

2009 AMC 8 Solutions

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1. Bridget bought a bag of apples at the grocery store. She gave half of the apples to Ann. Then she gave Cassie 3 apples, keeping 4 apples for herself. How many apples did Bridget buy?

A 3

B 4

C 7

D 11

E 14

Solution(s):

We can work backwards, starting with the 4 apples that Bridget kept for herself. Adding the 3 apples that she gave Cassie, she now has 7 apples.

Finally, we multiply this value by 2 since she gave half of her initial apples to Ann. $7 \cdot 2 = 14$, so Bridget started off with 14 apples.

Thus, **E** is the correct answer.

2. On average, for every 4 sports cars sold at the local dealership, 7 sedans are sold. The dealership predicts that it will sell 28 sports cars next month. How many sedans does it expect to sell?

A 7

B 32

C 35

D 49

E 112

Solution(s):

We can set up the following proportion:

$$\frac{4}{7} = \frac{28}{x}.$$

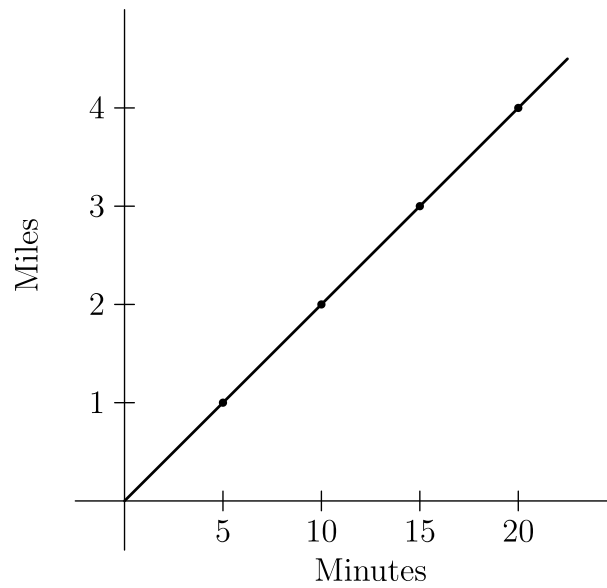
Cross multiplying, we get

$$4x = 7 \cdot 28$$

$$x = 49$$

Thus, **D** is the correct answer.

3. The graph shows the constant rate at which Suzanna rides her bike. If she rides a total of a half an hour at the same speed, how many miles would she have ridden?



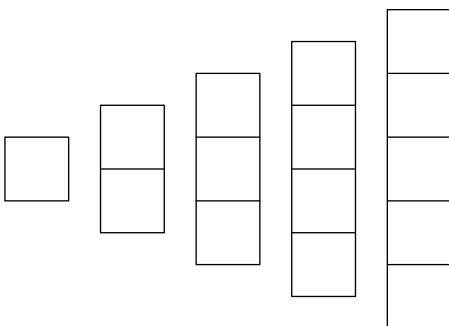
- ☐ A 5
- ☐ B 5.5
- ☒ C 6
- ☐ D 6.5
- ☐ E 7

Solution(s):

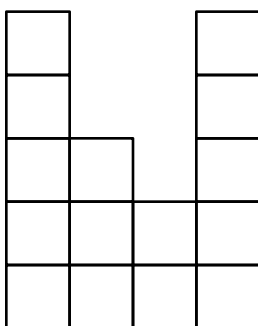
From the graph, we can see that Suzanna rides 3 miles in 15 minutes. This means that in 30 minutes, she will have ridden 6 miles.

Thus, **C** is the correct answer.

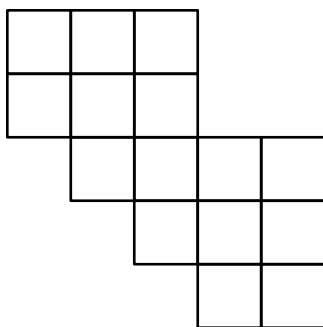
4. The five pieces shown below can be arranged to form four of the five figures below. Which figure **cannot** be formed?



