2000 AMC 8 Solutions

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1. Aunt Anna is 42 years old. Caitlin is 5 years younger than Brianna, and Brianna is half as old as Aunt Anna. How old is Caitlin?

A 15

B 16

C 17

D 21

E 37

Solution(s):

Brianna is $42 \div 2 = 21$ years old. Caitlin is therefore $21 - 5 = 16$ years old.

Thus, **B** is the correct answer.

2. Which of these numbers is less than its reciprocal?

A -2

B -1

C 0

D 1

E 2

Solution(s):

0 has no reciprocal, and 1 and -1 are their own reciprocals.

The reciprocal of 2 is $\frac{1}{2}$, but 2 is not less than $\frac{1}{2}$.

Therefore, as $-2 < -\frac{1}{2}$, we know that -2 is the only one of the answer choices that is less than its reciprocal.

Thus, **A** is the correct answer.

3. How many whole numbers lie in the interval between $\frac{5}{3}$ and 2π?

- A 2
- B 3
- C 4
- D 5
- E infinitely many

Solution(s):

The smallest whole number greater than $\frac{5}{3}$ is 2. The greatest whole number less than 2π is 6.

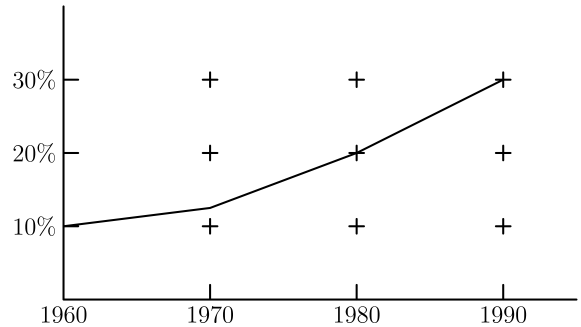
The whole numbers within this range are

$$2, 3, 4, 5, \text{ and } 6.$$

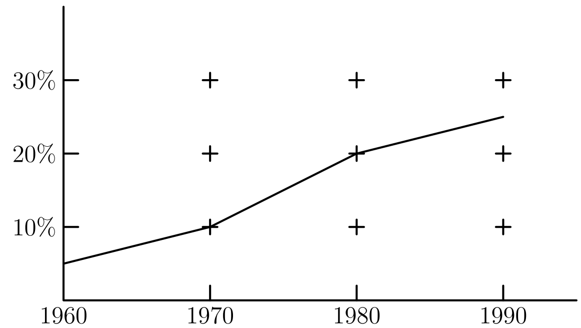
Thus, **D** is the correct answer.

4. In 1960 only 5% of the working adults in Carlin City worked at home. By 1970 the "at-home" work force had increased to 8%. In 1980 there were approximately 15% working at home, and in 1990 there were 30%. The graph that best illustrates this is

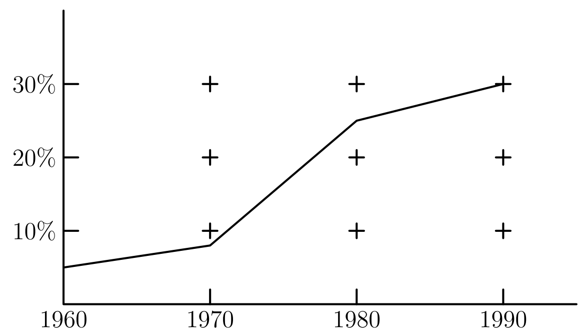
A



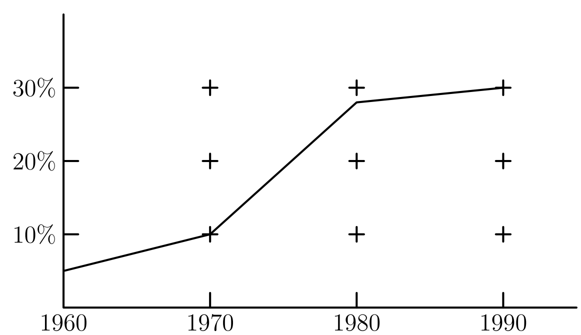
B



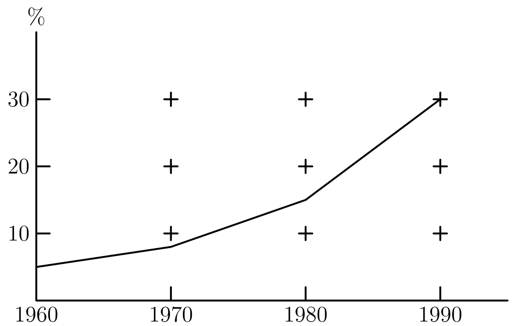
C



D



E



Solution(s):

The only graph that shows all the data points is graph **E**.

