

2011 AMC 8 Solutions



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1. Margie bought 3 apples at a cost of 50 cents per apple. She paid with a 5-dollar bill. How much change did Margie receive?

A \$1.50

B \$2.00

C \$2.50

D \$3.00

E \$3.50

Solution(s):

The apples costed a total of $3 \cdot 50 = 150$ cents, which equals 1.50\$. This means that Margie received $5 - 1.5 = 3.5$ dollars in change.

Thus, **E** is the correct answer.



2. Karl's rectangular vegetable garden is 20 feet by 45 feet, and Makenna's is 25 feet by 40 feet. Whose garden is larger in area?

- A Karl's garden is larger by 100 square feet.
- B Karl's garden is larger by 25 square feet.
- C The gardens are the same size.
- D Makenna's garden is larger by 25 square feet.
- E Makenna's garden is larger by 100 square feet.

Solution(s):

The area of Karl's garden is

$$20\text{ft} \cdot 45\text{ft} = 900\text{ft}^2.$$

The area of Makenna's garden is

$$25\text{ft} \cdot 40\text{ft} = 1000\text{ft}^2.$$

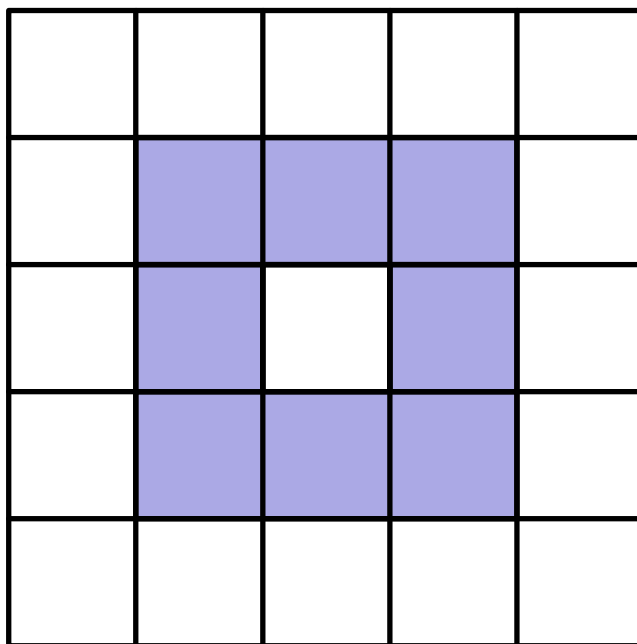
The difference of these area is

$$1000\text{ft}^2 - 900\text{ft}^2 = 100\text{ft}^2.$$

Therefore, Makenna's garden is 100ft^2 larger than Karl's.

Thus, **E** is the correct answer.

3. Extend the square pattern of 8 colored and 17 uncolored square tiles by attaching a border of colored tiles around the square. What is the ratio of colored tiles to uncolored tiles in the extended pattern?



A 8 : 17

B 25 : 49

C 36 : 25

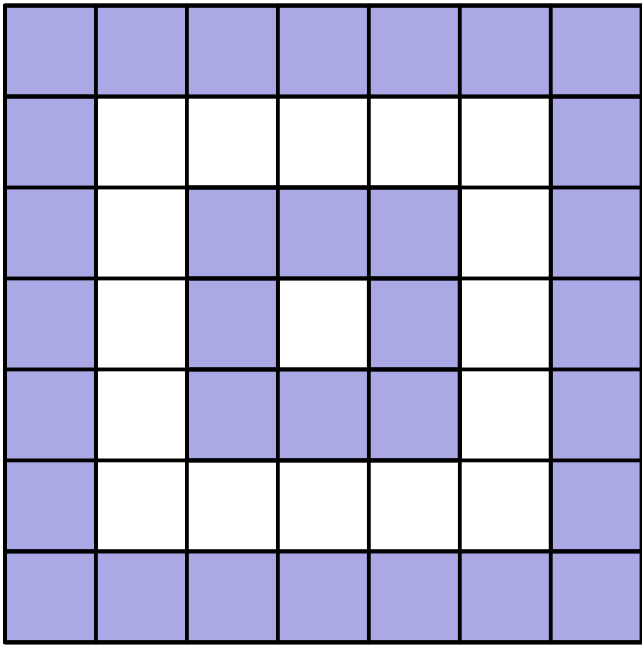
D 32 : 17

E 36 : 17

Solution(s):

In the extended figure, there are 32 colored tiles and 17 uncolored tiles. Therefore, the ratio of colored tiles to uncolored tiles is 32 : 17.

