2006 AMC 8 Solutions

Typeset by: LIVE, by Po-Shen Loh
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1. Mindy made three purchases for $\$1.98$ dollars, $\$5.04$ dollars, and $\$9.89$ dollars. What was her total, to the nearest dollar?

A $10

B $15

C $16

D $17

E $18

Solution(s):

Since all the values are already close to a whole number, we can just add the sums of the rounded numbers.

These prices round to $\$2$, $\$5$, and $\$10$, which add to $\$17$.

Thus, **D** is the correct answer.

2. On the AMC 8 contest Billy answers 13 questions correctly, answers 7 questions incorrectly and doesn't answer the last 5. What is his score?

- A 1
- B 6
- C 13
- D 19
- E 26

Solution(s):

Since the AMC 8 only awards 1 point for each correct question, Billy will get 13 points.

Thus, **C** is the correct answer.

3. Elisa swims laps in the pool. When she first started, she completed 10 laps in 25 minutes. Now, she can finish 12 laps in 24 minutes. By how many minutes has she improved her lap time?

- A $\frac{1}{2}$
- B $\frac{3}{4}$
- C 1
- D 2
- E 3

Solution(s):

Initially, Elisa swam 1 lap in

$$\frac{25}{10} = \frac{5}{2}$$

minutes. Now, she swims 1 lap in

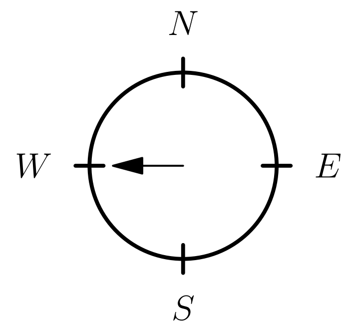
$$\frac{24}{12} = 2$$

minutes.

Therefore, she improved her lap time by $\frac{5}{2} - 2 = \frac{1}{2}$ minutes.

Thus, **A** is the correct answer.

4. Initially, a spinner points west. Chenille moves it clockwise $2\frac{1}{4}$ revolutions and then counterclockwise $3\frac{3}{4}$ revolutions. In what direction does the spinner point after the two moves?



A North

B East

C South

D West

E Northwest

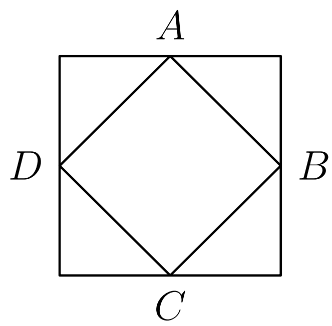
Solution(s):

If the spinner goes $2\frac{1}{4}$ revolutions clockwise and $3\frac{3}{4}$ revolutions counterclockwise, it goes $1\frac{1}{2}$ revolutions counterclockwise.

This means that the spinner will be pointing east.

Thus, **B** is the correct answer.

5. Points A, B, C and D are midpoints of the sides of the larger square. If the larger square has area 60, what is the area of the smaller square?



A 15

B 20

C 24

D 30

E 40

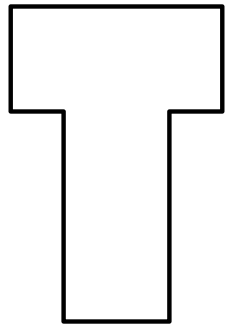
Solution(s):

Note that we can fold all the triangles in to perfectly cover the smaller square. This means that area of the smaller square is half the area of the larger square.

This makes the area of the smaller square $60 \div 2 = 30$.

Thus, **D** is the correct answer.

6. The letter T is formed by placing two 2×4 inch rectangles next to each other, as shown. What is the perimeter of the T, in inches?



A 12

B 16

C 20

D 22

E 24

Solution(s):

If we found the total perimeter of the two rectangles separately, we would have gotten

$$2(2(2+4)) = 2 \cdot 2 \cdot 6 = 24.$$

In the letter T, we can see that their intersection removes a piece of length 2 from each of the rectangles. Therefore, the perimeter of the T is

$$24 - 2 \cdot 2 = 20.$$

Thus, **C** is the correct answer.

7. Circle X has a radius of π. Circle Y has a circumference of 8π. Circle Z has an area of 9π. List the circles in order from smallest to largest radius.

