1998 AMC 8

Time limit: 40 minutes

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https://live.poshenloh.com/past-contests/amc8/1998

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1. For $x = 7$, which of the following is the smallest?

A $\frac{6}{x}$

B $\frac{6}{x+1}$

C $\frac{6}{x-1}$

D $\frac{x}{6}$

E $\frac{x+1}{6}$

2. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - b \cdot c$, what is the value of $\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$?

A -2

B -1

C 0

D 1

E 2

3. What is the value of the following expression?

$$\frac{\frac{3}{8}+\frac{7}{8}}{\frac{4}{5}}$$

A 1

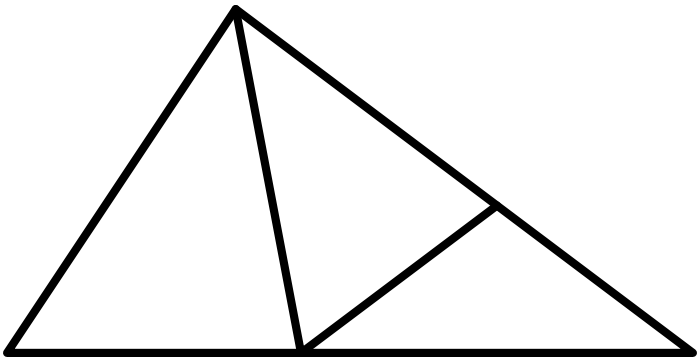
B $\frac{25}{16}$

C 2

D $\frac{43}{20}$

E $\frac{47}{16}$

4. How many triangles are in this figure? (Some triangles may overlap other triangles.)



A 9

B 8

C 7

D 6

E 5

5. Which of the following numbers is largest?

A 9.12344

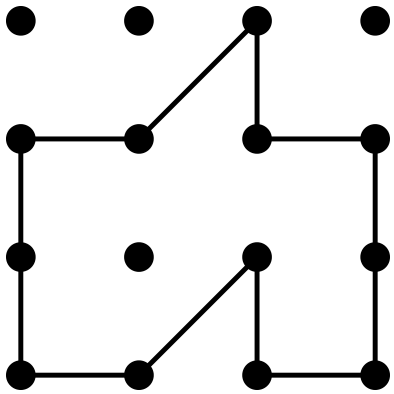
B $9.123\overline{4}$

C $9.12\overline{34}$

D $9.1\overline{234}$

E $9.\overline{1234}$

6. Dots are spaced one unit apart, horizontally and vertically. The number of square units enclosed by the polygon is



A 5

B 6

C 7

D 8

E 9

7. $100 \times 19.98 \times 1.998 \times 1000 =$

A $(1.998)^2$

B $(19.98)^2$

C $(199.8)^2$

D $(1998)^2$

E $(19980)^2$

8. A child's wading pool contains 200 gallons of water. If water evaporates at the rate of 0.5 gallons per day and no other water is added or removed, how many gallons of water will be in the pool after 30 days?

A 140

B 170

C 185

D 198.5

E 199.85

9. For a sale, a store owner reduces the price of a $\$10$ scarf by 20%. Later the price is lowered again, this time by one-half the reduced price. The price is now

A $\$2.00$

B $\$3.75$

C $\$4.00$

D $\$4.90$

E $\$6.40$

10. Each of the letters W, X, Y, and Z represents a different integer in the set $\{1, 2, 3, 4\}$, but not necessarily in that order. They satisfy

$$\frac{W}{X} - \frac{Y}{Z} = 1$$

What is the sum of W and Y?

A 3

B 4

C 5

D 6

E 7

11. Harry has 3 sisters and 5 brothers. His sister Harriet has S sisters and B brothers. What is the product of S and B?

A 8

B 10

C 12

D 15

E 18

12. What is the value of the following expression?

$$2\left(1-\frac{1}{2}\right)+3\left(1-\frac{1}{3}\right)+4\left(1-\frac{1}{4}\right)+\cdots+10\left(1-\frac{1}{10}\right)$$

A 45

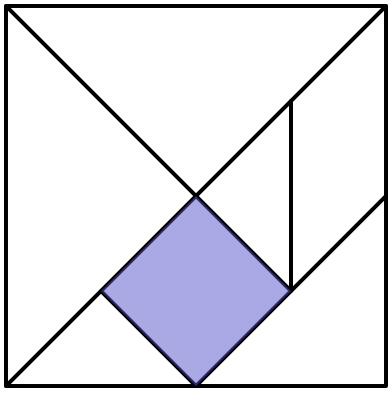
B 49

C 50

D 54

E 55

13. What is the ratio of the area of the shaded square to the area of the large square? (The figure is drawn to scale.)



A $\frac{1}{6}$

B $\frac{1}{7}$

C $\frac{1}{8}$

D $\frac{1}{12}$

E $\frac{1}{16}$

14. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

A 12

B 15

C 30

D 36

E 40

15. Problems 15, 16, and 17 all refer to the following:

Don't Crowd The Isles

In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is $24{,}900$ square miles.

Estimate the population of Nisos in the year 2050.