Thus, **D** is the correct answer.



4. Here is a list of the numbers of fish that Tyler caught in nine outings last summer:

2, 0, 1, 3, 0, 3, 3, 1, 2.

Which statement about the mean, median, and mode is true?

A median $<$ mean $<$ mode

B mean $<$ mode $<$ median

C mean $<$ median $<$ mode

D median $<$ mode $<$ mean

E mode $<$ median $<$ mean

Solution(s):

To find these values more easily, we can get the following ordered list:

0, 0, 1, 1, 2, 2, 3, 3, 3.

From this, we see that the mode is 3, the median is 2, and the mean is $15/9 = 5/3$.

Since $\frac{5}{3} < 2 < 3$, we get that:

mean $<$ median $<$ mode.

Thus, **C** is the correct answer.

5. What time was it 2011 minutes after midnight on January 1, 2011?

A January 1 at 9 : 31 PM

B January 1 at 11 : 51 PM

C January 2 at 3 : 11 AM

D January 2 at 9 : 31 AM

E January 2 at 6 : 01 PM

Solution(s):

The remainder when 2011 is divided by 60 is 31. This means that $2011 = 60 \cdot 33 + 31$, which means that 2011 minutes is the same as 33 hours and 31 minutes.

24 hours takes us to January 2, so we get that we are 9 hours and 31 minutes into January 2.

Thus, **D** is the correct answer.

6. In a town of 351 adults, every adult owns a car, motorcycle, or both. If 331 adults own cars and 45 adults own motorcycles, how many of the car owners do not own a motorcycle?

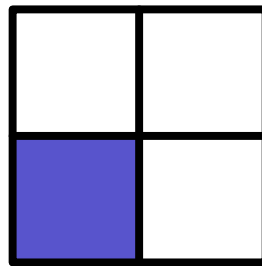
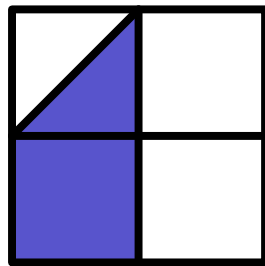
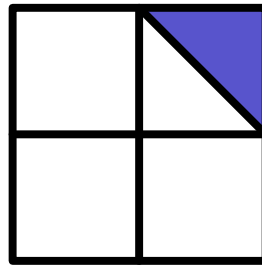
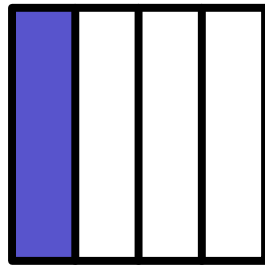
- A 20
- B 25
- C 45
- D 306
- E 351

Solution(s):

We know that 45 people own motorcycles, so $351 - 45 = 306$ people do not own motorcycles.

Thus, **D** is the correct answer.

7. Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially shaded. What percent of the total area is partially shaded?



A $12\frac{1}{2}$

B 20

C 25

D $33\frac{1}{3}$

E $37\frac{1}{2}$

Solution(s):

The top left and the bottom right shaded regions are both a quarter of each square. The top right is one-eighth, and the bottom left is three-eighths. Their combined area is

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{3}{8} = 1.$$

Therefore, the shaded regions combined equal the area of one square, so they are 25% of the total area.

Thus, **C** is the correct answer.

8. Bag A has three chips labeled 1, 3, and 5. Bag B has three chips labeled 2, 4, and 6. If one chip is drawn from each bag, how many different values are possible for the sum of the two numbers on the chips?

