2012 AMC 8 Solutions

Typeset by: LIVE, by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc8/2012/solutions

1. Rachelle uses 3 pounds of meat to make 8 hamburgers for her family. How many pounds of meat does she need to make 24 hamburgers for a neighbourhood picnic?

A 6

B $6\frac{2}{3}$

C $7\frac{1}{2}$

D 8

E 9

Solution:

If we have 8 hamburgers, we have $\frac{1}{3}$ of the 24 hamburgers. This means we have $\frac{1}{3}$ of the meat when we have 3 pounds. The total amount of meat is therefore $\frac{1}{\frac{1}{3}} \cdot 3 = 3 \cdot 3 = 9$.

Thus, the answer is **E**.

2. In the country of East Westmore, statisticians estimate there is a baby born every 8 hours and a death every day. To the nearest hundred, how many people are added to the population of East Westmore each year?

A 600

B 700

C 800

D 900

E 1000

Solution:

Since we have 1 birth every 8 hours, we have 3 births every 24 hours. Therefore, we have 3 births a day and 1 death a day. The ner change in population every day should be on average 2. Since we have 365 days in a year and 2 added to the population every year, the net change in population should be around $2 \cdot 365 = 730$. This is approximately 700.

Thus, the answer is **B**.

3. On February 13 *The Oshkosh Northwester* listed the length of daylight as 10 hours and 24 minutes, the sunrise was $6:57\text{AM}$, and the sunset as $8:15\text{PM}$. The length of daylight and sunrise were correct, but the sunset was wrong. When did the sun really set?

A $5{:}10\ \text{PM}$

B $5{:}21\ \text{PM}$

C $5{:}41\ \text{PM}$

D $5{:}57\ \text{PM}$

E $6{:}03\ \text{PM}$

Solution:

Since 10 hours after $6:57\text{AM}$ is $4:57\text{PM}$, we can then say 10 hours and 3 minutes after sunrise is $5:00\text{PM}$. We then have 21 more minutes until sunset, so sunset is $5:21\text{PM}$.

Thus, the answer is **B**.

4. Peter's family ordered a 12-slice pizza for dinner. Peter ate one slice and shared another slice equally with his brother Paul. What fraction of the pizza did Peter eat?

A $\frac{1}{24}$

B $\frac{1}{12}$

C $\frac{1}{8}$

D $\frac{1}{6}$

E $\frac{1}{4}$

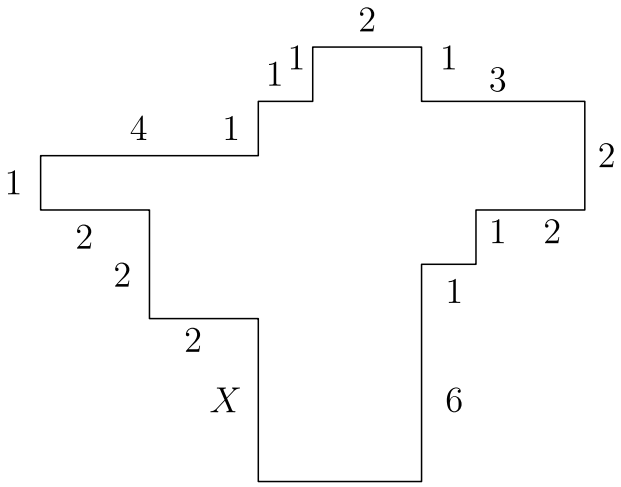
Solution:

Peter ate 1 full slice, and he ate $\frac{1}{2}$ of the slice that he split.

Therefore, he ate $\frac{1.5}{12}$ of the pizza, which is equivalent to $\frac{1}{8}$.

Thus, the answer is **C**.

5. In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is , X in centimeters?



A 1

B 2

C 3

D 4

E 5

Solution:

First, we can find the height of the object by getting the sum of the heights on the right. Therefore, the height is $1 + 2 + 1 + 6 = 10$.

Next, we can find the height of the object by getting the sum of the heights on the left. Therefore, the height is $1 + 1 + 1 + 2 + X = 5 + X$.

Since the heights are the same, we know $10 = 5 + X$, so $X = 5$.

Thus, the answer is **E**.

6. A rectangular photograph is placed in a frame that forms a border two inches wide on all sides of the photograph. The photograph measures 8 inches high and 10 inches wide. What is the area of the border, in square inches?