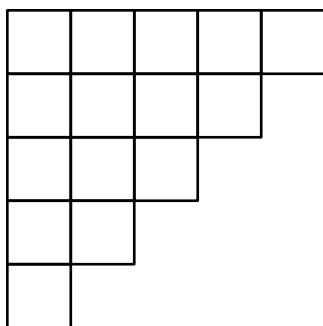
A



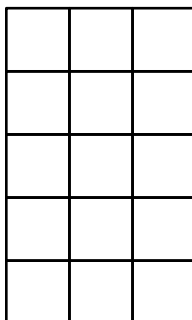
B



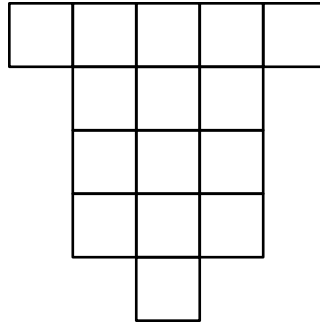
C



D



E



Solution(s):

Note that option **B** does not have any segments in it that are 5 blocks long. This means that it is impossible to arrange the 5 block long piece to fit within the figure.

Thus, **B** is the correct answer.

5. A sequence of numbers starts with 1, 2, and 3. The fourth number of the sequence is the sum of the previous three numbers in the sequence: $1 + 2 + 3 = 6$. In the same way, every number after the fourth is the sum of the previous three numbers. What is the eighth number in the sequence?

A 11

B 20

C 37

D 68

E 99

Solution(s):

We can create the following list to find all the numbers up till the eight number a_i denotes the i -th number in the sequence.

$a_1 = 1$, $a_2 = 2$, and $a_3 = 3$ from the problem statement.

$$a_4 = 1 + 2 + 3 = 6$$

$$a_5 = 2 + 3 + 6 = 11$$

$$a_6 = 3 + 6 + 11 = 20$$

$$a_7 = 6 + 11 + 20 = 37$$

$$a_8 = 11 + 20 + 37 = 68$$

Thus, **D** is the correct answer.

6. Steve's empty swimming pool will hold 24,000 gallons of water when full. It will be filled by 4 hoses, each of which supplies 2.5 gallons of water per minute. How many hours will it take to fill Steve's pool?

A 40

B 42

C 44

D 46

E 48

Solution(s):

The 4 hoses together fill the pool with

$$2.5 \cdot 4 = 10$$

gallons of water per minute.

To fill 24,000 gallons, it will take the hoses

$$24,000 \div 10 = 2,400$$

minutes to fill the pool.

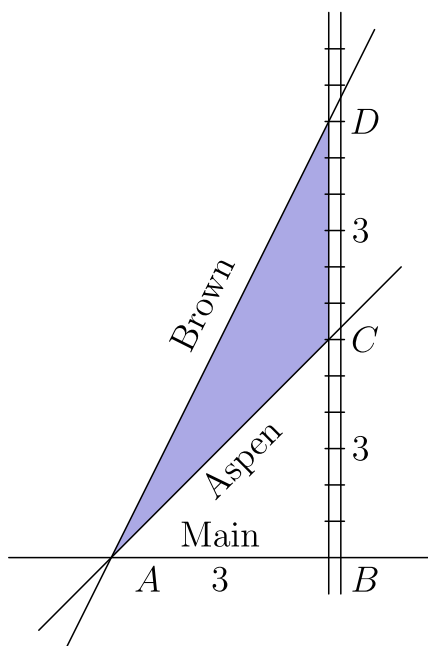
2,400 minutes is the same as

$$2,400 \div 60 = 40$$

hours.

Thus, **A** is the correct answer.

7. The triangular plot of ACD lies between Aspen Road, Brown Road and a railroad. Main Street runs east and west, and the railroad runs north and south. The numbers in the diagram indicate distances in miles. The width of the railroad track can be ignored. How many square miles are in the plot of land ACD ?



- A 2
- B 3
- C 4.5**
- D 6
- E 9

Solution(s):

The base of ADC is CD , which is 3. The altitude is 3 as well.