- A 4
- B 5
- C 6
- D 7
- E 9

Solution(s):

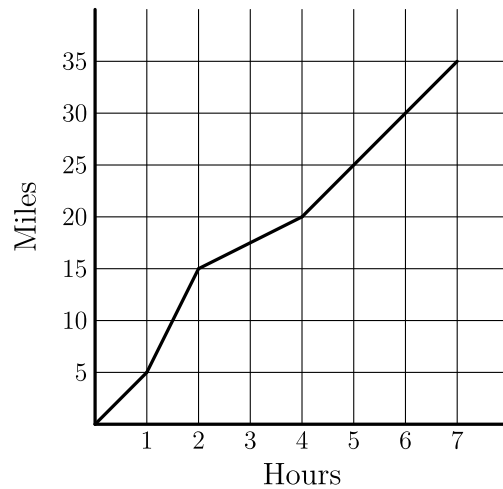
We can create a table to look at all the possible outcomes and their respective sums.

+	1	3	5
2	3	5	7
4	5	7	9
6	7	9	11

From this, we can see that there are 5 distinct values that we can get.

Thus, **B** is the correct answer.

9. Carmen takes a long bike ride on a hilly highway. The graph indicates the miles traveled during the time of her ride. What is Carmen's average speed for her entire ride in miles per hour?



- A 2
- B 2.5
- C 4
- D 4.5
- E 5

Solution(s):

Carmen travels 35 miles in 7 hours, so her average speed is $35/7 = 5$ miles per hour.

Thus, **E** is the correct answer.

10. The taxi fare in Gotham City is \$2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate \$0.20 for each additional 0.1 mile. You plan to give the driver a \$2 tip. How many miles can you ride for \$10?

A 3.0

B 3.25

C 3.3

D 3.5

E 3.75

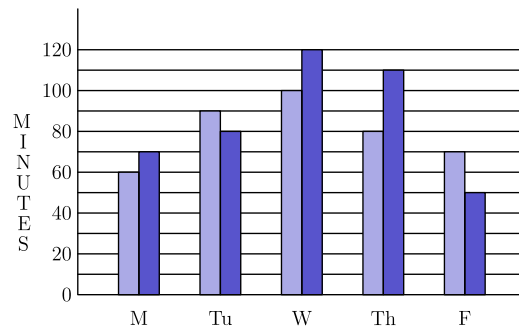
Solution(s):

There is a guaranteed \$2 tip, so we can subtract that from the total, leaving \$8. This is greater than \$2.40, so we can subtract that and add $\frac{1}{2}$ miles to the total distance.

We now have \$5.60 to use for additional miles. \$0.20 per 0.1 mile is the same as \$2 for 1 mile. That means one can ride for $5.60/2 = 2.8$ more miles with this much money. This leaves a total of $2.8 + \frac{1}{2} = 3.3$ miles.

Thus, **C** is the correct answer.

11. The graph shows the number of minutes studied by both Asha (black bar) and Sasha (grey bar) in one week. On the average, how many more minutes per day did Sasha study than Asha?



A 6

B 8

C 9

D 10

E 12

Solution(s):

We can calculate the difference in average minutes by looking at the differences per day.

Starting with Monday, the differences between Sasha and Asha are 10, -10 , 20, 30, and -20 . This is a total of 30 minutes. Therefore, the average difference is $30 \div 5 = 6$.

Thus, **A** is the correct answer.

12. Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the probability that Angie and Carlos are seated opposite each other?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

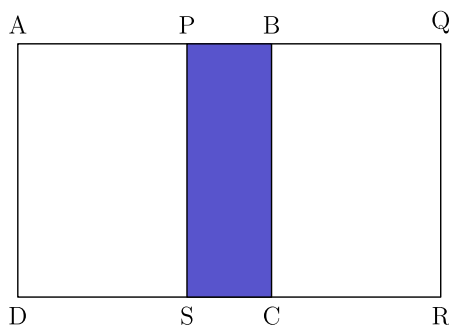
E $\frac{3}{4}$

Solution(s):

Consider that Angie's seat is chosen. Carlos has an equal probability of being in any of the other 3 seats. Only one of them is opposite Angie, however. Therefore, the probability is $\frac{1}{3}$.

Thus, **B** is the correct answer.

