

2001 AMC 8 Solutions

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1. Casey's shop class is making a golf trophy. He has to paint 300 dimples on a golf ball. If it takes him 2 seconds to paint one dimple, how many minutes will he need to do his job?

A 4

B 6

C 8

D 10

E 12

Solution(s):

It will take case $2 \cdot 300 = 600$ seconds to do the job. This is the same as $600 \div 60 = 10$ minutes.

Thus, **D** is the correct answer.

2. I'm thinking of two whole numbers. Their product is 24 and their sum is 11. What is the larger number?

A 3

B 4

C 6

D 8

E 12

Solution(s):

Let the numbers be x and y . Then we know that $xy = 24$ and $x + y = 11$.

From this, we get that $x = 11 - y$, and substituting yields

$$\begin{aligned}(11 - y)y &= 24 \\ y^2 - 11y + 24 &= 0 \\ (y - 3)(y - 8) &= 0.\end{aligned}$$

This means that y is either 3 or 8, and x is either 8 or 3 respectively. The larger number is 8.

Thus, **D** is the correct answer.

3. Granny Smith has \$63. Elberta has \$2 more than Anjou and Anjou has one-third as much as Granny Smith. How many dollars does Elberta have?

A 17

B 18

C 19

D 21

E 23

Solution(s):

Anjou has $\$63 \div 3 = \21 . This means that Elberta has $\$21 + \$2 = \$23$.

Thus, **E** is the correct answer.

4. The digits 1, 2, 3, 4, and 9 are each used once to form the smallest possible **even** five-digit number. The digit in the tens place is

A 1

B 2

C 3

D 4

E 9

Solution(s):

To get a smaller number, we want the larger digits to appear towards the right of the number.

This means that the units digit is 4 (it has to be either 2 or 4 since the number is even).

The next largest number is 9, which then has to go in the tens place.

Thus, **E** is the correct answer.

5. On a dark and stormy night Snoopy suddenly saw a flash of lightning. Ten seconds later he heard the sound of thunder. The speed of sound is 1088 feet per second and one mile is 5280 feet. Estimate, to the nearest half-mile, how far Snoopy was from the flash of lightning.

A 1

B $1\frac{1}{2}$

C 2

D $2\frac{1}{2}$

E 3

Solution(s):

In 10 seconds, the thunder was able to travel $10 \cdot 1088 = 10880$ feet.

Note that 2 miles is about $2 \cdot 5280 = 10560$ feet, which is close to what we found above.

Thus, **C** is the correct answer.

6. Six trees are equally spaced along one side of a straight road. The distance from the first tree to the fourth is 60 feet. What is the distance in feet between the first and last trees?

A 90

B 100

C 105

D 120

E 140

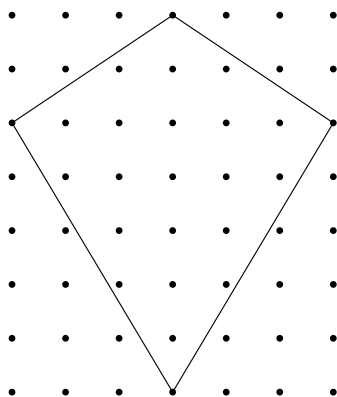
Solution(s):

There are 3 gaps between the first and fourth trees, which means that one gap is $60 \div 3 = 20$ feet long.

There are 5 gaps between the first and last trees. This means that they are $5 \cdot 20 = 100$ feet apart.

Thus, **B** is the correct answer.

7. To promote her school's annual Kite Olympics, Genevieve makes a small kite and a large kite for a bulletin board display. The kites look like the one in the diagram. For her small kite Genevieve draws the kite on a one-inch grid. For the large kite she triples both the height and width of the entire grid.



What is the number of square inches in the area of the small kite?

A 21

B 22

C 23

D 24

E 25

Solution(s):

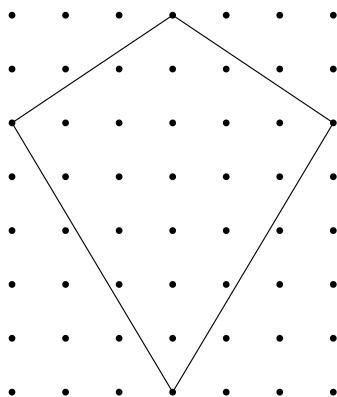
Recall that the area of a kite is the product of its diagonals divided by 2.