Therefore, the area of ADC is $\frac{1}{2} \cdot 3 \cdot 3 = 4.5$.

Thus, **C** is the correct answer.

8. The length of a rectangle is increased by 10% percent and the width is decreased by 10% percent. What percent of the old area is the new area?

A 90

B 99

C 100

D 101

E 110

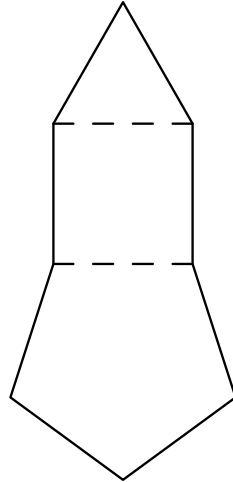
Solution(s):

Let l be the old width of the rectangle and w be the old width. The new length is then $1.1l$, and the new width is $.9w$. This means that the new area is $1.1 \times .9lw = .99lw$.

This shows that the new area is 99% of the original area.

Thus, **B** is the correct answer.

9. Construct a square on one side of an equilateral triangle. On one non-adjacent side of the square, construct a regular pentagon, as shown. On a non-adjacent side of the pentagon, construct a hexagon. Continue to construct regular polygons in the same way, until you construct an octagon. How many sides does the resulting polygon have?



A 21

B 23

C 25

D 27

E 29

Solution(s):

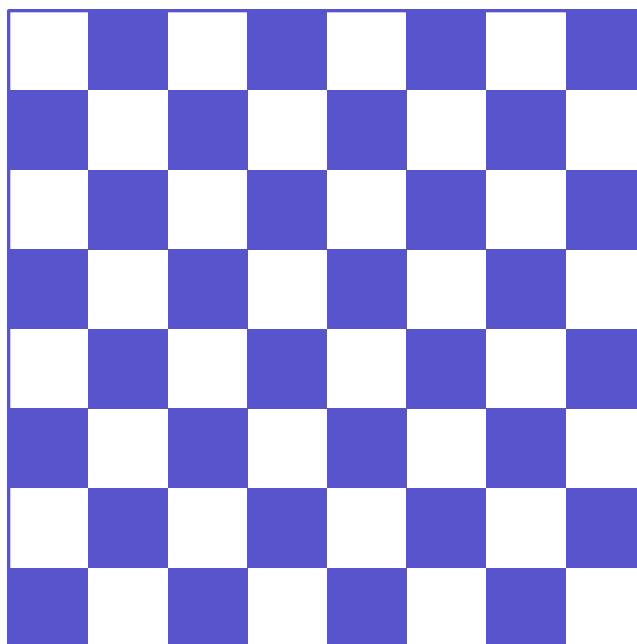
Notice that every polygon in the middle of the chain (every shape except for the triangle and octagon) contribute all but 1 of their sides to the overall polygon. The triangle and octagon contribute all but 1 of their sides.

Therefore, the total number of sides is

$$\begin{aligned} & (3 - 1) + (4 - 2) + (5 - 2) + \\ & (6 - 2) + (7 - 2) + (8 - 1) \\ & = 2 + 2 + 3 + 4 + 5 + 7 \\ & = 23. \end{aligned}$$

Thus, **B** is the correct answer.

10. On a checkerboard composed of 64 unit squares, what is the probability that a randomly chosen unit square does **not** touch the outer edge of the board?



A $\frac{1}{16}$

B $\frac{7}{16}$

C $\frac{1}{2}$

D $\frac{9}{16}$

E $\frac{49}{64}$

Solution(s):

There are

$$(8 - 2)^2 = 6^2 = 36$$

squares on the interior.

This means that the probability of choosing one of these squares is

$$\frac{36}{64} = \frac{9}{16}.$$

Thus, **D** is the correct answer.

11. The Amaco Middle School bookstore sells pencils costing a whole number of cents. Some seventh graders each bought a pencil, paying a total of 1.43 dollars. Some of the 30 sixth graders each bought a pencil, and they paid a total of 1.95 dollars. How many more sixth graders than seventh graders bought a pencil?