13. Two congruent squares, $ABCD$ and $PQRS$, have side length 15. They overlap to form the 15 by 25 rectangle $AQRD$ shown. What percent of the area of rectangle $AQRD$ is shaded?



- A 15
- B 18
- C 20
- D 24
- E 25

Solution(s):

We get that

$$\begin{aligned} SC &= DC + SR - DR \\ &= 15 + 15 - 25 \\ &= 5. \end{aligned}$$

This means that the area of $PBSC$ is $5 \cdot 15 = 75$. The area of $AQRD$ is $25 \cdot 15 = 375$.

$$\frac{75}{375} = \frac{1}{5}, \text{ which is } 20\%.$$

Thus, **C** is the correct answer.

14. There are 270 students at Colfax Middle School, where the ratio of boys to girls is 5 : 4. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is 4 : 5. The two schools hold a dance and all students from both schools attend. What fraction of the students at the dance are girls?

A $\frac{7}{18}$

B $\frac{7}{15}$

C $\frac{22}{45}$

D $\frac{1}{2}$

E $\frac{23}{45}$

Solution(s):

The total number of girls is

$$\frac{4}{9} \cdot 270 + \frac{5}{9} \cdot 180 =$$

$$120 + 100 = 220.$$

There are $270 + 180 = 450$ students total, so the fraction of girls is $\frac{220}{450} = \frac{22}{45}$.

Thus, **C** is the correct answer.

15. How many digits are in the product $4^5 \cdot 5^{10}$?

A 8

B 9

C 10

D 11

E 12

Solution(s):

To find the number of digits, we can try to express this number in terms of powers of 10.

We get that

$$\begin{aligned}4^5 \cdot 5^{10} &= 2^{10} \cdot 5^{10} \\ &= 10^{10}.\end{aligned}$$

This shows that the desired number is 1 followed by 10 zeros, for a total of 11 digits.

Thus, **D** is the correct answer.

16. Let A be the area of the triangle with sides of length 25, 25, and 30. Let B be the area of the triangle with sides of length 25, 25, and 40. What is the relationship between A and B ?

A $A = \frac{9}{16}B$

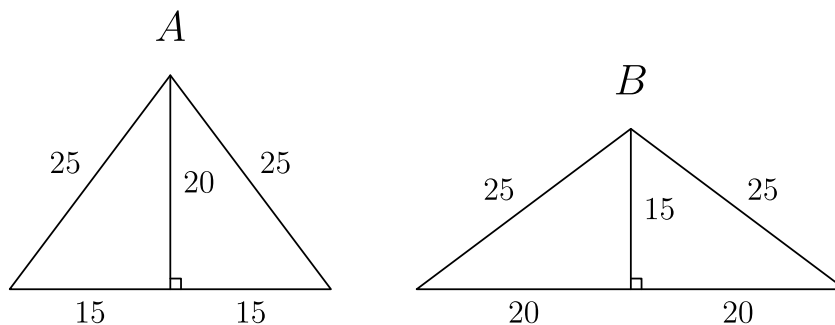
B $A = \frac{3}{4}B$

C $A = B$

D $A = \frac{4}{3}B$

E $A = \frac{16}{9}B$

Solution(s):



Since these triangles are isosceles, we can drop altitudes to create two congruent right triangles as shown in the diagram.

Using the Pythagorean theorem, we get that altitude of the triangle with area A equals

$$\sqrt{25^2 - 15^2} = 20.$$

Similarly, we get that the altitude of the triangle with area B equals

$$\sqrt{25^2 - 20^2} = 15.$$

With these altitudes, we can calculate the areas of the triangles. We get that

$$A = \frac{1}{2} \cdot 20 \cdot 30 = 300.$$

Similarly,

$$B = \frac{1}{2} \cdot 15 \cdot 40 = 300.$$

Therefore, $A = B$.

Thus, **C** is the correct answer.

17. Let w , x , y , and z be whole numbers. If

$$2^w \cdot 3^x \cdot 5^y \cdot 7^z = 588,$$

then what does

$$2w + 3x + 5y + 7z$$

equal?

A 21

B 25

C 27

D 35

E 56

Solution(s):

To find the desired exponents, note that all the bases are prime numbers. This means that finding the prime factorization will be helpful.