- A X, Y, Z
- **B** Z, X, Y
- C Y, X, Z
- D Z, Y, X
- E X, Z, Y

Solution(s):

Recall that $C = 2\pi r$ and $A = \pi r^2$. Using these formulas we get that the radius of Y is

$$8\pi \div (2\pi) = 4.$$

We also get that the radius of Z is

$$\sqrt{9\pi \div \pi} = 3.$$

As π is greater than 3 and less than 4, the correct order is Z, X, Y.

Thus, the answer is **B**.

8. The table shows some of the results of a survey by radio station KAMC. What percentage of the males surveyed listen to the station?

	Listen	Don't Listen	Total
Male	?	26	?
Female	58	?	96
Total	136	64	200

A 39

B 48

C 52

D 55

E 75

Solution(s):

The total number of males surveyed is the total number surveyed by the total number of women surveyed:

$$200 - 96 = 104.$$

The percentage of males that listen to the station is 100% minus the percent that don't listen to the station:

$$100\% - 100 \cdot \frac{26}{104}\%$$

$$= 100\% - 25\%$$

$$= 75\%.$$

Thus, **E** is the correct answer.

9. What is the product of

$$\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \cdots \times \frac{2006}{2005}?$$

A 1

B 1002

C 1003

D 2005

E 2006

Solution(s):

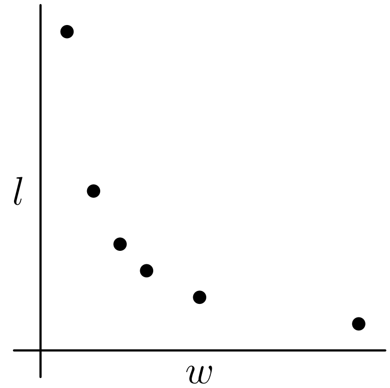
Note that the numerator of every fraction cancels with the denominator of the following fraction. This leaves two numbers:

$$\frac{2006}{2} = 1003.$$

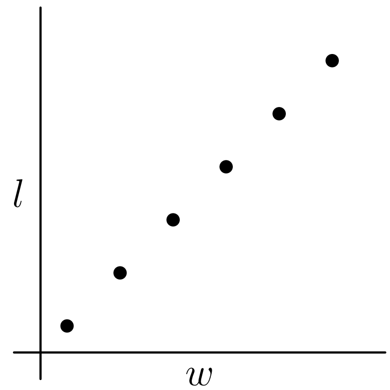
Thus, **C** is the correct answer.

10. Jorge's teacher asks him to plot all the ordered pairs (w, l) of positive integers for which w is the width and l is the length of a rectangle with area 12. What should his graph look like?

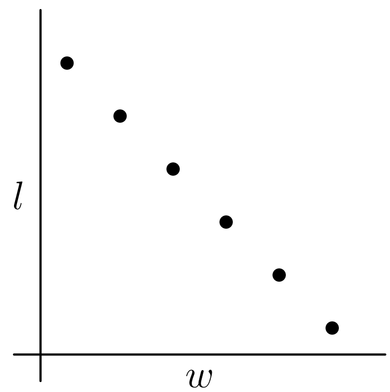
A



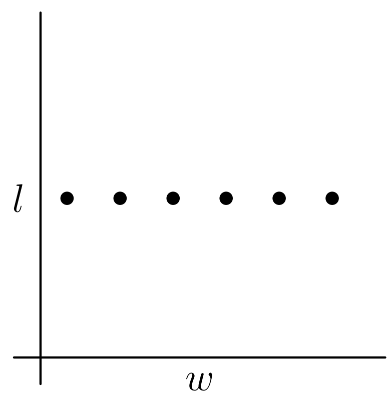
B



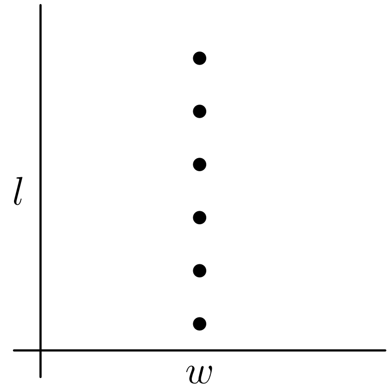
C



D



E



Solution(s):

We know that $wl = 12$, so $w = \dfrac{12}{l}$.