A 1

B 2

C 3

D 4

E 5

Solution(s):

The number of seventh graders that bought a pencil is 143 divided by the price of a pencil. Similarly, the number of sixth graders that bought a pencil is 195 divided by the price of a pencil.

This means that the price of a pencil divides both 143 and 195. Prime factorizing, we get

$$143 = 11 \cdot 13$$

and

$$195 = 3 \cdot 5 \cdot 13.$$

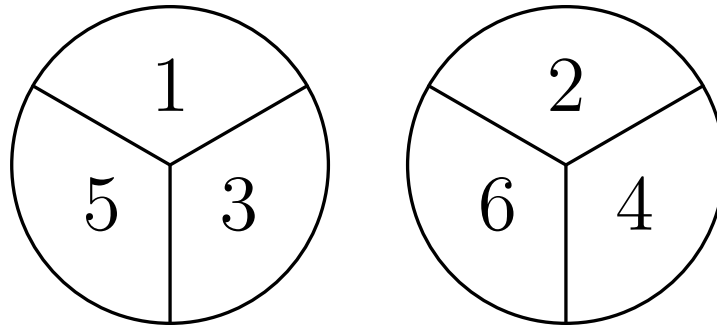
The only numbers that divide both 143 and 195 are 1 and 13.

If 1 cent was the price of the pencil, that means 195 sixth graders bought pencils, which is not possible. Therefore, the price of a pencil is 13 cents.

This means that $143 \div 13 = 11$ seventh graders bought a pencil, and $195 \div 13 = 15$ sixth graders bought a pencil. Therefore, 4 more sixth graders than seventh graders bought pencils.

Thus, **D** is the correct answer.

12. The two spinners shown are spun once and each lands on one of the numbered sectors. What is the probability that the sum of the numbers in the two sectors is prime?



A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{7}{9}$

E $\frac{5}{6}$

Solution(s):

We can find the sum of the two numbers in every possible outcome.

$$\begin{gather*} 1 + 2 = 3 \quad 1 + 4 = 5 \quad 1 + 6 = 7 \quad 3 + 2 = 5 \quad 3 + 4 = 7 \quad 3 + 6 = 9 \\ 5 + 2 = 7 \quad 5 + 4 = 9 \quad 5 + 6 = 11. \end{gather*}$$

There are only 2 outcomes where the sum is not prime (the two instances when the sum is 9). Therefore, the probability that the sum is prime is $\frac{7}{9}$.

Thus, **D** is the correct answer.

13. A three-digit integer contains one of each of the digits 1, 3, and 5. What is the probability that the integer is divisible by 5?

A $\frac{1}{6}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{5}{6}$

Solution(s):

Then number is equally likely to end in a 1, 3, or 5. It is only divisible if the last digit is a 5, which happens with a $\frac{1}{3}$ probability.

Thus, **B** is the correct answer.

14. Austin and Temple are 50 miles apart along Interstate 35. Bonnie drove from Austin to her daughter's house in Temple, averaging 60 miles per hour. Leaving the car with her daughter, Bonnie rode a bus back to Austin along the same route and averaged 40 miles per hour on the return trip. What was the average speed for the round trip, in miles per hour?

A 46

B 48

C 50

D 52

E 54

Solution(s):

The trip from Austin to Temple took $50 \div 60 = \frac{5}{6}$ hours. The trip from Temple to Austin took $50 \div 40 = \frac{5}{4}$ hours. This means that the total time for the round trip was $\frac{5}{6} + \frac{5}{4} = \frac{25}{12}$ hours.

The total distance of the round trip was $2 \cdot 50 = 100$ miles. Therefore, the average speed for the round trip was $100 \div \frac{25}{12} = 48$ miles per hour.

Thus, **B** is the correct answer.

15. A recipe that makes 5 servings of hot chocolate requires 2 squares of chocolate, $\frac{1}{4}$ cup sugar, 1 cup water and 4 cups milk. Jordan has 5 squares of chocolate, 2 cups of sugar, lots of water, and 7 cups of milk. If he maintains the same ratio of ingredients, what is the greatest number of servings of hot chocolate he can make?