We get that

$$588 = 2^2 \cdot 3^1 \cdot 7^2.$$

From this, it is clear that $w = 2$, $x = 1$, $y = 0$, and $z = 2$ ($y = 0$ since that makes the 5^y term equal 1).

Therefore,

$$\begin{aligned} & 2w + 3x + 5y + 7z \\ &= 2 \cdot 2 + 3 \cdot 1 + 5 \cdot 0 + 7 \cdot 2 \\ &= 21. \end{aligned}$$

Thus, **A** is the correct answer.

18. A fair 6 sided die is rolled twice. What is the probability that the first number that comes up is greater than or equal to the second number?

A $\frac{1}{6}$

B $\frac{5}{12}$

C $\frac{1}{2}$

D $\frac{7}{12}$

E $\frac{5}{6}$

Solution(s):

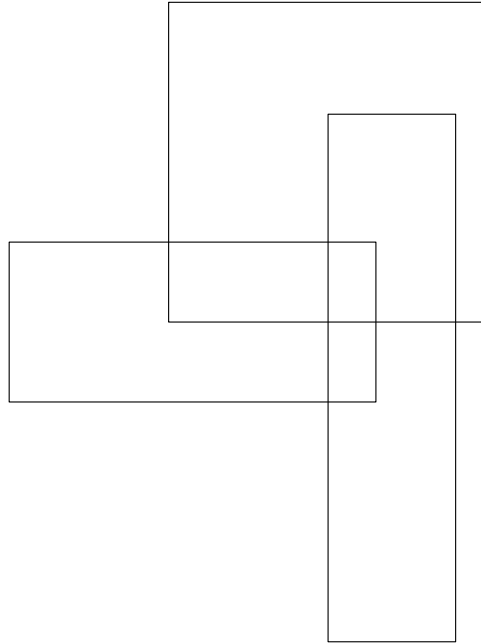
There are 3 possible outcomes when rolling a die twice: the first number is greater than the second, both numbers are equal, or the first number is less than the second number. The first and third outcomes have the same probability since they are symmetric.

The second outcome has a $\frac{1}{6}$ chance of happening, since the first number can be anything, and the second number must equal first number. The other two outcomes have a combined probability of $1 - \frac{1}{6} = \frac{5}{6}$. This means that each outcome has a $\frac{5}{6} \div 2 = \frac{5}{12}$ chance of happening.

The desired probability is the first outcome plus the second outcome, for a total probability of $\frac{5}{12} + \frac{1}{6} = \frac{7}{12}$.

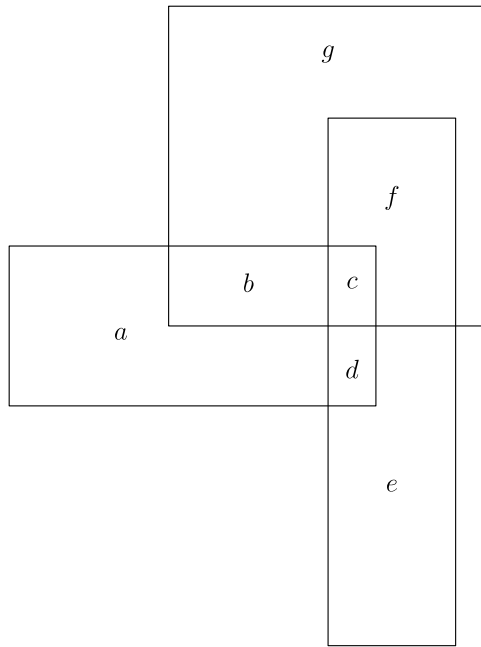
Thus, **D** is the correct answer.