Therefore, the area of the small kite is

$$\frac{6 \cdot 7}{2} = \frac{42}{2} = 21.$$

Thus, **A** is the correct answer.

8. To promote her school's annual Kite Olympics, Genevieve makes a small kite and a large kite for a bulletin board display. The kites look like the one in the diagram. For her small kite Genevieve draws the kite on a one-inch grid. For the large kite she triples both the height and width of the entire grid.



Genevieve puts bracing on her large kite in the form of a cross connecting opposite corners of the kite. How many inches of bracing material does she need?

- A 30
- B 32
- C 35
- D 38
- E 39

Solution(s):

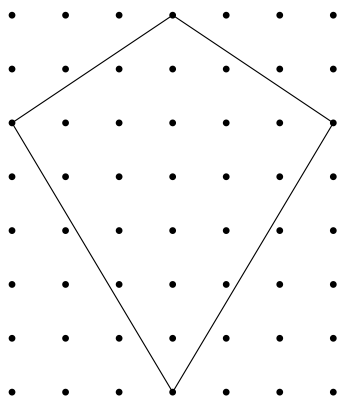
The long diagonal is 7 units, and the short one is 6 units.

In the large kite, one unit is 3 inches, so the total amount of bracing material needed is

$$3(7 + 6) = 3 \cdot 13 = 39 \text{ in.}$$

Thus, **E** is the correct answer.

9. To promote her school's annual Kite Olympics, Genevieve makes a small kite and a large kite for a bulletin board display. The kites look like the one in the diagram. For her small kite Genevieve draws the kite on a one-inch grid. For the large kite she triples both the height and width of the entire grid.



The large kite is covered with gold foil. The foil is cut from a rectangular piece that just covers the entire grid. How many square inches of waste material are cut off from the four corners?

- A 63
- B 72
- C 180
- D 189
- E 264

Solution(s):

The area of the entire grid would be

$$3 \cdot 7 \cdot 3 \cdot 6 = 378.$$

The area of the kite is one-half this area from the formula for the area of the kite, so the wasted material is $378 \div 2 = 189$.

Thus, **D** is the correct answer.

10. A collector offers to buy state quarters for 2000% of their face value. At that rate how much will Bryden get for his four state quarters?

A \$20

B \$50

C \$200

D \$500

E \$2000

Solution(s):

Four state quarters is the same as on \$1.

2000% of this is

$$\$1 \cdot \frac{2000}{100} = \$1 \cdot 20 = \$20.$$

Thus, **A** is the correct answer.

11. Points A , B , C and D have these coordinates: $A(3, 2)$, $B(3, -2)$, $C(-3, -2)$ and $D(-3, 0)$. What is the area of quadrilateral $ABCD$?

A 12

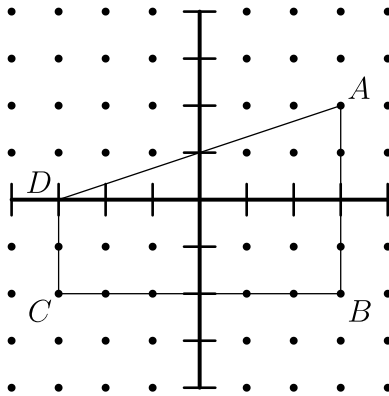
B 15

C 18

D 21

E 24

Solution(s):



We can see that $ABCD$ is a trapezoid since $\overline{DC} \parallel \overline{AB}$.

Recall that the formula for the area of a trapezoid is

$$A = \frac{1}{2}(b_1 + b_2)h.$$

Plugging in

$$b_1 = DC = 2,$$

$$b_2 = AB = 4,$$

and

$$h = CB = 6,$$

we get that

$$A = \frac{1}{2}(2 + 4) \cdot 6$$
$$= \frac{1}{2} \cdot 6 \cdot 6 = 18.$$

Thus, **C** is the correct answer.