A $5\frac{1}{8}$

B $6\frac{1}{4}$

C $7\frac{1}{2}$

D $8\frac{3}{4}$

E $9\frac{7}{8}$

Solution(s):

We need to find which of the ingredients is the limiting factor.

With 5 squares of chocolate, Jordan can only make $\frac{5}{2}$ times what the recipe requires.

With 2 cups of sugar, Jordan can only make $\frac{2}{\frac{1}{4}} = 8$ times the required amount.

With 7 cups of milk, Jordan can only make $\frac{7}{4}$ times what is needed.

Therefore, the number of servings is limited by the amount of milk. The amount of servings Jordan can make is

$$5 \cdot \frac{7}{4} = \frac{35}{4} = 8\frac{3}{4}.$$

Thus, **D** is the correct answer.

16. How many 3-digit positive integers have digits whose product equals 24?

A 12

B 15

C 18

D 21

E 24

Solution(s):

The only triples of integers less than 10 that multiply to 24 are

$(1, 3, 8), (1, 4, 6), (2, 2, 6),$

and $(2, 3, 4)$.

The triples with 3 distinct numbers can be rearranged to form 6 distinct 3-digit positive integers. The other triple can be arranged to form 3 distinct 3-digit positive integers.

This leaves a total of $3 \cdot 6 + 3 = 21$ integers.

Thus, **D** is the correct answer.

17. The positive integers x and y are the two smallest positive integers for which the product of 360 and x is a square and the product of 360 and y is a cube. What is the sum of x and y ?

A 80

B 85

C 115

D 165

E 610

Solution(s):

For a number to be a perfect square, every exponent in the prime factorization must be even. For it to be a cube, the exponents must be divisible by 3.

We can factor 360 to get

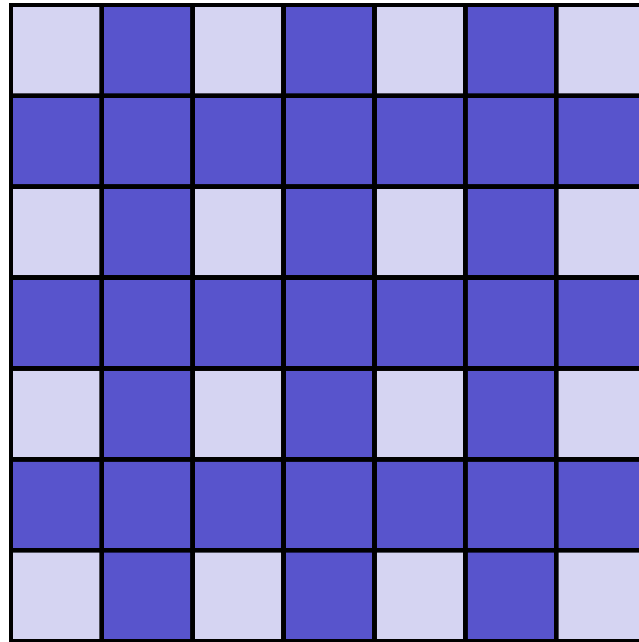
$$360 = 2^3 \cdot 3^2 \cdot 5.$$

For $360x$ to be a perfect square and x to be minimized, x must have one factor of 2 and one factor of 5. Therefore, we can let $x = 10$.

For $360y$ to be a cube, y must have one factor of 3 and two factors of 5. Therefore, we can let $y = 75$, suggesting $x + y = 85$.

Thus, **B** is the correct answer.

18. The diagram represents a 7-foot-by-7-foot floor that is tiled with 1-square-foot light tiles and dark tiles. Notice that the corners have dark tiles. If a 15-foot-by-15-foot floor is to be tiled in the same manner, how many dark tiles will be needed?



- A 49
- B 57
- C 64**
- D 96
- E 126

Solution(s):

Looking at the example given, note that there are 4^2 dark tiles. This is because there are 4 rows that contain 4 dark tiles each.