A 600

B 800

C 1000

D 2000

E 3000

16. Estimate the year in which the population of Nisos will be approximately 6000.

A 2050

B 2075

C 2100

D 2125

E 2150

17. In how many years, approximately, from 1998 will the population of Nisos be as much as Queen Irene has proclaimed that the islands can support?

A 50 years

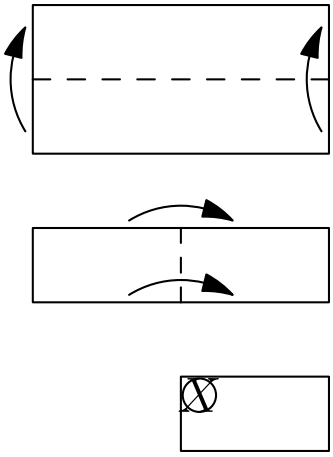
B 75 years

C 100 years

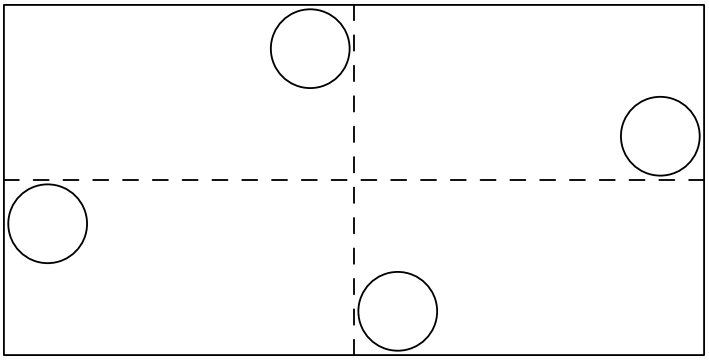
D 125 years

E 150 years

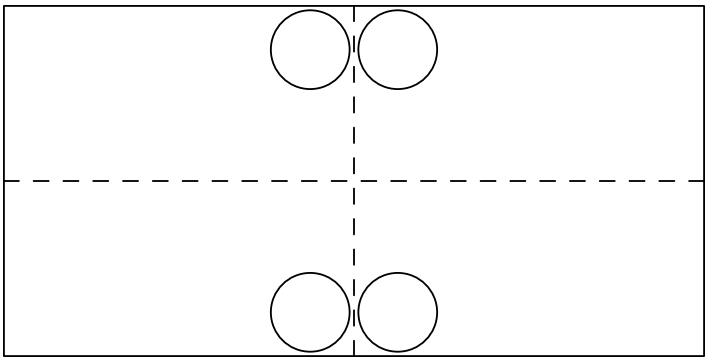
18. As indicated by the diagram below, a rectangular piece of paper is folded bottom to top, then left to right, and finally, a hole is punched at X. What does the paper look like when unfolded?



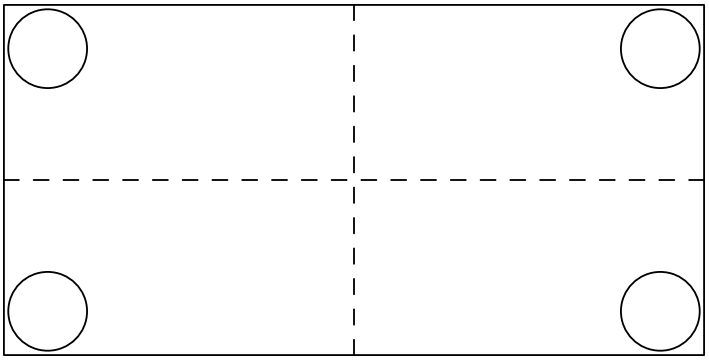
A



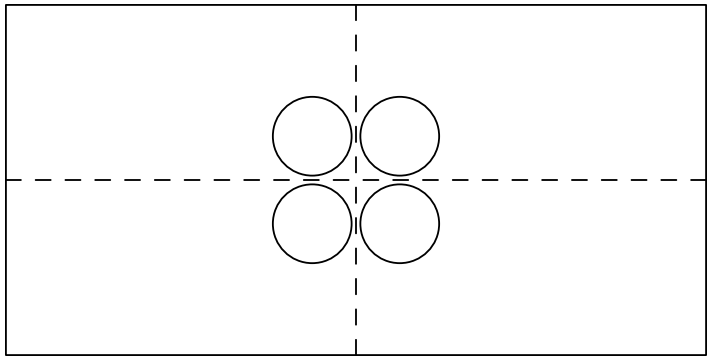
B



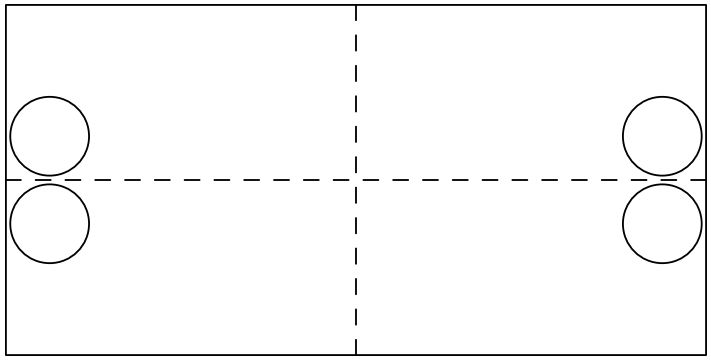
C



D



E



19. Tamika selects two different numbers at random from the set $\{8, 9, 10\}$ and adds them. Carlos takes two different numbers at random from the set $\{3, 5, 6\}$ and multiplies them. What is the probability that Tamika's result is greater than Carlos' result?