A 36

B 40

C 64

D 72

E 88

Solution:

If we add 2 inches on each side, we add 4 inches total on both sides. This means that the dimensions of outer part of the frame is 12×14. The area of this is $12 \cdot 14 = 168$.

However, we must take out the area of the inner part of the frame which has area $8 \cdot 10 = 80$.

Therefore, the total area is $168 - 80 = 88$.

Thus, the answer is **E**.

7. Isabella must take four 100-point tests in her math class. Her goal is to achieve an average grade of 95 on the tests. Her first two test scores were 97 and 91. After seeing her score on the third test, she realized she can still reach her goal. What is the lowest possible score she could have made on the third test?

A 90

B 92

C 95

D 96

E 97

Solution:

If the average is 95, then the sum of the tests are $95 \cdot 4 = 380$. Since we have the first two tests, the sum of the last two tests is 380 minus the first two scores.

This makes the sum of the last two scores equal to $380 - 91 - 97 = 192$. Her last two scores therefore hace a sum of 192.

Given the sum of the tests we try to minimize one score, then we must maximize the other test. Therefore, we maximize the fourth test by making it 100. This would make the third test equal to $192 - 100 = 92$.

Thus, the answer is **B**.

8. A shop advertises everything is "half price in today's sale." In addition, a coupon gives a 20% discount on sale prices. Using the coupon, the price today represents what percentage off the original price?

A 10

B 33

C 40

D 60

E 70

Solution:

Let p be the original price. If everything is half off, we have the new price as $0.5p$.

Having a 20% discount makes it such that we keep 80% of the price, so the price is $0.5p \cdot 0.8 = 0.4p$.

This would have $0.6p$ off, so we get a discount of $\frac{0.6p}{p} = 0.6$, which is 60%.

Thus, the answer is **D**.

9. The Fort Worth Zoo has a number of two-legged birds and a number of four-legged mammals. On one visit to the zoo, Margie counted 200 heads and 522 legs. How many of the animals that Margie counted were two-legged birds?

A 61

B 122

C 139

D 150

E 161

Solution:

Let f be the number of animals with 4 legs and let t be the number of animals with 2 legs.

Counting the number of legs yields $2t + 4f = 522$ and counting the number of heads yields $t + f = 200$. This means $4t + 4f = 800$, and subtracting the first equation from the second yields $2t = 278 \implies t = 139$. This means therer are 139 two-legged birds.

Thus, the answer is **C**.

10. How many 4-digit numbers greater than 1000 are there that use the four digits of 2012?

A 6

B 7

C 8

D 9

E 12

Solution:

First, we can't have the 0 in the thousands position. Therefore, we have 3 spots we can put it. Then, we have 3 avaiable positions for the 1, and then the two 2s are placed. This makes it such that we have $3 \cdot 3 = 9$ combinations.

Thus, the answer is **D**.

11. The mean, median, and unique mode of the positive integers $3, 4, 5, 6, 6, 7$, and x are all equal. What is the value of x?

A 5

B 6

C 7

D 11

E 12

Solution:

Every value except 6 shown has appears once while 6 appears twice. If any of the other values are chosen, then we have two modes, which means we don't have unique modes. Otherwise, the only value that shows up more than once is 6 making that the unique mode. This also means 6 is the mean. Since there are 7 elements, the sum of the elements is $6 \cdot 7 = 42$. The sum is also

$$\begin{aligned} &3 + 4 + 5 + 6 + 6 + 7 + x \\ &= 31 + x \\ &= 42, \end{aligned}$$

so $x = 11$.

Thus, the answer is **D**.

12. What is the units digit of 13^{2012}?

A 1

B 3

C 5

D 7

E 9

Solution:

We have to find $13^{2012} \mod 10$. The following is true:

$$\begin{aligned} 13^{2012} &\equiv 3^{2012} \mod 10 \\ &\equiv 81^{503} \mod 10 \\ &\equiv 1^{503} \mod 10 \\ &\equiv 1 \mod 10 \end{aligned}$$

This means 13^{2012} has the same units digit as 1, so the units digit of 13^{2012} is 1.

Thus, the answer is **A**.

13. Jamar bought some pencils costing more than a penny each at the school bookstore and paid $\$1.43$. Sharona bought some of the same pencils and paid $\$1.87$. How many more pencils did Sharona buy than Jamar?