For a 15-foot-by-15-foot floor, there are going to be 8 rows with 8 dark tiles in each row. This will give us $8^2 = 64$ dark tiles.

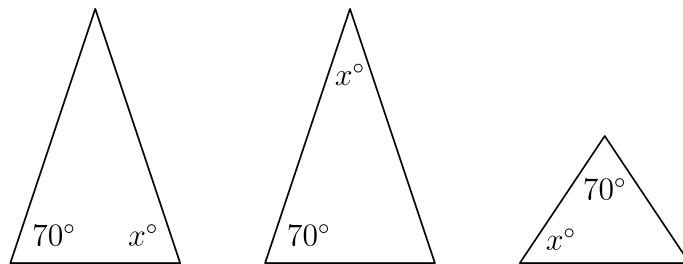
Thus, **C** is the correct answer.

19. Two angles of an isosceles triangle measure 70° and x° . What is the sum of the three possible values of x ?

- A 95
- B 125
- C 140
- D 165**
- E 180

Solution(s):

All the following possibilities are shown below.



In the first scenario, we get $x = 70$ by the properties of the isosceles triangle.

In the second scenario, we get that

$$70 \cdot 2 + x = 180,$$

from which we get that $x = 40$.

From the third scenario, we get that

$$2x + 70 = 180,$$

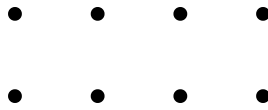
from which we get that $x = 55$.

The sum of these values yields

$$70 + 40 + 55 = 165.$$

Thus, **D** is the correct answer.

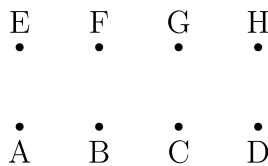
20. How many non-congruent triangles have vertices at three of the eight points in the array shown below?



- ☐ A 5
- ☐ B 6
- ☐ C 7
- ☒ D 8
- ☐ E 9

Solution(s):

We can label the points to find all the unique triangles.



All the possible distinct triangles are

$AEF, AEG, AEH, AFG, AFH,$
 $AGH, AFC,$ and AFD .

Any other triangle will be congruent to one of the 8 listed above.

Thus, **D** is the correct answer.

21. Andy and Bethany have a rectangular array of numbers with 40 rows and 75 columns. Andy adds the numbers in each row. The average of his 40 sums is A . Bethany adds the numbers in each column. The average of her 75 sums is B . What is the value of $\frac{A}{B}$?

☐ A $\frac{64}{225}$

☐ B $\frac{8}{15}$

☐ C 1

☒ D $\frac{15}{8}$

☐ E $\frac{225}{64}$

Solution(s):

Each person includes every number in the array in their sums exactly once. This means that before they divide in the average, their sums are the exact same. This means that $40A = 75B$. Therefore, $\frac{A}{B} = \frac{75}{40} = \frac{15}{8}$.

Thus, **D** is the correct answer.

22. How many whole numbers between 1 and 1000 do **not** contain the digit 1?

A 512

B 648

C 720

D 728

E 800

Solution(s):

We can case on the number of digits.

There are 8 one digit numbers excluding 1.

There are $8 \cdot 9 = 72$ two digit numbers that lack the digit 1.

There are $8 \cdot 9 \cdot 9 = 648$ three digit numbers that do not include 1.

This yields a total of

$$8 + 72 + 648 = 728$$

numbers that do not contain the digit 1.

Thus, **D** is the correct answer.

23. On the last day of school, Mrs. Wonderful gave jelly beans to her class. She gave each boy as many jelly beans as there were boys in the class. She gave each girl as many jelly beans as there were girls in the class. She brought 400 jelly beans, and when she finished, she had six jelly beans left. There were two more boys than girls in her class. How many students were in her class?

A 26

B 28

C 30

D 32

E 34

Solution(s):

Let b be the number of boys in the class and g be the number of girls. From the problem, we get that $b = g + 2$.

If each boy gets b jelly beans, then Mrs. Wonderful will give out a total of b^2 jelly beans to all the boys. Similarly, she will give out g^2 jelly beans to all the girls.