Thus, **E** is the correct answer.

5. Each principal of Lincoln High School serves exactly one 3-year term. What is the maximum number of principals this school could have during an 8-year period?

A 2

B 3

C 4

D 5

E 8

Solution(s):

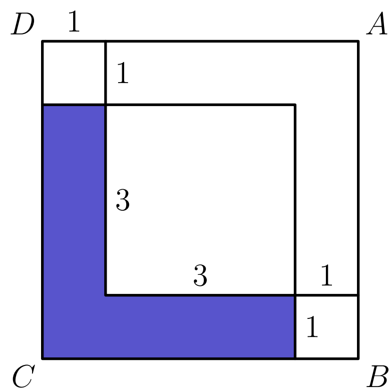
To maximize the number of principals, assume that the first year of this period is the final year of some principal's term.

Then, there can be 2 more principals for 6 years, followed by another principal who works the final year.

This is 4 principals.

Thus, **C** is the correct answer.

6. Figure $ABCD$ is a square. Inside this square three smaller squares are drawn with side lengths as labeled. The area of the shaded L-shaped region is



A 7

B 10

C 12.5

D 14

E 15

Solution(s):

We can subtract out the areas of the top unit square, the bottom right unit square, and the top right 4×4 square.

The area of $ABCD$ is therefore

$$5^2 - 2 \cdot 1^2 - 4^2 = 7.$$

Thus, **A** is the correct answer.

7. What is the minimum possible product of three different numbers of the set

$$\{-8, -6, -4, 0, 3, 5, 7\}?$$

A -336

B -280

C -210

D -192

E 0

Solution(s):

To get a negative product using three numbers, you can either multiply one negative number and two positives, or three negatives.

The only 2 viable options are

$$-8 \cdot -6 \cdot -4 = -192$$

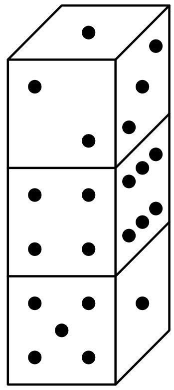
and

$$-8 \cdot 5 \cdot 7 = -280.$$

The latter is clearly the lesser value.

Thus, **B** is the correct answer.

8. Three dice with faces numbered 1 through 6 are stacked as shown. Seven of the eighteen faces are visible, leaving eleven faces hidden (back, bottom, between). The total number of dots NOT visible in this view is



A 21

B 22

C 31

D 41

E 53

Solution(s):

The sum of the numbers on one die is

$$1+2+3+4+5+6=21.$$

Therefore, the sum of the numbers on all 3 dice is $3 \cdot 21=63$.

The visible numbers add up to

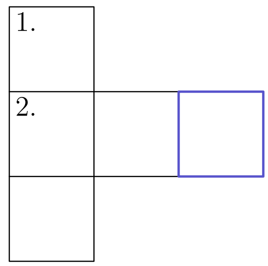
$$1+1+2+3+4+5+6=22.$$

This makes the sum of the unseen numbers $63-22=41$.

Thus, **D** is the correct answer.

9. Three-digit powers of 2 and 5 are used in this "cross-number" puzzle. What is the only possible digit for the outlined square?

Across:	**Down:**
2. 2^m	**1.** 5^n



A 0

B 2

C 4

D 6

E 8

Solution(s):

The only 3-digit powers of 5 are 125 and 625. This means that the 2 spot is filled with a 2.

The only 2-digit power beginning with a 2 is 256, so the outlined square is filled with a 6.

Thus, **D** is the correct answer.

10. Ara and Shea were once the same height. Since then Shea has grown 20% while Ara has grown half as many inches as Shea. Shea is now 60 inches tall. How tall, in inches, is Ara now?

A 48

B 51

C 52

D 54

E 55

Solution(s):

Let x be Ara and Shea's initial height. Then we get that

$$\begin{aligned} 1.2x &= 60 \\ x &= 50. \end{aligned}$$

This means that Shea grew 10 inches, which means that Ara grew $10 \div 2 = 5$ inches, making her $50 + 5 = 55$ inches tall.

Thus, **E** is the correct answer.