This shows that w and l are inversely proportional, which can only be represented by a non-linear graph.

Thus, **A** is the correct answer.

11. How many two-digit numbers have digits whose sum is a perfect square?

A 13

B 16

C 17

D 18

E 19

Solution(s):

There is 1 number whose digit sum is $1 : 10$.

There are 4 numbers whose digit sum is $4 : 13, 22, 31$, and 40.

There are 9 numbers whose digit sum is $9 : 18, 27, 36, 45, 54, 63, 72, 81$, and 90.

There are 3 numbers whose digit sum is $16 : 79, 88$, and 97.

Therefore, there are 17 numbers that satisfy the problem statement.

Thus, **C** is the correct answer.

12. Antonette gets 70% on a 10-problem test, 80% on a 20-problem test and 90% on a 30-problem test. If the three tests are combined into one 60-problem test, which percent is closest to her overall score?

- A 40
- B 77
- C 80
- D 83
- E 87

Solution(s):

Antonette got $.7 \cdot 10 = 7$ questions write on the first test. Similarly, she got $.8 \cdot 20 = 16$ and $.9 \cdot 30 = 27$ problems right on her second and third tests respectively.

Adding these up yields a total of 50 correct questions. Her score on the 60-problem test would have been a

$$100 \cdot \frac{50}{60} = 100 \cdot .8\overline{3} \approx 83\%.$$

Thus, **D** is the correct answer.

13. Cassie leaves Escanaba at $8:30$ AM heading for Marquette on her bike. She bikes at a uniform rate of 12 miles per hour. Brian leaves Marquette at $9:00$ AM heading for Escanaba on his bike. He bikes at a uniform rate of 16 miles per hour. They both bike on the same 62-mile route between Escanaba and Marquette. At what time in the morning do they meet?

- A $10:00$
- B $10:15$
- C $10:30$
- D $11:00$
- E $11:30$

Solution(s):

By the time Brian starts biking, Marquette has already traveled $\frac{1}{2} \cdot 12 = 6$ miles. This means that Brian and Marquette must then travel a total of $62 - 6 = 56$ miles.

Combined, the two bike at $12 + 16 = 28$ miles per hour. This means that they can travel 56 miles in $56 \div 28 = 2$ hours.

This means that they meet at $11:00$.

Thus, **D** is the correct answer.

14. The students in Mrs. Reed's English class are reading the same 760-page novel. Three friends, Alice, Bob and Chandra, are in the class. Alice reads a page in 20 seconds, Bob reads a page in 45 seconds and Chandra reads a page in 30 seconds.

If Bob and Chandra both read the whole book, Bob will spend how many more seconds reading than Chandra?

A $7,600$

B $11,400$

C $12,500$

D $15,200$

E $22,800$

Solution(s):

Bob will take $760 \cdot 45$ seconds to read the book, and Chandra will take $760 \cdot 30$ seconds.

The difference between the time they spent reading is

$$760 \cdot 45 - 760 \cdot 30$$

$$= 760(45 - 30)$$

$$= 760 \cdot 15$$

$$= 11,400$$

seconds.

Thus, **B** is the correct answer.

15. The students in Mrs. Reed's English class are reading the same 760-page novel. Three friends, Alice, Bob and Chandra, are in the class. Alice reads a page in 20 seconds, Bob reads a page in 45 seconds and Chandra reads a page in 30 seconds.

Chandra and Bob, who each have a copy of the book, decide that they can save time by "team reading" the novel. In this scheme, Chandra will read from page 1 to a certain page and Bob will read from the next page through page 760, finishing the book. When they are through they will tell each other about the part they read. What is the last page that Chandra should read so that she and Bob spend the same amount of time reading the novel?

A 425

B 444

C 456

D 484

E 506

Solution(s):

Let x be the number of pages that Chandra will read. Then Bob will read $760 - x$ pages.

For them to read for the same amount of time,

$$\begin{aligned} 30x &= 45(760 - x) \\ 30x &= 45 \cdot 760 - 45x \\ 75x &= 45 \cdot 760 \\ 5x &= 3 \cdot 760 \\ x &= 3 \cdot 152 \\ x &= 456. \end{aligned}$$

Thus, **C** is the correct answer.

