1999 AMC 8 Solutions

Typeset by: LIVE, by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc8/1999/solutions

1. $(6\ ?\ 3) + 4 - (2 - 1) = 5$. To make this statement true, the question mark between the 6 and the 3 should be replaced by

- A $\div$
- B $\times$
- C $+$
- D $-$
- E None of these

Solution(s):

We have that

$$4 - (2 - 1) = 4 - 1 = 3.$$

Then

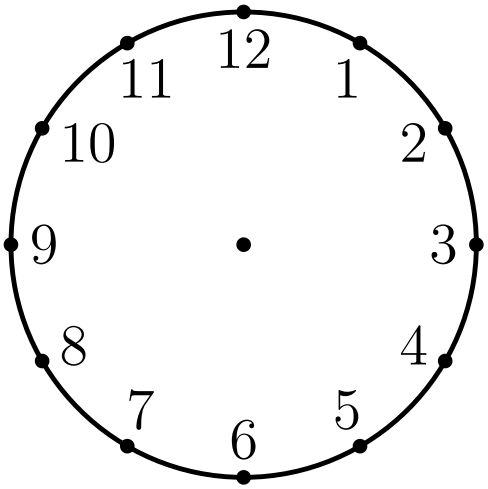
$$(6\ ?\ 3) + 3 = 5$$

$$6\ ?\ 3 = 2.$$

The only choice that works is $\div$.

Thus, **A** is the correct answer.

2. What is the degree measure of the smaller angle formed by the hands of a clock at 10 o'clock?



A 30

B 45

C 60

D 75

E 90

Solution(s):

At 10 o'clock, we have that the hour hand is at 10 and the minute hand is at 12.

This means that the hands are a $\frac{1}{6}$ of the entire circle apart. This is equal to

$$360^\circ \cdot \frac{1}{6} = 60^\circ.$$

Thus, **C** is the correct answer.

3. Which triplet of numbers has a sum NOT equal to 1?

A $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$

B $(2, -2, 1)$

C $(0.1, 0.3, 0.6)$

D $(1.1, -2.1, 1.0)$

E $(-\frac{3}{2}, -\frac{5}{2}, 5)$

Solution(s):

We have that **A** is the same as

$$\frac{3}{6} + \frac{2}{6} + \frac{1}{6} = 1.$$

B reduces to

$$2 - 2 + 1 = 1.$$

C adds up to

$$.1 + .3 + .6 = 1.$$

D however adds to

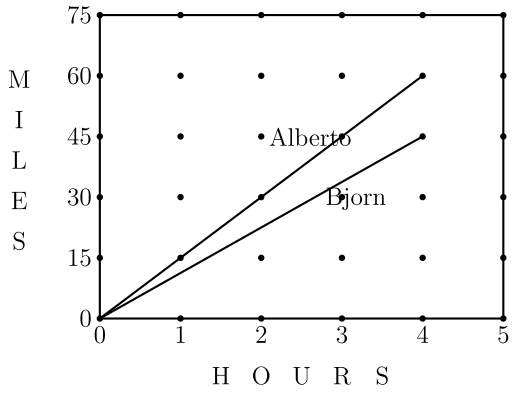
$$1.1 - 2.1 + 1.0 = 0.$$

To make sure, we check that **E** has a sum of

$$-\frac{8}{2} + 5 = 5 - 4 = 1.$$

Thus, **D** is the correct answer.

4. The diagram shows the miles traveled by bikers Alberto and Bjorn. After four hours, about how many more miles has Alberto biked than Bjorn?



A 15

B 20

C 25

D 30

E 35

Solution(s):

Looking at the graph, we have that Alberto has biked 60 miles and Bjorn 45 miles after 4 hours.

The difference between the two distances is $60 - 45 = 15$ miles.

Thus, **A** is the correct answer.

5. A rectangular garden 50 feet long and 10 feet wide is enclosed by a fence. To make the garden larger, while using the same fence, its shape is changed to a square. By how many square feet does this enlarge the garden?