12. If

$$a \otimes b = \frac{a + b}{a - b},$$

then

$$(6 \otimes 4) \otimes 3 =$$

A 4

B 13

C 15

D 30

E 72

Solution(s):

We can evaluate it as follows

$$(6 \otimes 4) \otimes 3 = \frac{6 + 4}{6 - 4} \otimes 3$$
$$= 5 \otimes 3$$
$$= \frac{5 + 3}{5 - 3}$$
$$= 4.$$

Thus, **A** is the correct answer.

13. Of the 36 students in Richelle's class, 12 prefer chocolate pie, 8 prefer apple, and 6 prefer blueberry. Half of the remaining students prefer cherry pie and half prefer lemon. For Richelle's pie graph showing this data, how many degrees should she use for cherry pie?

A 10

B 20

C 30

D 50

E 72

Solution(s):

The number of students that prefer cherry or lemon pie is

$$36 - 12 - 8 - 6 = 10.$$

Half of these like cherry pie, which is $10 \div 2 = 5$.

The number of degrees for cherry pie would then be

$$360^\circ \cdot \frac{5}{36} = 5 \cdot 10^\circ = 50^\circ.$$

Thus, **D** is the correct answer.

14. Tyler has entered a buffet line in which he chooses one kind of meat, two different vegetables and one dessert. If the order of food items is not important, how many different meals might he choose?

Meat: beef, chicken, pork

Vegetables: baked beans, corn, potatoes, tomatoes

Dessert: brownies, chocolate cake, chocolate pudding, ice cream

- A 4
- B 24
- C 72
- D 80
- E 144

Solution(s):

He has 3 choices for the meat and 4 for the dessert.

He has 4 options for the first vegetable and 3 for the second. Since order doesn't matter the number of options for the vegetables is

$$\frac{4 \cdot 3}{2} = \frac{12}{2} = 6.$$

This gives him a total of

$$3 \cdot 6 \cdot 4 = 72$$

combinations of meals.

Thus, **C** is the correct answer.

15. Homer began peeling a pile of 44 potatoes at the rate of 3 potatoes per minute. Four minutes later Christen joined him and peeled at the rate of 5 potatoes per minute. When they finished, how many potatoes had Christen peeled?

A 20

B 24

C 32

D 33

E 40

Solution(s):

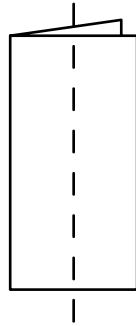
Homer had peeled $3 \cdot 4 = 12$ potatoes by the time Christen joined him, leaving $44 - 12 = 32$ potatoes.

Together, Homer and Christen peel $5 + 3 = 8$ potatoes per minute, taking them $32 \div 8 = 4$ minutes to peel the rest.

In 4 minutes, Christen peeled $4 \cdot 5 = 20$ potatoes.

Thus, **A** is the correct answer.

16. A square piece of paper, 4 inches on a side, is folded in half vertically. Both layers are then cut in half parallel to the fold. Three new rectangles are formed, a large one and two small ones. What is the ratio of the perimeter of one of the small rectangles to the perimeter of the large rectangle?



A $\frac{1}{3}$

B $\frac{1}{2}$

C $\frac{3}{4}$

D $\frac{4}{5}$

E $\frac{5}{6}$

Solution(s):

The square is folded in half to create a 4×2 rectangle.

Cutting the folded paper in half yields 2 smaller 4×1 rectangles and one 4×2 rectangle.

The ratio of the perimeter of the small to rectangle to that of the large rectangle is therefore

$$\frac{2(4 + 1)}{2(4 + 2)} = \frac{5}{6}.$$

Thus, **E** is the correct answer.

17. For the game show *Who Wants To Be a Millionaire?*, the dollar values of each question are shown in the following table (where K = 1000).

Question	1	2	3		
Value	100	200	300		
	4	5	6	7	8
	500	1K	2K	4K	8K
	9	10	11	12	
	16K	32K	64K	125K	
	13	14	15		
	250K	500K	1000K		

Between which two questions is the percent increase of the value the smallest?

- A From 1 to 2
- B From 2 to 3
- C From 3 to 4
- D From 11 to 12
- E From 14 to 15

Solution(s):

Note that most of the increases are doubling. The only exceptions are 2 to 3, 3 to 4, and 11 to 12.