A $\frac{4}{9}$

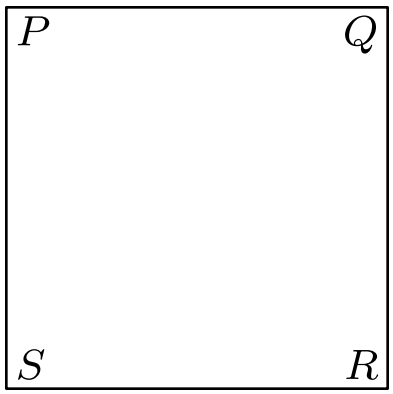
B $\frac{5}{9}$

C $\frac{1}{2}$

D $\frac{1}{3}$

E $\frac{2}{3}$

20. Let $PQRS$ be a square piece of paper. P is folded onto R and then Q is folded onto S. The area of the resulting figure is 9 square inches. Find the perimeter of square $PQRS$.



A 9

B 16

C 18

D 24

E 36

21. A $4 \times 4 \times 4$ cubical box contains 64 identical small cubes that exactly fill the box. How many of these small cubes touch a side or the bottom of the box?

A 48

B 52

C 60

D 64

E 80

22. Terri produces a sequence of positive integers by following three rules. She starts with a positive integer, then applies the appropriate rule to the result, and continues in this fashion.

Rule 1 : If the integer is less than 10, multiply it by 9.

Rule 2 : If the integer is even and greater than 9, divide it by 2.

Rule 3 : If the integer is odd and greater than 9, subtract 5 from it.

For example, consider the sample sequence $23, 18, 9, 81, 76, \ldots$.

Find the 98th term of the sequence that begins $98, 49, \ldots$.

A 6

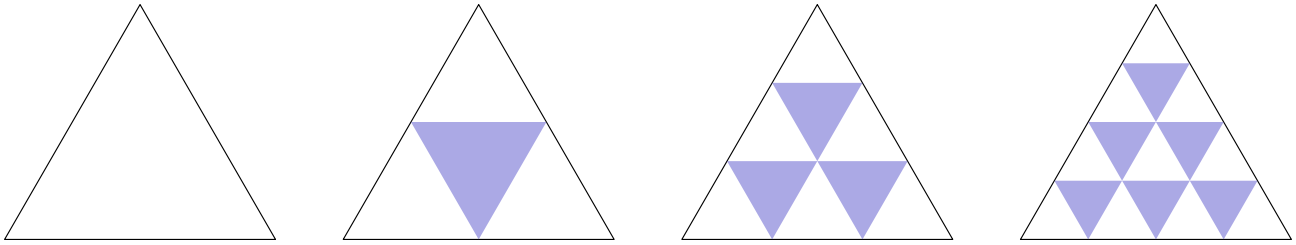
B 11

C 22

D 27

E 54

23. If the pattern in the diagram continues, what fraction of the interior would be shaded in the eighth triangle?



A $\frac{3}{8}$

B $\frac{5}{27}$

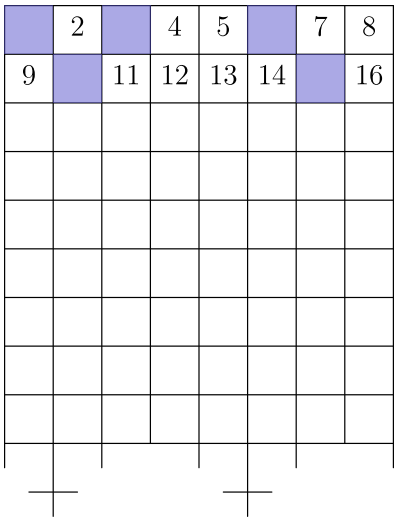
C $\frac{7}{16}$

D $\frac{9}{16}$

E $\frac{11}{45}$

24. A rectangular board of 8 columns has squares numbered beginning in the upper left corner and moving left to right so row one is numbered 1 through 8, row two is 9 through 16, and so on. A student shades square 1, then skips one square and shades square 3, skips two squares and shades square 6, skips 3 squares and shades square 10, and continues in this way until there is at least one shaded square in each column.

What is the number of the shaded square that first achieves this result?



A 36

B 64

C 78

D 91

E 120

25. Three generous friends, each with some cash, redistribute their money as follows: Ami gives enough money to Jan and Toy to double the amount that each has. Jan then gives enough to Ami and Toy to double their amounts. Finally, Toy gives Ami and Jan enough to double their amounts. If Toy has $\$36$ when they begin and $\$36$ when they end, what is the total amount that all three friends have?

A $\$108$

B $\$180$

C $\$216$

D $\$252$

E $\$288$

Solutions: https://live.poshenloh.com/past-contests/amc8/1998/solutions