16. The students in Mrs. Reed's English class are reading the same 760-page novel. Three friends, Alice, Bob and Chandra, are in the class. Alice reads a page in 20 seconds, Bob reads a page in 45 seconds and Chandra reads a page in 30 seconds.

Before Chandra and Bob start reading, Alice says she would like to team read with them. If they divide the book into three sections so that each reads for the same length of time, how many seconds will each have to read?

A 6400

B 6600

C 6800

D 7000

E 7200

Solution(s):

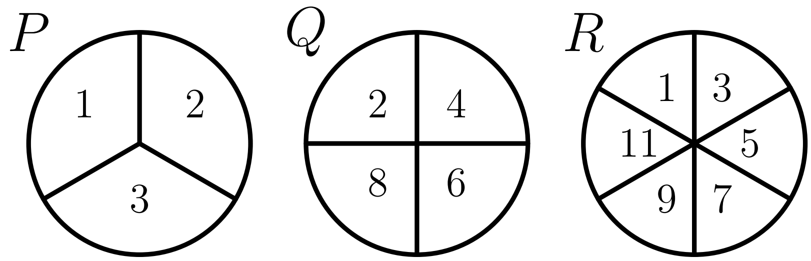
If all 3 individuals read the same amount of time, then the number of pages Bob, Chandra, and Alice will read will be in the ratio $4 : 6 : 9$ respectively.

This means that they will read $160, 240$, and 360 pages respectively. Since they all read for the same amount of time, we can just calculate how long it takes for Bob to read his portion.

Bob will take $45 \cdot 160 = 7200$ seconds to read his portion.

Thus, **E** is the correct answer.

17. Jeff rotates spinners P, Q and R and adds the resulting numbers. What is the probability that his sum is an odd number?



A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

Solution(s):

Note that adding an even number to a number does not affects is parity. Therefore, whatever the second spinner lands on will not impact whether the sum is odd.

The only way for a number on P and a number on R to add to an odd number is if P lands on 2. Otherwise, it would be the sum of two odd numbers, which is even.

P lands on 2 with a $\frac{1}{3}$ probability.

Thus, **B** is the correct answer.

18. A cube with 3-inch edges is made using 27 cubes with 1-inch edges. Nineteen of the smaller cubes are white and eight are black. If the eight black cubes are placed at the corners of the larger cube, what fraction of the surface area of the larger cube is white?

A $\frac{1}{9}$

B $\frac{1}{4}$

C $\frac{4}{9}$

D $\frac{5}{9}$

E $\frac{19}{27}$

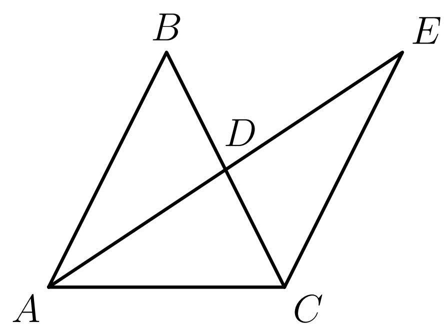
Solution(s):

Since each face has the same black and white surface area, we can analyze what fraction of one side is white.

On one side, there are 9 unit squares. 4 of them are black. This means that $\frac{5}{9}$ of each face is white.

Thus, **D** is the correct answer.

19. Triangle ABC is an isosceles triangle with $\overline{AB} = \overline{BC}$. Point D is the midpoint of both $\overline{BC}$ and $\overline{AE}$, and $\overline{CE}$ is 11 units long. Triangle ABD is congruent to triangle ECD. What is the length of $\overline{BD}$?



- **A** 4
- **B** 4.5
- **C** 5
- **D** 5.5
- **E** 6

Solution(s):

By the congruency condition, we know that

$$AB = EC = 11.$$

Also from the isosceles condition, we know that

$$BC = AB = 11.$$

Since D is the midpoint of $\overline{BC}$, we know that

$$BD = BC \div 2 = 5.5$$

Thus, **D** is the correct answer.

20. A singles tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 3 games, Kendra won 2 games and Lara won 2 games, how many games did Monica win?

A 0

B 1

C 2

D 3

E 4

Solution(s):

In every match, there was exactly one winner. There are $6 \cdot 5 \div 2 = 15$ games and therefore 15 wins.

There are already

$$4 + 3 + 3 + 2 + 2 = 13$$

wins accounted for, so Monica won $15 - 13 = 2$ games.