11 to 12 is close to doubling, so we can ignore that. 2 to 3 is a

$$\frac{300 - 200}{200} = \frac{1}{2} = 50\%$$

increase. 3 to 4 is a

$$\frac{500 - 300}{300} = \frac{2}{3} \approx 67\%$$

increase.

Thus, **B** is the correct answer.

18. Two dice are thrown. What is the probability that the product of the two numbers is a multiple of 5?

A $\frac{1}{36}$

B $\frac{1}{18}$

C $\frac{1}{6}$

D $\frac{11}{36}$

E $\frac{1}{3}$

Solution(s):

The only way for the product to be a multiple of 5 is if at least one of the rolls is 5.

We can do complementary counting. The probability that neither rolls is a 5 is

$$\frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}.$$

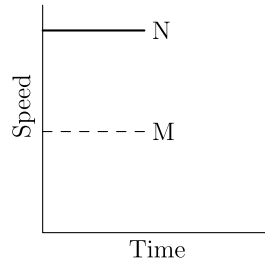
Therefore, the probability that at least one roll is a 5 is

$$1 - \frac{25}{36} = \frac{11}{36}.$$

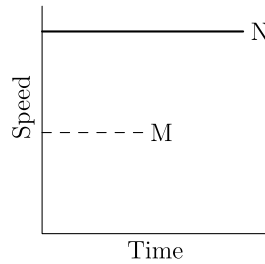
Thus, **D** is the correct answer.

19. Car M traveled at a constant speed for a given time. This is shown by the dashed line. Car N traveled at twice the speed for the same distance. If Car N 's speed and time are shown as solid line, which graph illustrates this?

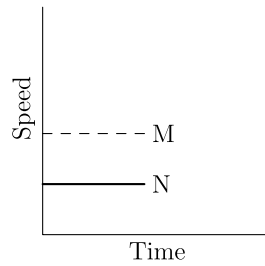
A



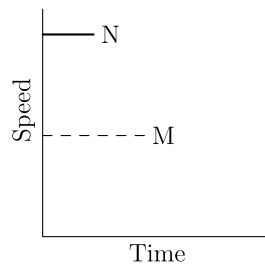
B



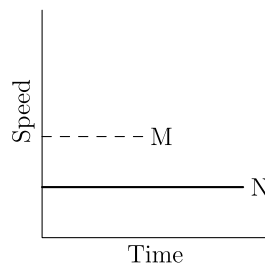
C



D



E



Solution(s):

Car N travels at twice the speed of car M , so it is represented by a point that is twice as high on the speed axis as car M .

Since car N travels at the same distance as car M but at a faster speed, it takes half the time to complete the trip.

Therefore, the line representing car N 's speed is half the length of the line representing car M 's speed.

Both lines are horizontal because the speeds are constant. Examining the given graphs, the only one that meets these conditions is graph **D**.

Thus, **D** is the correct answer.

- 20.** Kaleana shows her test score to Quay, Marty and Shana, but the others keep theirs hidden. Quay thinks, "At least two of us have the same score." Marty thinks, "I didn't get the lowest score." Shana thinks, "I didn't get the highest score." List the scores from lowest to highest for Marty (M), Quay (Q) and Shana (S).

A S,Q,M

B Q,M,S

C Q,S,M

D M,S,Q

E S,M,Q

Solution(s):

Since Quay only knows Kaleana's score, we get that Quay and Kaleana have the same score, so $K = Q$.

Marty knows that his score is higher than Kaleana's, so $M > K$. Shana knows that her score is lower than Kaleana's, so $S < K$.

Substituting Q for K , we get the following inequality: $S < Q < M$.

Thus, **A** is the correct answer.

21. The mean of a set of five different positive integers is 15. The median is 18. The maximum possible value of the largest of these five integers is

A 19

B 24

C 32

D 35

E 40

Solution(s):

The median of the set of numbers is the third largest number, which is 18. There are two numbers less than 18 and two numbers greater than it.

The mean of the set is 15, so the sum of all the numbers is $5 \cdot 15 = 75$. In order to maximize the largest number with this sum, the other numbers must be as small as possible.