A 2

B 3

C 4

D 5

E 6

Solution:

Let c be the price of pencils in cents. Then, c is divisible by 187 and 143. This means c is divisible by $gcd(143, 187) = 11$. Since the cost is more than a penny and the only divisors of 11 are 1 and 11, the cost must be 11 cents. Since Sharona paid 44 more cents than Jamal and pencils are 11 cents, she buys $\frac{44}{11} = 4$ more pencils.

Thus, the answer is **C**.

14. In the BIG N, a middle school football conference, each team plays every other team exactly once. If a total of 21 conference games were played during the 2012 season, how many teams were members of the BIG N conference?

A 6

B 7

C 8

D 9

E 10

Solution:

Each of the N teams play $N-1$ games. However, 2 teams play each game, so multiplying N and $N-1$ would be twice the number of games. Therefore, we know $N(N-1)=42$. This leads to

$$N^2-N+0.25=42.25$$

Which implies

$$(N-0.5)^2=6.5^2.$$

In turn, this suggests:

$$N-0.5=6.5$$

$$N=7$$

Thus, the answer is **B**.

15. The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?

A 40 and 50

B 51 and 55

C 56 and 60

D 61 and 65

E 66 and 99

Solution:

Let the number be x. Since it leaves a remainder of 2 when divided by $3, 4, 5, 6$, we know $x-2$ is a multiple of $3, 4, 5,$ and 6. This means $x-2$ is a multiple of $lcm(3,4,5,6)$ which is 60. Therefore, $x-2$ must be a multiple of 60. The next number such that this occurs is when $x-2=60 \implies x=62$.

Thus, the answer is **D**.

16. Each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 is used only once to make two five-digit numbers so that they have the largest possible sum. Which of the following could be one of the numbers?

A 76531

B 86724

C 87431

D 96240

E 97403

Solution:

To construct two five digit numbers, first digits of the numbers from the left must be as great as possible. Therefore, the leftmost unused number must be either of the greatest two numbers. This means the first digit must be either of $9, 8$, the second digit must be either of $7, 6$, the third digit must be either of $5, 4$, the fourth digit must ber either of $3, 2$ and the last digit must be either of $1, 0$.

The only of the given numbers that satisfy this is 87431.

Thus, the answer is **C**.

17. A square with an integer side length is cut into 10 squares, all of which have integer side length and at least 8 of which have area 1. What is the smallest possible value of the length of the side of the original square?

A 3

B 4

C 5

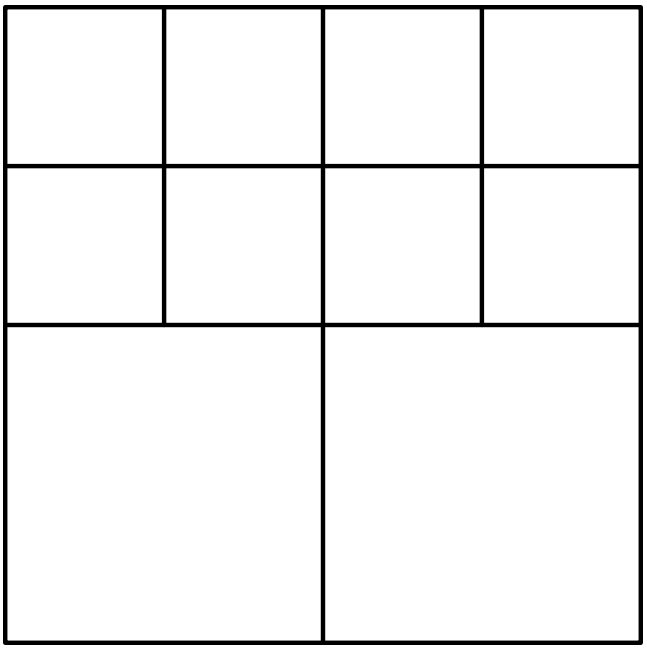
D 6

E 7

Solution:

Since all the 10 squares have integer side length, they each must have a side length greater than or equal ro 1. This means the total area must be over $10 \cdot 1 = 10$. Therefore, the square can't have a side length less than or equal to 3 or else it would have an area less than 9.

We can make a configuration with side length 4 however with the following configuration.



Thus, the answer is **B**.

18. What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50?

A 3127

B 3133

C 3137

D 3139

E 3149

Solution:

Since our number isn't prime, it must be the product of at least 2 numbers that aren't 1. Moreover, these factors must not have prime factors less than 50, so our number must be the product of primes greater than 50. Also, our number is not a square, so it must be the product of distint primes.