Thus, **C** is the correct answer.

21. An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. The aquarium is filled with water to a depth of 37 cm. A rock with volume $1000\ \text{cm}^3$ is then placed in the aquarium and completely submerged. By how many centimeters does the water level rise?

- A 0.25
- B 0.5
- C 1
- D 1.25
- E 2.5

Solution(s):

The original volume of the water was

$$100 \cdot 40 \cdot 37 = 168,000\ \text{cm}^3.$$

The volume after the rock was added is

$$168,000 + 1,000 = 169,000\ \text{cm}^3.$$

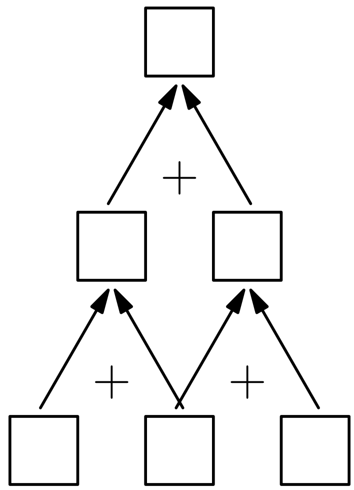
This means the new height is

$$169,000 \div 100 \div 40 = 37.25\ \text{cm}.$$

This means that water level rose by $.25$ cm.

Thus, **A** is the correct answer.

22. Three different one-digit positive integers are placed in the bottom row of cells. Numbers in adjacent cells are added and the sum is placed in the cell above them. In the second row, continue the same process to obtain a number in the top cell. What is the difference between the largest and smallest numbers possible in the top cell?



A 16

B 24

C 25

D 26

E 35

Solution(s):

If the lower cells contain a, b, and c, the middle row will have $a+b$ and $b+c$.

This means that the top row will have $a+2b+c$. To minimize this, we can let $b=1$, $a=2$, and $c=3$. This yields a top element of 7.

To maximize, this we can let $b=9$, $a=8$, and $c=7$. This yields a top element of 33. The desired difference is $33-7=26$.

Thus, **D** is the correct answer.

23. A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

- A 0
- B 1
- C 2
- D 3
- E 5

Solution(s):

The positive integers that leave a remainder of 4 when divided by 6 are

$$4, 10, 16, 22, 28, 34, 40, \cdots .$$

The positive integers that leave a remainder of 3 when divided by 5 are

$$3, 8, 13, 18, 23, 28, 33, \cdots .$$

From this, we can see that the smallest number of coins that work is 28. This leaves a remainder of 0 when divided by 7.

Thus, **A** is the correct answer.

24. In the multiplication problem below A, B, C, D are different digits. What is $A + B$?

$$\begin{array}{cccc} & A & B & A \\ \times & & C & D \\ \hline C & D & C & D \end{array}$$

A 1

B 2

C 3

D 4

E 9

Solution(s):

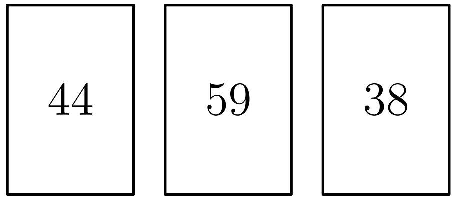
Note that

$$CDCD = 101 \cdot CD.$$

This forces $ABA = 101$. Then $A = 1$, $B = 0$, and $A + B = 1$.

Thus, **A** is the correct answer.

25. Barry wrote 6 different numbers, one on each side of 3 cards, and laid the cards on a table, as shown. The sums of the two numbers on each of the three cards are equal. The three numbers on the hidden sides are prime numbers. What is the average of the hidden prime numbers?



A 13

B 14

C 15

D 16

E 17

Solution(s):

Note that if we add an odd number to 59, we will get an even number. To make the other two sums even numbers, the primes on the back of 44 and 38 must both be even.

They also have to be different, however. Since there is only one even prime, the number on the back of 59 must be an even prime, namely 2.

This means that the sum of the front and back of all the cards is $59 + 2 = 61$. This makes the other two primes $61 - 44 = 17$ and $61 - 38 = 23$.

The average of the primes is therefore

$$\frac{2 + 17 + 23}{3} = \frac{42}{3} = 14.$$

Thus, **B** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/2006