A 100

B 200

C 300

D 400

E 500

Solution(s):

The current length of the fence is

$$2(50+10) = 2 \cdot 60 = 120$$

feet. If all the sides become the same, then each side length is $120 \div 4 = 30$ feet. As such, the area of the square is then $30^2 = 900$ square feet. Note that the original area of the garden is $50 \cdot 10 = 500$ square feet.

Therefore, the difference is $900 - 500 = 400$ square feet.

Thus, **D** is the correct answer.

6. Bo, Coe, Flo, Jo, and Moe have different amounts of money. Neither Jo nor Bo has as much money as Flo. Both Bo and Coe have more than Moe. Jo has more than Moe, but less than Bo. Who has the least amount of money?

A Bo

B Coe

C Flo

D Jo

E Moe

Solution(s):

Let B, C, F, J, and M represent the amount of money that Bo, Coe, Flo, Jo, and Moe have respectively.

Then

$$J < F \text{ and } B < F.$$

We also have

$$B > M \text{ and } C > M.$$

Finally, we are given

$$M < J < B.$$

From the first and second inequalities, we can get that $F > M$.

This shows that everyone has more money than Moe.

Thus, **E** is the correct answer.

7. The third exit on a highway is located at milepost 40 and the tenth exit is at milepost 160. There is a service center on the highway located three-fourths of the way from the third exit to the tenth exit. At what milepost would you expect to find this service center?

A 90

B 100

C 110

D 120

E 130

Solution(s):

There are $160 - 40 = 120$ mileposts between the third and tenth exits.

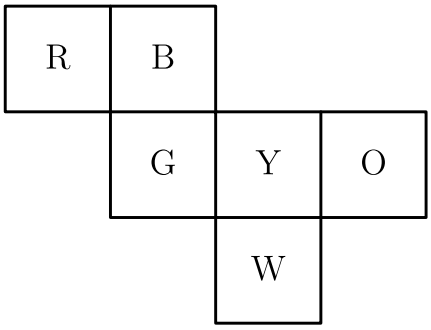
This means $\frac{3}{4}$ of the way is at the

$$40 + \frac{3}{4} \cdot 120 = 130$$

milepost.

Thus, **E** is the correct answer.

8. Six squares are colored, front and back, (R = red, B = blue, O = orange, Y = yellow, G = green, and W = white). They are hinged together as shown, then folded to form a cube. The face opposite the white face is



A B

B G

C O

D R

E Y

Solution(s):

Consider the cube with the yellow face facing upwards.

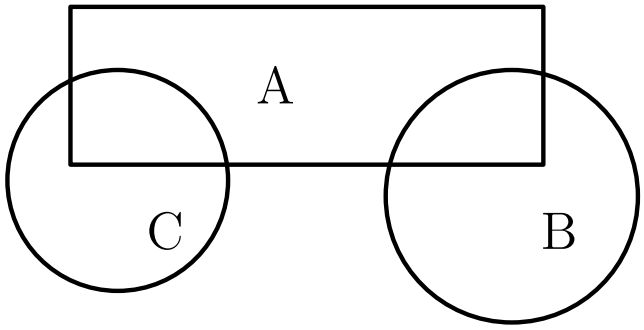
Then we can fold the white, green, and orange faces down.

Since the blue face is attached to the green face, it will end up folding backwards.

This means that the blue face will end up facing opposite the white face.

Thus, **A** is the correct answer.

9. Three flower beds overlap as shown. Bed A has 500 plants, bed B has 450 plants, and bed C has 350 plants. Beds A and B share 50 plants, while beds A and C share 100. The total number of plants is



A 850

B 1000

C 1150

D 1300

E 1450

Solution(s):

Note there are $50 + 100 = 150$ plants that are in two beds and there are no plants in all three beds.

The total number of plants is then

$$500 + 450 + 350 - 150 = 1150.$$

We subtract to get rid of the plants that we counted twice.

Thus, **C** is the correct answer.