Therefore,

$$\begin{aligned}b^2 + g^2 &= 400 - 6 \\(g + 2)^2 + g^2 &= 394 \\2g^2 + 4g + 4 &= 394 \\g^2 + 2g - 195 &= 0 \\(g + 15)(g - 13) &= 0.\end{aligned}$$

Since g cannot be negative, we get that $g = 13$. This means that $b = 15$, so $b + g = 28$.

Thus, **B** is the correct answer.

24. The letters A , B , C and D represent digits.

If
$$\begin{array}{r} A \ B \\ + \ C \ A \\ \hline D \ A \end{array}$$
 and
$$\begin{array}{r} A \ B \\ - \ C \ A \\ \hline A \end{array}$$
, what digit does D represent?

☐ A 5

☐ B 6

☐ C 7

☐ D 8

☒ E 9

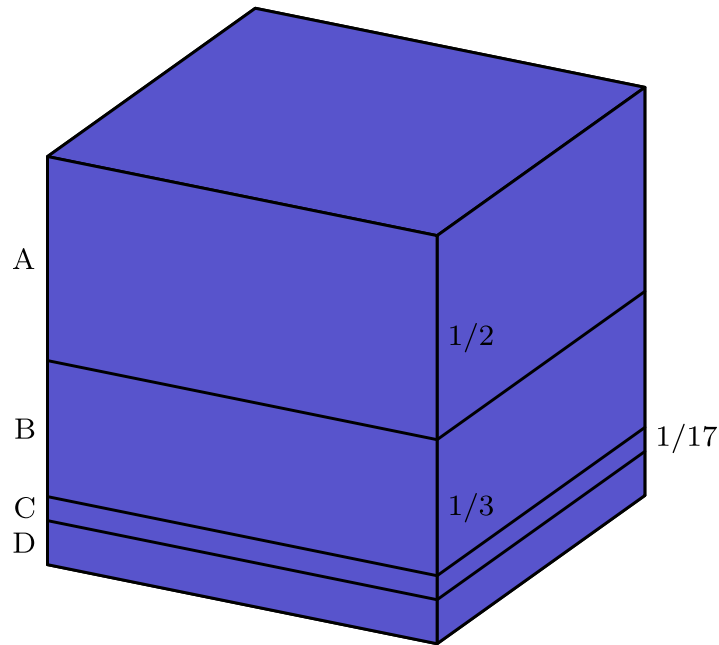
Solution(s):

Since $A + B = A$, we get that $B = 0$. Using this and the second equation, we also get that $A = 5$. This follows from carrying over a 10 to B , resulting in $10 - A = A$. Therefore, $AB = 50$.

Since $50 - C5 = 5$, we also know that $C = 4$. Then $D = C + 5 = 9$.

Thus, **E** is the correct answer.

- 25.** A one-cubic-foot cube is cut into four pieces by three cuts parallel to the top face of the cube. The first cut is $\frac{1}{2}$ foot from the top face. The second cut is $\frac{1}{3}$ foot below the first cut, and the third cut is $\frac{1}{17}$ foot below the second cut. From the top to the bottom the pieces are labeled A , B , C , and D . The pieces are then glued together end to end as shown in the second diagram. What is the total surface area of this solid in square feet?



A 6

B 7

C $\frac{419}{51}$

D $\frac{158}{17}$

E 11

Solution(s):

We can look at this shape from all 6 directions to add up the surface area from each view to get the total surface area.

Looking at it from the ends gives us the face of A , which has an area of $\frac{1}{2} \cdot 1 = \frac{1}{2} \text{ ft}^2$.

Each side has the same area as if we were to stack up all the pieces into a unit cube and take the area of one side. This would yield 1 ft^2 .

The top and bottom each have 4 unit squares, for a total of 4 ft^2 from each view.

Adding this all up gives us a total surface area of

$$\frac{1}{2} \cdot 2 + 1 \cdot 2 + 4 \cdot 2 = 11 \text{ ft}^2.$$

Thus, **E** is the correct answer.

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