11. The number 64 has the property that it is divisible by its unit digit. How many whole numbers between 10 and 50 have this property?

A 15

B 16

C 17

D 18

E 20

Solution(s):

We can do casework based on the units digit x.

$x = 0:$

0 solutions since no number is divisible by 0.

$x = 1:$

4 solutions since every number is divisible by 1.

$x = 2:$

4 solutions since every number that ends in 2 is divisible by 2.

$x = 3:$

1 solution since 23 and 43 are not divisible by 3, but 33 is.

$x = 4:$

2 solutions since 24 and 44 are divisible, but 34 is not.

$x = 5:$

4 solutions since every number that ends in 5 is divisible by 5.

$x = 6:$

1 solutions since 36 is divisible, but 26 and 46 are not.

$x = 7:$

0 solutions since $27, 37$ and 47 are all not divisible.

$x = 8:$

1 solutions since 48 is divisible, but 28 and 38 are not.

$x=9:$

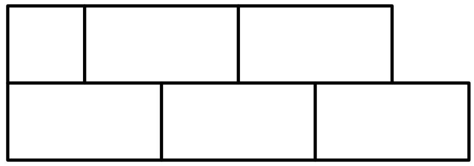
0 solutions since $29, 39$ and 49 are not divisible.

The total number of numbers is

$$3 \cdot 4+3 \cdot 1+2=17.$$

Thus, **C** is the correct answer.

12. A block wall 100 feet long and 7 feet high will be constructed using blocks that are 1 foot high and either 2 feet long or 1 foot long (no blocks may be cut). The vertical joins in the blocks must be staggered as shown, and the wall must be even on the ends. What is the smallest number of blocks needed to build this wall?



- A 344
- B 347
- C 350
- D 353
- E 356

Solution(s):

The total number of rows in the wall is 7, with each row being 1 foot high.

To use the minimum number of bricks, rows $1, 3, 5,$ and 7 will have the same pattern as the bottom row in the picture, which requires 50 bricks to construct.

Rows $2, 4,$ and 6 will have the same pattern as the upper row in the picture, which has 49 2-foot bricks in the middle and 1 1-foot bricks on each end, for a total of 51 bricks.

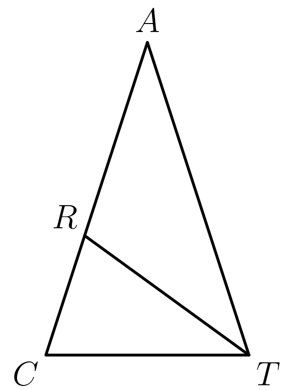
When you add up 4 rows of 50 bricks and 3 rows of 51 bricks, you get a total of

$$450 + 351 = 200 + 153 = 353$$

bricks.

Thus, **D** is the correct answer.

13. In triangle CAT, we have $\angle ACT = \angle ATC$ and $\angle CAT = 36^\circ$. If $\overline{TR}$ bisects $\angle ATC$, then $\angle CRT =$



- **A** 16°
- **B** 51°
- **C** 72°
- **D** 90°
- **E** 108°

Solution(s):

We get

$$\begin{aligned} \angle CAT + \angle ATC + \angle ACT &= 180 \\ 36 + 2 \cdot \angle ATC &= 180 \\ \angle ATC &= 72^\circ. \end{aligned}$$

Due to bisection, we also know that

$$\angle RTC = 72 \div 2 = 36^\circ.$$

Finally, we see that

$$\begin{aligned} \angle RTC + \angle TCR + \angle CRT &= 180 \\ 36 + 72 + \angle CRT &= 180 \\ \angle CRT &= 72^\circ. \end{aligned}$$

Thus, **C** is the correct answer.

14. What is the units digit of $19^{19} + 99^{99}$?

A 0

B 1

C 2

D 8

E 9

Solution(s):

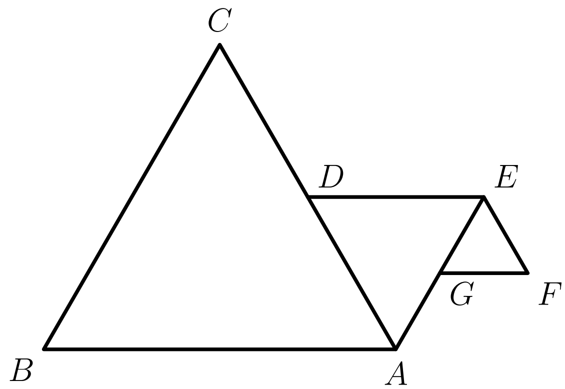
Note that the units digit of an exponent depends only upon the units digit of the base.