10. A complete cycle of a traffic light takes 60 seconds. During each cycle the light is green for 25 seconds, yellow for 5 seconds, and red for 30 seconds. At a randomly chosen time, what is the probability that the light will NOT be green?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{5}{12}$

D $\frac{1}{2}$

E $\frac{7}{12}$

Solution(s):

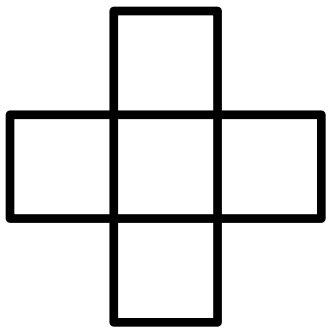
During a given cycle, the light is not green for $30 + 5 = 35$ seconds.

Then the probability that it is not green is

$$\frac{35}{60} = \frac{7}{12}.$$

Thus, **E** is the correct answer.

11. Each of the five numbers $1, 4, 7, 10$, and 13 is placed in one of the five squares so that the sum of the three numbers in the horizontal row equals the sum of the three numbers in the vertical column. The largest possible value for the horizontal or vertical sum is



A 20

B 21

C 22

D 24

E 30

Solution(s):

Let x be the number in the middle. Then the sum of the horizontal and vertical sums is

$$1 + 4 + 7 + 10 + 13 + x,$$

since x is counted twice. To maximize this, we want to maximize x, so we let $x = 13$.

Then, from the above, expression, we have that the largest possible value of the sum of the horizontal and vertical sums is $35 + 13 = 48$.

The sum of one direction is then $48 \div 2 = 24$.

Thus, **D** is the correct answer.

12. The ratio of the number of games won to the number of games lost (no ties) by the Middle School Middies is $11/4$. To the nearest whole percent, what percent of its games did the team lose?

A 24%

B 27%

C 36%

D 45%

E 73%

Solution(s):

The total number of games is $11 + 4 = 15$, so the percent of games lost is

$$\frac{4}{15} \cdot 100\% = \frac{4 \cdot 20}{3}\% \approx 27\%.$$

Thus, **B** is the correct answer

13. The average age of the 40 members of a computer science camp is 17 years. There are 20 girls, 15 boys, and 5 adults. If the average age of the girls is 15 and the average age of the boys is 16, what is the average age of the adults?

A 26

B 27

C 28

D 29

E 30

Solution(s):

The sum of the ages of everybody at the camp is $40 \cdot 17 = 680$.

The sum of the ages of the girls is $20 \cdot 15 = 300$ and of the boys is $15 \cdot 16 = 240$.

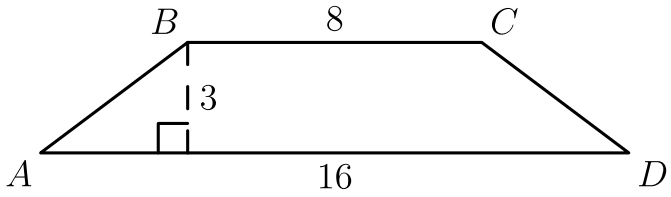
As such, we know that the ages of the adults must be $680 - 300 - 240 = 140$.

And therefore, the average age of the adults is then

$$140 \div 5 = 28.$$

Thus, **C** is the correct answer.

14. In trapezoid $ABCD$, the sides AB and CD are equal. The perimeter of $ABCD$ is



A 27

B 30

C 32

D 34

E 48

Solution(s):

Let H be where the altitude from B to $\overline{AD}$ intersects $\overline{AD}$.

Then we have that

$$AH = \frac{16-8}{2} = 4,$$

since $AB = CD$.

We then have that

$$AB = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Then the perimeter is

$$8 + 16 + 2 \cdot 5 = 34.$$

Thus, **D** is the correct answer.

15. Bicycle license plates in Flatville each contain three letters. The first is chosen from the set $\{C, H, L, P, R\}$, the second from $\{A, I, O\}$, and the third from $\{D, M, N, T\}$.

When Flatville needed more license plates, they added two new letters. The new letters may both be added to one set or one letter may be added to one set and one to another set. What is the largest possible number of *additional* license plates that can be made by adding two letters?