This means our number is the product of distinct primes greater than 50. The smallest primes greater than 50 are 53 and 59, so our number is $53 \cdot 59 = 3127$.

Thus, our answer is **A**.

19. In a jar of red, green, and blue marbles, all but 6 are red marbles, all but 8 are green, and all but 4 are blue. How many marbles are in the jar?

A 6

B 8

C 9

D 10

E 12

Solution:

Let r, g, b be the number of red marbles, green marbles, and blue marbles respectively. We then know

$$r+g+b-r=6,$$

$$r+g+b-g=8,$$

$$r+g+b-b=4$$

by the statements given. Adding these equations yields $3(r+g+b)-(r+g+b)=18$. This would mean $2(r+g+b)=18$, so $r+g+b=9$. Therefore, the sum of all of the marbles is 9.

Therefore, the sum is **C**.

20. What is the correct ordering of the three numbers $\frac{5}{19}$, $\frac{7}{21}$, and $\frac{9}{23}$, in increasing order?

A $\frac{9}{23} < \frac{7}{21} < \frac{5}{19}$

B $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$

C $\frac{9}{23} < \frac{5}{19} < \frac{7}{21}$

D $\frac{5}{19} < \frac{9}{23} < \frac{7}{21}$

E $\frac{7}{21} < \frac{5}{19} < \frac{9}{23}$

Solution:

We can start with $19 < 21 < 23$.

Then, $\frac{1}{19} > \frac{1}{21} > \frac{1}{23}$ since we take the recipricol of positive numbers.

Then, multiplying by the negative constant -14 yields $-\frac{14}{19} < -\frac{14}{21} < -\frac{14}{23}$ since that switches the direction of the inequalities.

Adding one to each of them then yields $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$.

Thus, the answer is **B**.

21. Marla has a large white cube that has an edge of 10 feet. She also has enough green paint to cover 300 square feet. Jenica uses all the paint to create a white square centered on each face, surrounded by a green border. What is the area of one of the white squares, in square feet?

A $5\sqrt{2}$

B 10

C $10\sqrt{2}$

D 50

E $50\sqrt{2}$

Solution:

The total surface area is $6 \cdot 10 \cdot 10 = 600$. Therefore, $600 - 300 = 300$ square feet aren't covered. This would be $\frac{300}{6} = 50$ square feet per face.

Thus, the answer is **D**.

22. Let R be a set of nine distinct integers. Six of the elements are $2, 3, 4, 6, 9$, and 14. What is the number of possible values of the median of R?

A 4

B 5

C 6

D 7

E 8

Solution:

A number is the median if there are 4 numbers in the set that are greater than it and 4 numbers that are less than or equal to it. Note that since we have distinct integers, the condition is now that the median is the number with 4 numbers greater than it and 4 numbers less than it.

If we have a number greater than 9 then it can't be the median as there is at least 5 numbers less than it.

If we have a number less than 3 then it can't be the median as there is at least 5 numbers greater than it.

Every number from 3 to 9 can be either placed or have numbers placed around it such that there are 4 numbers greater than it and 4 numbers less than it. Therefore, they can all be medians. This makes us have 7 medians.

Thus, the answer is **D**.

23. An equilateral triangle and a regular hexagon have equal perimeters. If the triangle's area is 4, what is the area of the hexagon?

A 4

B 5

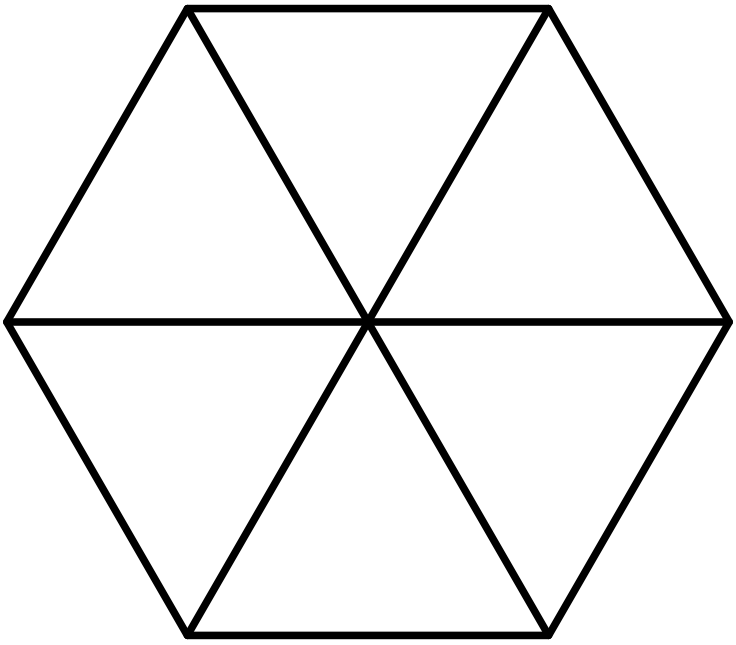
C 6

D $4\sqrt{3}$

E $6\sqrt{3}$

Solution:

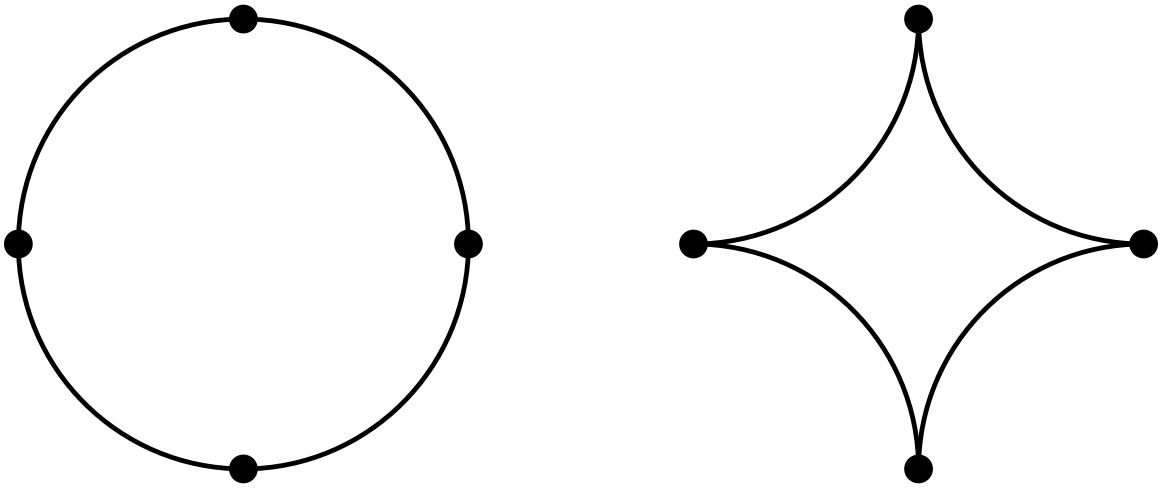
Let the side length of the triangle be s. This means the perimeter is $3s$. Therefore, the side length for the hexagon is $\frac{3s}{6} = \frac{s}{2}$.



A hexagon can be made of 6 equilateral triangles with side length $\frac{s}{2}$ as shown above. Each triangle is the original triangle scaled down by $\frac{1}{2}$, so the area is scaled down by $(\frac{1}{2})^2 = \frac{1}{4}$. Therefore, the area of each of these triangles is $4 \cdot \frac{1}{4} = 1$. Since there are 6 of them, the area is $6 \cdot 1 = 6$.

Thus, the answer is **C**.

24. A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?



A $\frac{4-\pi}{\pi}$

B $\frac{1}{\pi}$

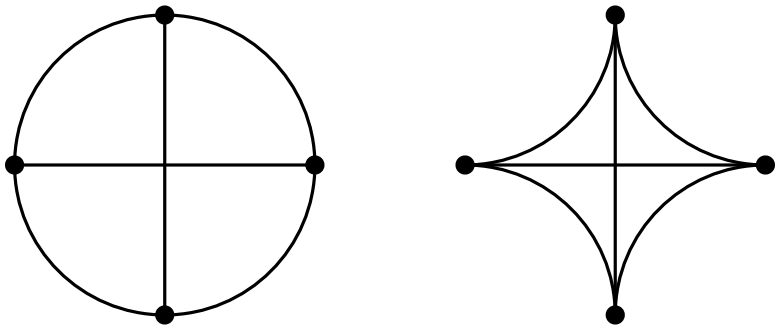
C $\frac{\sqrt{2}}{\pi}$

D $\frac{\pi-1}{\pi}$

E $\frac{3}{\pi}$

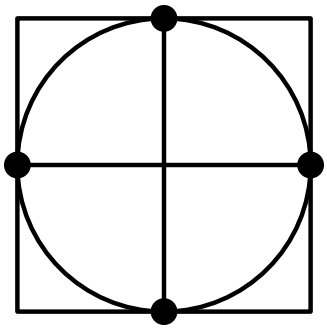
Solution:

The total area of the circle is $\pi \cdot (2)^2 = 4\pi$.