Experimenting, we get that 9 to even power ends with a 1 and to an odd power ends with a 9.

Therefore, 19^{19} ends with a 9 and 99^{99} also ends with a 9. Adding them together yields a number that ends in 8.

Thus, **D** is the correct answer.

15. Triangles ABC, ADE, and EFG are all equilateral. Points D and G are midpoints of $\overline{AC}$ and $\overline{AE}$, respectively. If $AB = 4$, what is the perimeter of figure $ABCDEFG$?



A 12

B 13

C 15

D 18

E 21

Solution(s):

The large equilateral triangle has side length 4, the middle one has side length 2, and the smaller one has side length 1.

The perimeter is therefore

$$\begin{gathered} CB + CD + DE + EF \\ FG + GA + AB \\ = 4 + 4 + 2 + 2 + 1 \\ +1 + 1 = 15. \end{gathered}$$

Thus, **C** is the correct answer.

16. In order for Mateen to walk a kilometer (1000m) in his rectangular backyard, he must walk the length 25 times or walk its perimeter 10 times. What is the area of Mateen's backyard in square meters?

A 40

B 200

C 400

D 500

E 1000

Solution(s):

We can see that the length is $1000 \div 25 = 40$ m, and the perimeter is $1000 \div 10 = 100$ m.

Note that the perimeter is 2 times the sum of the length and width.

This means that the width is

$$100 \div 2 - 40 = 10 \text{ m},$$

and the area is

$$40 \cdot 10 = 400 \text{ m}^2.$$

Thus, **C** is the correct answer.

17. The operation $\otimes$ is defined for all nonzero numbers by

$$a \otimes b = \frac{a^2}{b}.$$

Determine

$$[(1 \otimes 2) \otimes 3] - [1 \otimes (2 \otimes 3)].$$

A $-\frac{2}{3}$

B $-\frac{1}{4}$

C 0

D $\frac{1}{4}$

E $\frac{2}{3}$

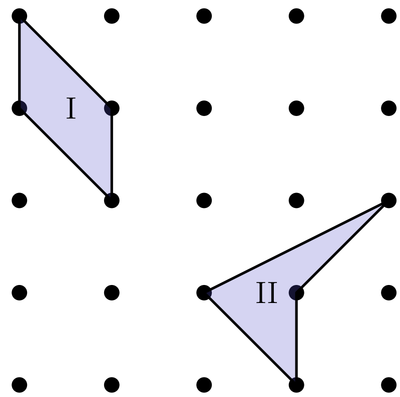
Solution(s):

We can calculate it as follows.

$$\begin{aligned}
&[(1 \otimes 2) \otimes 3] - [1 \otimes (2 \otimes 3)] \\
&= [\frac{1^2}{2} \otimes 3] - [1 \otimes \frac{2^2}{3}] \\
&= [\frac{1}{2} \otimes 3] - [1 \otimes \frac{4}{3}] \\
&= \frac{(1/2)^2}{3} - \frac{1^2}{(4/3)} \\
&= \frac{1}{4} \cdot \frac{1}{3} - \frac{3}{4} \\
&= \frac{1}{12} - \frac{3}{4} \\
&= -\frac{2}{3}.
\end{aligned}$$

Thus, **A** is the correct answer.

18. Consider these two geoboard quadrilaterals. Which of the following statements is true?



A The area of quadrilateral I is more than the area of quadrilateral II.

B The area of quadrilateral I is less than the area of quadrilateral II.

C The quadrilaterals have the same area and the same perimeter.

D The quadrilaterals have the same area, but the perimeter of I is more than the perimeter of II.

E The quadrilaterals have the same area, but the perimeter of I is less than the perimeter of II.

Solution(s):

Assume that the pegs on this grid are separated by 1 unit.

Note that region I is a parallelogram with base 1 and height 1, makings its area $1 \cdot 1 = 1$.