A 24

B 30

C 36

D 40

E 60

Solution(s):

There are currently

$$5 \cdot 3 \cdot 4 = 60$$

license plates that can be made.

If both letters are added to the first set, then there are

$$7 \cdot 3 \cdot 4 = 84$$

possible plates.

If they are both added to the second, there are

$$5 \cdot 5 \cdot 4 = 100$$

plates.

If they are added to the third, there are

$$5 \cdot 3 \cdot 6 = 90$$

choices.

If one is added to the first set and the other to the second set, there are

$$6 \cdot 4 \cdot 4 = 96$$

plates.

If the other is added to the third set, we get

$$6 \cdot 3 \cdot 5 = 90$$

possible plates.

Finally, if the letters are added to the second and third sets, there are

$$5 \cdot 4 \cdot 5 = 100$$

plates.

We see that 100 is the greatest number of plates that we can achieve. This is an additional $100 - 60 = 40$ plates.

Thus, **D** is the correct answer.

16. Tori's mathematics test had 75 problems: 10 arithmetic, 30 algebra, and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of the algebra, and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% of the problems right.

How many more problems would she have needed to answer correctly to earn a 60% passing grade?

A 1

B 5

C 7

D 9

E 11

Solution(s):

She answered $10 \cdot .7 = 7$ arithmetic questions correctly. She answered $30 \cdot .4 = 12$ algebra ones correctly. She also got $35 \cdot .6 = 21$ geometry questions correctly.

This means she got a total of $7 + 12 + 21 = 40$ questions correct.

As such, in order to get a 60%, Tori must have answered

$$75 \cdot .6 = 45$$

questions correctly. This means that she would have needed to answer an additional $45 - 40 = 5$ questions correctly.

Thus, **B** is the correct answer.

17. Problems $17, 18$, and 19 refer to the following:

Cookies For a Crowd

At Central Middle School the 108 students who take the AMC $\rightarrow 8$ meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookies this year. Their recipe, which makes a pan of 15 cookies, lists these items: $1\frac{1}{2}$ cups of flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipes.

Walter can buy eggs by the half-dozen. How many half-dozens should he buy to make enough cookies? (Some eggs and some cookies may be left over.)

A 1

B 2

C 5

D 7

E 15

Solution(s):

Since the students eat an average of 2 cookies each, they will eat a total of $108 \cdot 2 = 216$ cookies.

Each recipe makes 15 cookies, which means we need

$$\left\lceil \frac{216}{15} \right\rceil = 15$$

full recipes to make enough cookies.

Each pan requires 2 eggs, which means we need $15 \cdot 2 = 30$ eggs. There are 6 eggs in a half-dozen, so we need $30 \div 6 = 5$ half-dozens.

Thus, **C** is the correct answer.

18. They learn that a big concert is scheduled for the same night and attendance will be down 25%. How many recipes of cookies should they make for their smaller party?

A 6

B 8

C 9

D 10

E 11

Solution(s):

There will now only be

$$108 \cdot \left(1 - \frac{1}{4}\right) = 108 \cdot \frac{3}{4} = 81$$

people at the party. This means we need

$$\left\lceil \frac{81 \cdot 2}{15} \right\rceil = \left\lceil \frac{162}{15} \right\rceil = 11$$

recipes.

Thus, **E** is the correct answer.

19. The drummer gets sick. The concert is cancelled. Walter and Gretel must make enough pans of cookies to supply 216 cookies. There are 8 tablespoons in a stick of butter. How many sticks of butter will be needed? (Some butter may be left over, of course.)

A 5

B 6

C 7

D 8

E 9

Solution(s):

To make 216 cookies, they have to make

$$\left\lceil \frac{216}{15} \right\rceil = 15$$

pans. Since each pan requires 3 tablespoons of butter, all the pans will need $15 \cdot 3 = 45$ tablespoons.

They will then need

$$\left\lceil \frac{45}{8} \right\rceil = 6$$

sticks of butter.

Thus, **B** is the correct answer.

20. Figure 1 is called a "stack map." The numbers tell how many cubes are stacked in each position. Fig. 2 shows these cubes, and Fig. 3 shows the view of the stacked cubes as seen from the front.