19. How many rectangles are in this figure?



- A 8
- B 9
- C 10
- D 11
- E 12

Solution(s):

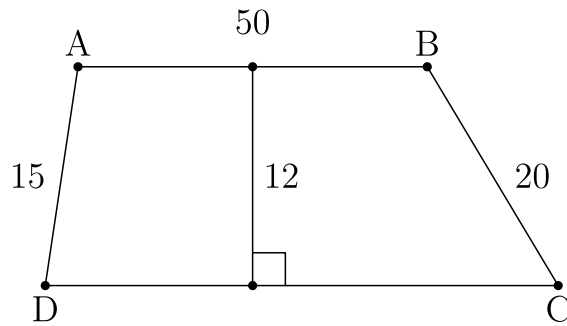


We can split the figure into these regions to make it easier to count the rectangles.

The rectangles in this figure are b , c , d , ab , bc , cd , cf , de , $abcd$, $cdef$, and $bcfg$. These form 11 rectangles.

Thus, **D** is the correct answer.

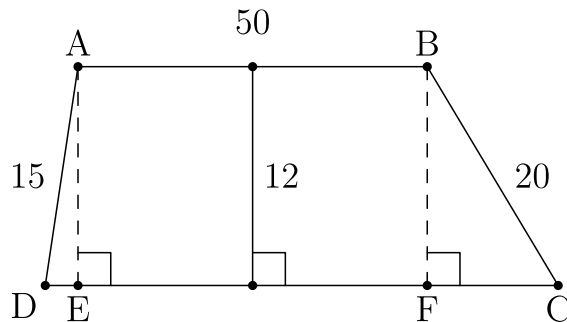
20. Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 50$, $BC = 20$, and the altitude is 12. What is the area of the trapezoid?



- A 600
- B 650
- C 700
- D 750
- E 800

Solution(s):

We can drop the following altitudes to more easily find the area.



We can use the Pythagorean to get that

$$DE = \sqrt{15^2 - 12^2} = 9$$

and

$$FC = \sqrt{20^2 - 12^2} = 16.$$

We also know that

$$EF = AB = 50,$$

so

$$DC = DE + EF + FC = 75.$$

Then the area of $ABCD$ is

$$\begin{aligned} \frac{1}{2} \cdot (DC + 50) \cdot 12 &= 6 \cdot 125 \\ &= 750. \end{aligned}$$

Thus, **D** is the correct answer.

21. Students guess that Norb's age is 24, 28, 30, 32, 36, 38, 41, 44, 47, and 49. Norb says, "At least half of you guessed too low, two of you are off by one, and my age is a prime number." How old is Norb?

- A 29
- B 31
- C 37
- D 43
- E 48

Solution(s):

The first part of the statement means that Norb's age is greater than 36.

The second part means that Norb's age is either between 36 and 38 or between 47 and 49.

Since 37 is prime and 48 is not, Norb's age is 37.

Thus, **C** is the correct answer.

22. What is the tens digit of 7^{2011} ?

A 0

B 1

C 3

D 4

E 7

Solution(s):

Solution 1

To find the tens digit, we can simply find the tens digit when taking the number $\pmod{100}$. Since $7^4 = 2401 \equiv 1 \pmod{100}$, then $7^{2011} \equiv 7^{4 \cdot 503} \cdot 7^3 \equiv 7^3 \equiv 343 \equiv 43$.

Solution 2

If we look at the tens digits of powers of 7, we get 01, 07, 49, 43, 01, and from there the pattern repeats.

This means that the tens digits repeat in periods of 4. Since 2011 leaves a remainder of 3 when divided by 4, its tens digit will be 4.

Thus, **D** is the correct answer.

23. How many 4-digit positive integers have four different digits, where the leading digit is not zero, the integer is a multiple of 5, and 5 is the largest digit?

A 24

B 48

C 60

D 84

E 108

Solution(s):

For a number to be divisible by 5, the units digit must be either 0 or 5.

If the units digit is 0, one of the other three digits must be 5. The remaining two digits must be chosen from $\{1, 2, 3, 4\}$. There are 6 ways to choose the pair, and there are 6 ways to arrange the three digits for a total of $6 \cdot 6 = 36$ numbers.

If the units digit is 5, there are 4 ways to choose the thousands digit. There are $4 \cdot 3 = 12$ ways to choose the other 2 digits. This leaves a total of $4 \cdot 12 = 48$ numbers for this case.

Combining both cases, we get the total number of such integers is $36 + 48 = 84$.