We can split region II into 2 triangles. Both with base 1 and height 1. This makes the sum of the areas

$$2 \cdot \frac{1}{2} \cdot 1 \cdot 1 = 1.$$

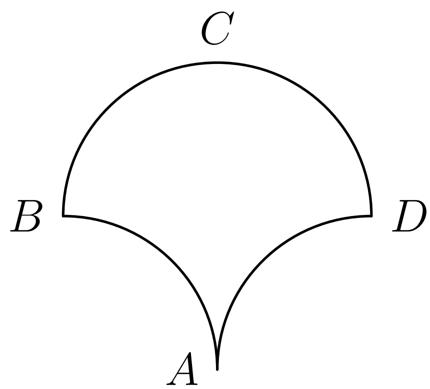
This shows that both regions have the same area.

Note that each region has 2 sides that are of length $\sqrt{2}$. Region I has 2 unit sides, whereas region II only has 1.

The other side of region II is clearly greater than 1, which shows that region II has the greater perimeter.

Thus, **E** is the correct answer.

19. Three circular arcs of radius 5 units bound the region shown. Arcs AB and AD are quarter-circles, and arc BCD is a semicircle. What is the area, in square units, of the region?



A 25

B $10+5\pi$

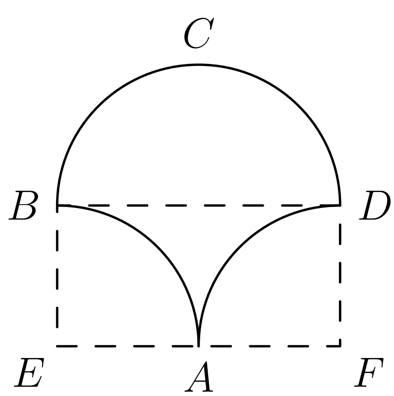
C 50

D $50+5\pi$

E 25π

Solution(s):

Create a rectangle that covers the bottom half of the figure as shown below.



Then, we get that

$$[ABCD] = [ABD] + [BCD].$$

We also know that

$$[ABD] = [BDEF] - [ABE]$$

$$-[ADF].$$

ABE and ADF are both quartercircles that form a semicircle with the same area as CBD.

This means that

$$[ABD] = [BDEF] - [BCD]$$

and

$$[ABCD] = [BCD] + [BDEF]$$

$$-[BCD]$$

$$= [BDEF] = 10 \cdot 5 = 50.$$

Thus, **C** is the correct answer.

20. You have nine coins: a collection of pennies, nickels, dimes, and quarters having a total value of $\$1.02$, with at least one coin of each type. How many dimes must you have?

- A 1
- B 2
- C 3
- D 4
- E 5

Solution(s):

Since we know that we have one coin of each type, we have

$$1 + 5 + 10 + 25 = 41$$

cents already accounted for. We need to figure out what makes up the remaining $102 - 41 = 61$ cents.

Note that we have 5 coins left. This means that we only have one penny, since otherwise we would need 6 pennies.

Now we have 4 coins for 60 cents.

Note that we need at least one quarter, since otherwise the maximum we could make is 40 cents with 4 dimes.

One quarter leaves 35 cents, which cannot be accomplished with 3 coins (3 dimes would only achieve 30 cents).

This means that there are 2 quarters, leaving 10 cents. We can see that 2 nickels can make this amount.

Therefore, as only the first one was used, we must only have one dime.

Thus, **A** is the correct answer.

21. Keiko tosses one penny and Ephraim tosses two pennies. The probability that Ephraim gets the same number of heads that Keiko gets is

A $\frac{1}{4}$

B $\frac{3}{8}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

Solution(s):

They can each either get one or zero heads.

The probability that Keiko gets one head is $\frac{1}{2}$. The probability that Ephraim gets one head is

$$\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

(heads then tails or tails than heads).

This means the probability that they both get one head is

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

The probably that Keiko gets zero heads is $\frac{1}{2}$. The probability that Ephraim gets zero heads is if both flips are tails:

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Therefore, the probability that both flip zero heads is

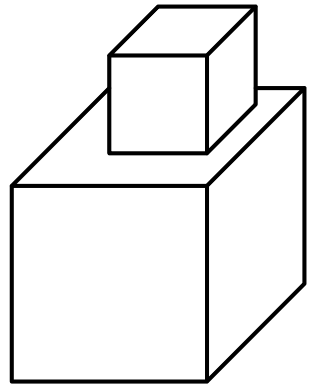
$$\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}.$$

Adding up both scenarios yields a total probability of

$$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}.$$

Thus, **B** is the correct answer.