Which of the following is the front view for the stack map in Fig. 4?

3	4
2	1

Figure 1

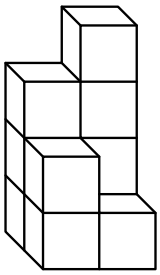


Figure 2

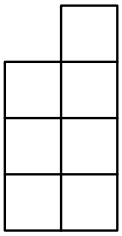
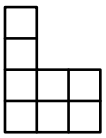


Figure 3

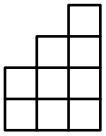
2	2	4
1	3	1

Figure 4

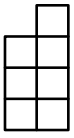
A



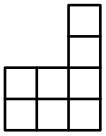
B



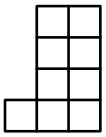
C



D



E



Solution(s):

Note that the height of the column is the maximum height of the front and back columns.

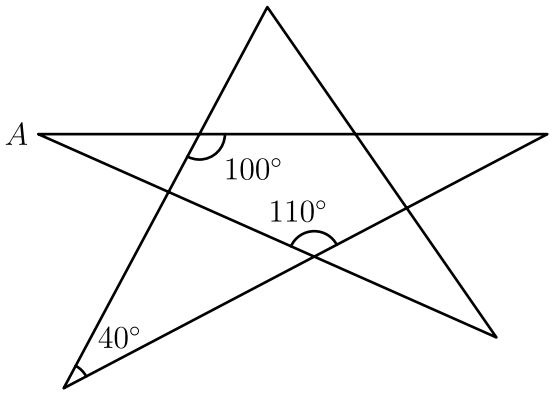
In the left column, we have that the back column is taller with height 2.

In the middle column, we have that the front column is taller with height 3.

Finally, the right column has height 4.

Thus, **B** is the correct answer.

21. The degree measure of angle A is



A 20

B 30

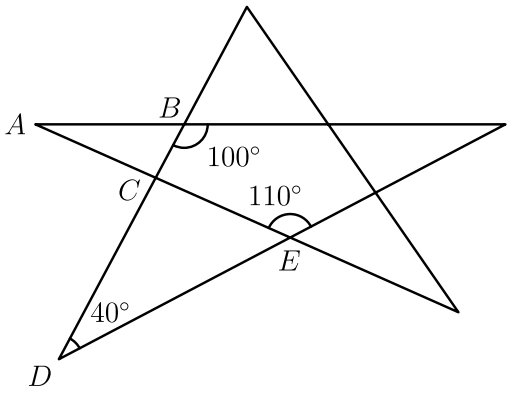
C 35

D 40

E 45

Solution(s):

Label the vertices as below.



We then have that

$$\angle ABC = 180^\circ - 100^\circ = 80^\circ$$

by supplementary angles. Then we have

$$\angle CED = 180^\circ - 110^\circ = 70^\circ$$

again by supplementary angles. Using the sum of the interior angles of a triangle is 180°, we get

$$\begin{aligned}\angle ECD &= 180^\circ - 70^\circ - 40^\circ \\ &= 70^\circ.\end{aligned}$$

Then, using vertical angles, we have

$$\angle ACB = \angle ECD = 70^\circ.$$

Finally, using the sum of the interior angles of a triangle, we get

$$\angle A = 180^\circ - 80^\circ - 70^\circ = 30^\circ.$$

Thus, **B** is the correct answer.

22. In a far-off land three fish can be traded for two loaves of bread and a loaf of bread can be traded for four bags of rice. How many bags of rice is one fish worth?

A $\frac{3}{8}$

B $\frac{1}{2}$

C $\frac{3}{4}$

D $2\frac{2}{3}$

E $3\frac{1}{3}$

Solution(s):

Let f be the worth of a fish, l for a loaf of bread, and r for rice.

