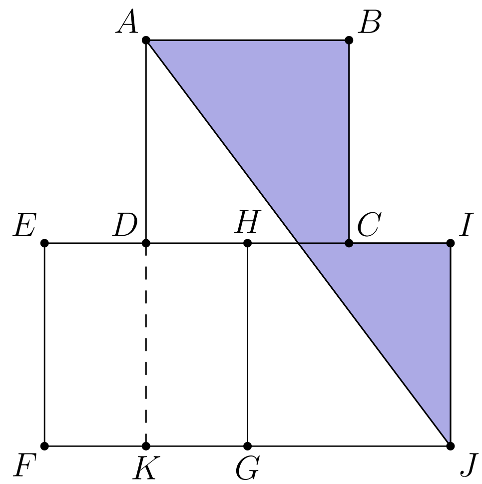
B —

C —

D —

E —

Solution(s):



As we are seeking to find a ratio between areas, we can arbitrarily define the side lengths of each of the squares to be This makes the sum of the areas of the squares equal to

To find the area of we can find the area of and subtract it from the total area. We can split into and

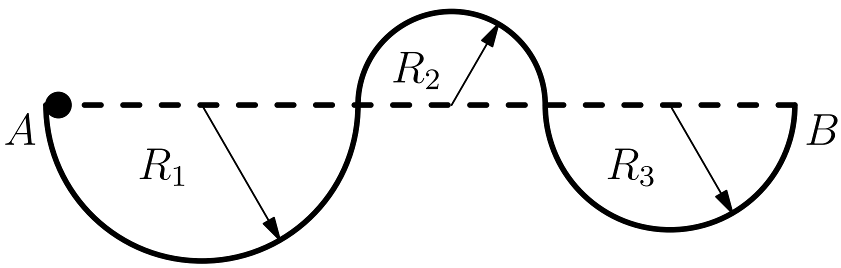
The area of is — — The area of is — — — The total area is the — — Therefore, the area of is

The ratio is then —

Thus, **C** is the correct answer.

25. A ball with diameter inches starts at point to roll along the track shown. The track is comprised of semicircular arcs whose radii are inches, inches, and inches, respectively. The ball always remains in contact with the track and does not slip.

What is the distance the center of the ball travels over the course from to



A

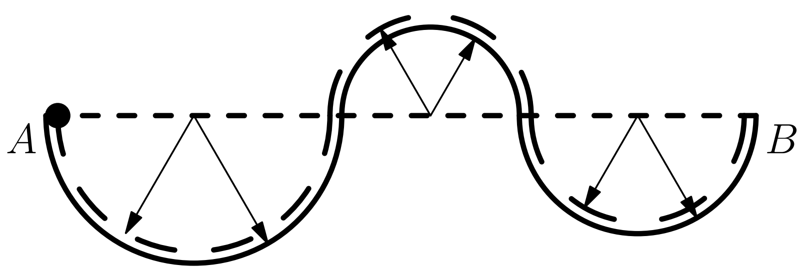
B

C

D

E

Solution(s):



Since the diameter of the ball is its radius is Looking at the diagram above, we see the semicircles that the center of the ball travels through. The first and third semicircles are smaller than the solid semicircles, but the second is larger.

Since the radius of the semicircle is the dashed semicircles have radii that are off from the solid semicircles. From this, we get that the radii of the dashed semicircles are and That means the total distance the center traveled is

Thus, **A** is the correct answer.

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