The two numbers less than 18 must be positive and distinct, so they must be 1 and 2.

The number immediately after 18 must also be as small as possible, so it must be 19.

Therefore, the remaining number, the maximum possible value in the set, is

$$75 - 1 - 2 - 18 - 19 = 35.$$

Thus, **D** is the correct answer.

22. On a twenty-question test, each correct answer is worth 5 points, each unanswered question is worth 1 point and each incorrect answer is worth 0 points. Which of the following scores is **NOT** possible?

A 90

B 91

C 92

D 95

E 97

Solution(s):

The highest possible score occurs if you get everything question right for a total of $20 \cdot 5 = 100$ points.

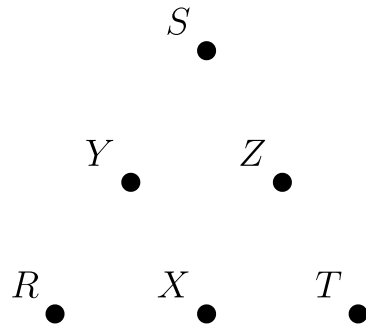
The next highest score is if there are 19 right answers and one unanswered question for a total of

$$19 \cdot 5 + 1 \cdot 1 = 96.$$

This means that any score between these 2 numbers is impossible.

Thus, **E** is the correct answer.

23. Points R , S and T are vertices of an equilateral triangle, and points X , Y and Z are midpoints of its sides. How many noncongruent triangles can be drawn using any three of these six points as vertices?



- A 1
- B 2
- C 3
- D 4
- E 20

Solution(s):

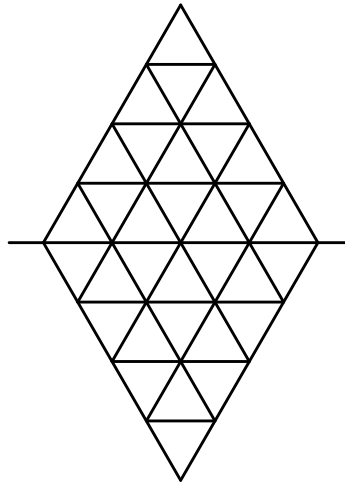
Split this triangle into half (draw a line from S to X).

Note that any noncongruent triangle except for $\triangle RST$ itself can be formed from these 4 points. Any other triangle can be reflected or rotated to get one of these triangles.

Counting the number of triangles in this half, we get 3, and adding on $\triangle RST$, we get a total of 4 triangles.

Thus, **D** is the correct answer.

24. Each half of this figure is composed of 3 red triangles, 5 blue triangles and 8 white triangles. When the upper half is folded down over the centerline, 2 pairs of red triangles coincide, as do 3 pairs of blue triangles. There are 2 red-white pairs. How many white pairs coincide?



- A 4
- B 5
- C 6
- D 7
- E 9

Solution(s):

Each half of the group has 3 red triangles, 5 blue triangles, and 8 white triangles.

2 pairs of red triangles are used, leaving 1 red triangle on each half. 3 pairs of blue triangles are used, leaving 2 blue triangles on each half.

2 pairs of red-white triangles are used, using 1 red triangle and 1 white triangle on each half (one side cannot use both red triangles since they each only have 1).

This leaves 4 pairs of blue-white triangles since any more blue-blue pairs are not allowed (otherwise there would be more than 3 pairs).

This leaves 5 white-white pairs.

Thus, **B** is the correct answer.

25. There are 24 four-digit whole numbers that use each of the four digits 2, 4, 5, and 7 exactly once. Only one of these four-digit numbers is a multiple of another one. Which of the following is it?

A 5724

B 7245

C 7254

D 7425

E 7542

Solution(s):

Let x be the number that we are looking for.

Note that x cannot be 4 times another number, since the smallest value of x would then be

$$2457 \cdot 4 = 9828,$$

which is past the range of the set.

Now, let's see if x is 3 times another number. The smallest possible value would then be

$$2457 \cdot 3 = 7371,$$

and the largest would be

$$2475 \cdot 3 = 7425,$$

since multiplying any other number in the set 3 would go past the range of the set.

We see that 7425 is indeed a number in the set.

Thus, **D** is the correct answer.

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