Then we are given

$$3f = 2l \text{ and } l = 4r.$$

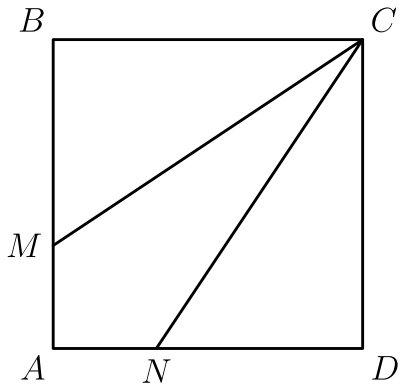
Substituting, we get

$$3f = 2 \cdot 4r = 8r$$

$$f = \frac{8}{3}r.$$

Thus, **D** is the correct answer.

23. Square $ABCD$ has sides of length 3. Segments CM and CN divide the square's area into three equal parts. How long is segment CM?



A $\sqrt{10}$

B $\sqrt{12}$

C $\sqrt{13}$

D $\sqrt{14}$

E $\sqrt{15}$

Solution(s):

The area of the square is $3^2 = 9$, which means that the area of one region is $9 \div 3 = 3$.

This means the area of $\triangle BMC$ is 3, which means that

$$\frac{1}{2} \cdot 3 \cdot BM = 3$$

$$BM = 2.$$

Since $\triangle BMC$ is right, we have

$$CM = \sqrt{2^2 + 3^2} = \sqrt{13}.$$

Thus, **C** is the correct answer.

24. When 1999^{2000} is divided by 5, the remainder is

A 4

B 3

C 2

D 1

E 0

Solution(s):

Note that to find the remainder when divided by 5, we only care about the units digit.

This means we only have to observe how the powers of the units digit work, namely the powers of 9.

Then, looking at powers of 9, we see that the units digit alternates between 9 and 1 :

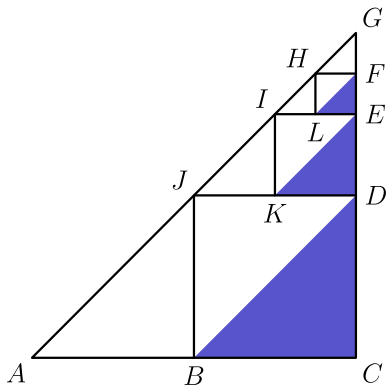
$$9, 81, 729, \cdots$$

This means that 1999^{2000} ends in a 1 since the power is even.

The remainder when divided by 5 is then 1, since it is 1 more than a multiple of 10.

Thus, **D** is the correct answer.

25. Points B, D, and J are midpoints of the sides of right triangle ACG. Points K, E, I are midpoints of the sides of triangle JDG, etc. If the dividing and shading process is done 100 times (the first three are shown) and $AC = CG = 6$, then the total area of the shaded triangles is nearest



A 6

B 7

C 8

D 9

E 10

Solution(s):

Note that all the triangles are isosceles right triangles. We can find the following side lengths:

$$CD = \frac{CG}{2} = 3 = DG,$$

$$DE = \frac{DG}{2} = \frac{3}{2} = EG,$$

and

$$EF = \frac{EG}{2} = \frac{3}{4}.$$

We then have that the areas of the 3 triangles are

$$\frac{1}{2} \cdot 3^2 = \frac{9}{2},$$

$$\frac{1}{2} \cdot \left(\frac{3}{2}\right)^2 = \frac{9}{8},$$

and

$$\frac{1}{2} \cdot \left(\frac{3}{4}\right)^2 = \frac{9}{32}.$$

The sum of these areas is

$$\frac{9}{2} + \frac{9}{8} + \frac{9}{32} = \frac{189}{32} \approx 5.9.$$

This can act as underestimate for the areas of all 100 triangles, since it does not include some areas.

Now, we have that the other 97 triangles are all contained with $\triangle FGH$.

This means that we can use the area of $\triangle FGH$ is an overestimate for all the other triangles.

The area of $\triangle FGH$ is

$$\frac{1}{2} \cdot \left(\frac{3}{4}\right)^2 = \frac{9}{32}.$$

Then

$$\frac{189}{32} + \frac{9}{32} = \frac{198}{32} \approx 6.2.$$

Then, the true value is between 5.9 and 6.2, which means that 6 is nearest it.

Thus, **A** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/1999