Thus, **D** is the correct answer.

24. In how many ways can 10001 be written as the sum of two primes?

A 0

B 1

C 2

D 3

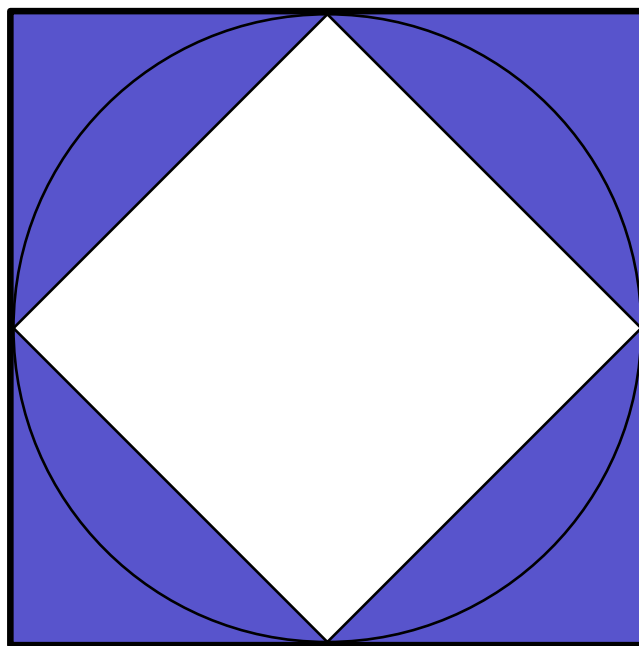
E 4

Solution(s):

For two numbers to add to an odd number, one of them must be odd and the other even. Thus only even prime is 2, so the other number is forced to be 9999. 9999 is not prime, however, so 10001 cannot be written as the sum of two primes.

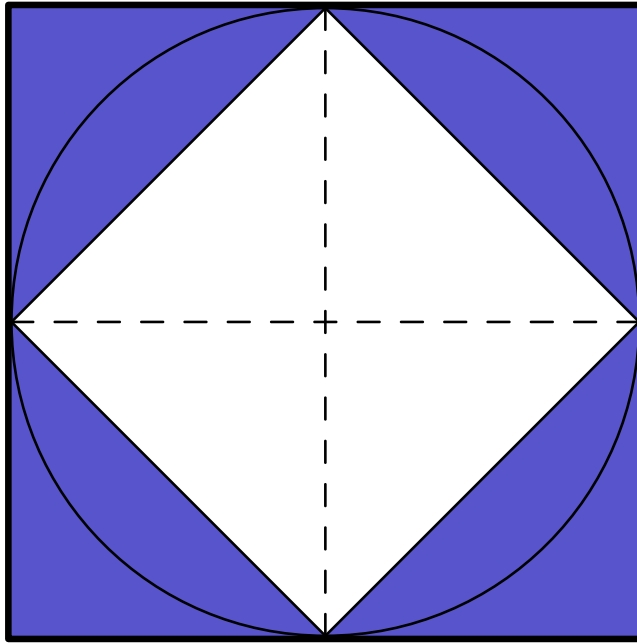
Thus, **A** is the correct answer.

25. A circle with radius 1 is inscribed in a square and circumscribed about another square as shown. Which fraction is closest to the ratio of the circle's shaded area to the area between the two squares?



- A $\frac{1}{2}$
- B 1
- C $\frac{3}{2}$
- D 2
- E $\frac{5}{2}$

Solution(s):



The circle's shaded area is equal to the area of the circle minus the area of the smaller square. The side length of the inner square can be calculate using the Pythagorean Theorem to get

$$\sqrt{1^2 + 1^2} = \sqrt{2}.$$

Therefore, the area of the inner square is $\sqrt{2}^2 = 2$. The area of the circle's shaded area is then $1^2\pi - 2 = \pi - 2$.

The area of the outside square is $2^2 = 4$, so the area of the shaded area between the two squares is $4 - 2 = 2$.

The desired fraction is

$$\frac{\pi - 2}{2} \approx \frac{3.14 - 2}{2} \approx \frac{1}{2}.$$

Thus, **A** is the correct answer.

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