Now, to find the area of the star, we can find the total area of both shapes combined. To do this, we split the as shown above. Then, we rearrange the

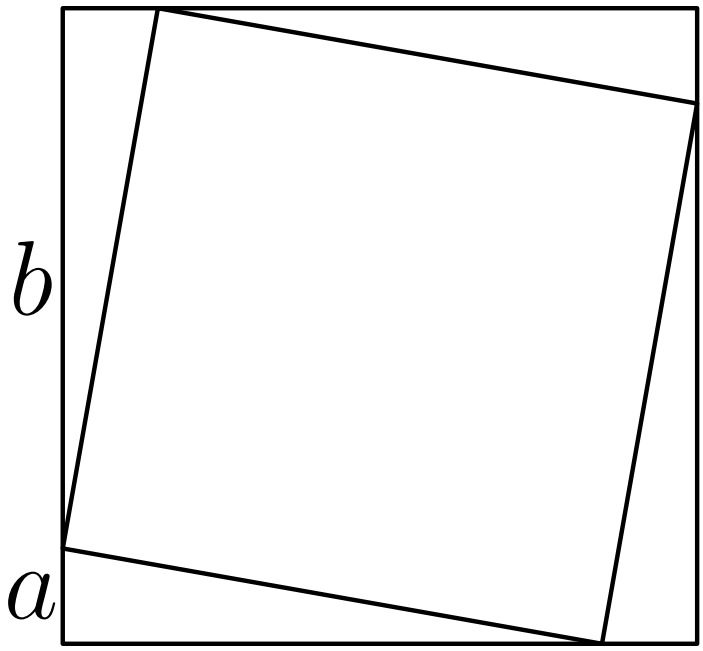
partitions as done below.



This makes a square of side length 4, so its area is $4 \cdot 4 = 16$. Then, we take out the area of the circle, so the area of the star is $16 - 4\pi$. This makes the ratio equal to $\frac{16 - 4\pi}{4\pi} = \frac{4 - \pi}{\pi}$.

Thus the answer is **A**.

25. A square with area 4 is inscribed in a square with area 5, with each vertex of the smaller square on a side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length a, and the other of length b. What is the value of ab?



A $\frac{1}{5}$

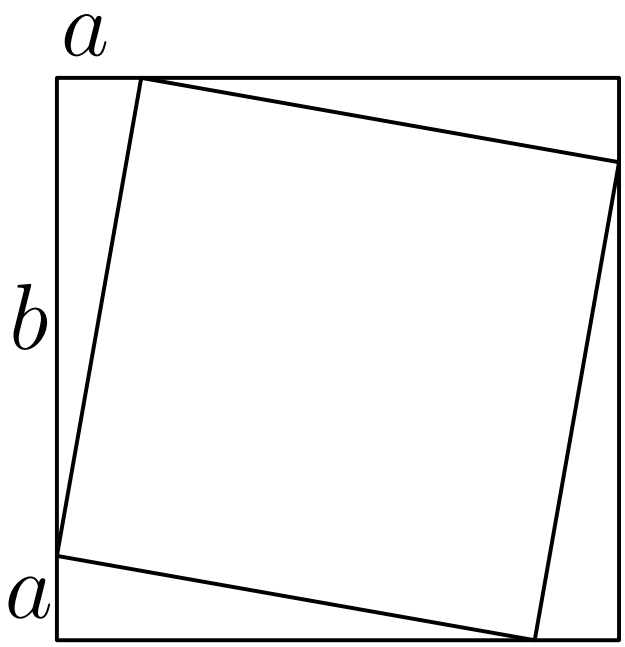
B $\frac{2}{5}$

C $\frac{1}{2}$

D 1

E 4

Solution:



Since all the triangles can be made from each other by rotating them around, they are all congruent. Therefore, we can place the a as we have. The total area of the triangles is $5-4=1$, so we have 4 congruent triangles with a combined area of 1. This means the area of each triangle is $\frac{1}{4}$. The area of each triangle is also $\frac{ab}{2}$, so $\frac{ab}{2}=\frac{1}{4}$. This means $ab=\frac{1}{2}$.

Thus, the correct answer is **C**.

Problems: https://live.poshenloh.com/past-contests/amc8/2012