22. A cube has edge length 2. Suppose that we glue a cube of edge length 1 on top of the big cube so that one of its faces rests entirely on the top face of the larger cube. The percent increase in the surface area (sides, top, and bottom) from the original cube to the new solid formed is closest to



- A 10
- B 15
- C 17
- D 21
- E 25

Solution(s):

The original surface area is just

$$6 \cdot 2^2 = 6 \cdot 4 = 24.$$

Note that the top face of the unit cube plus the visible area of the top face of the larger cube is the same as the area of one face of the larger cube.

This means that the unit square on top only adds 4 unit squares to the total surface area, making the increase 4.

The percent increase is therefore

$$100 \cdot \frac{4}{24} = 100 \cdot \frac{1}{6} \approx 16.7\%.$$

Thus, **C** is the correct answer.

23. There is a list of seven numbers. The average of the first four numbers is 5, and the average of the last four numbers is 8. If the average of all seven numbers is $6\frac{4}{7}$, then the number common to both sets of four numbers is

A $5\frac{3}{7}$

B 6

C $6\frac{4}{7}$

D 7

E $7\frac{3}{7}$

Solution(s):

The sum of the first four numbers is $4 \cdot 5 = 20$. The sum of the last four numbers is $4 \cdot 8 = 32$.

The sum of all seven numbers is $7 \cdot 6\frac{4}{7} = 46$. We know that the number common to both sets is included in both of first two sums.

This means that the sum of the first two sums includes every number once, except for the common number which is included twice.

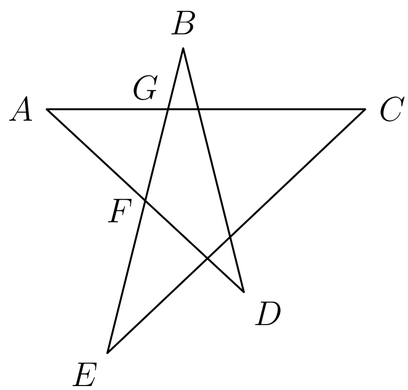
The third sum, however, only includes every number once. This means that the sum of the first two sums minus the third sum yields our desired number.

Therefore, the common number is

$$20 + 32 - 46 = 52 - 46 = 6.$$

Thus, **B** is the correct answer.

24. If $\angle A = 20^\circ$ and $\angle AFG = \angle AGF$, then $\angle B + \angle D =$



A 48°

B 60°

C 72°

D 80°

E 90°

Solution(s):

We know that

$$\begin{aligned} 180^\circ &= \angle A + \angle AFG + \angle AGF \\ &= 20^\circ + 2 \cdot \angle AFG \\ 160^\circ &= 2 \cdot AFG \\ 80^\circ &= \angle AFG. \end{aligned}$$

Now, we get that

$$\angle BFD = 180^\circ - \angle AFG$$

$$= 180^\circ - 80^\circ = 100^\circ.$$

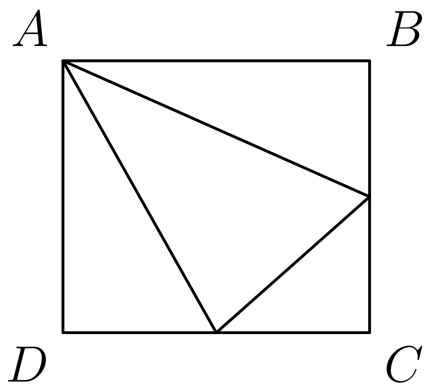
Note that $\angle B$ and $\angle D$ are two angles of $\triangle BFD$.

This means that

$$\begin{aligned} 180^\circ &= \angle BFD + \angle B + \angle D \\ \angle B + \angle D &= 180^\circ - 100^\circ = 80^\circ. \end{aligned}$$

Thus, **D** is the correct answer.

25. The area of rectangle $ABCD$ is 72 units squared. If point A and the midpoints of $\overline{BC}$ and $\overline{CD}$ are joined to form a triangle, the area of that triangle is



A 21

B 27

C 30

D 36

E 40

Solution(s):

We can find the area of the three right triangles and subtract them from the area of the rectangle to get the desired area.

The three triangles have the following areas:

$$\frac{1}{2} \cdot 9 \cdot 4 = 18,$$
$$\frac{1}{2} \cdot 8 \cdot \frac{9}{2} = 18,$$
$$\frac{1}{2} \cdot 4 \cdot \frac{9}{2} = 9.$$

The sum of these areas is

$$18 + 18 + 9 = 45.$$

The desired area is therefore

$$72 - 45 = 27.$$

Thus, **B** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/2000