1997 AMC 8 Solutions

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1.

$$\frac{1}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{7}{10000} =$$

A 0.0026

B 0.0197

C 0.1997

D 0.26

E 1.997

Solution(s):

Converting all the fractions to decimals, we get

$$.1 + .09 + .009 + .0007 = .1997.$$

Thus, **C** is the correct answer.

2. Ahn chooses a two-digit integer, subtracts it from 200, and doubles the result. What is the largest number Ahn can get?

A 200

B 202

C 220

D 380

E 398

Solution(s):

To get the largest number, we would want to subtract the smallest number possible from 200.

The smallest two-digit number is 10. Having Ahn choose this number will give us

$$2(200 - 10) = 2 \cdot 190 = 380$$

as the final result.

Thus, **D** is the correct answer.

3. Which of the following numbers is the largest?

- A 0.97
- B 0.979
- C 0.9709
- D 0.907
- E 0.9089

Solution(s):

We have that all the tenths place digits are the same, so we then look at the hundredths digit.

The largest hundredths digit is 7, so we can limit the answer choices to the ones with this value.

The largest thousandths digit is 9, which is achieved by $.979$.

Thus, **B** is the correct answer.

4. Julie is preparing a speech for her class. Her speech must last between one-half hour and three-quarters of an hour. The ideal rate of speech is 150 words per minute. If Julie speaks at the ideal rate, which of the following number of words would be an appropriate length for her speech?

A 2250

B 3000

C 4200

D 4350

E 5650

Solution(s):

One-half hour is 30 minutes, in which Julie can speak

$$150 \cdot 30 = 4500$$

words. Three-quarters of an hour is 45 minutes, in which Julie can speak

$$150 \cdot 45 = 6750$$

words. The only answer choice in between these two values is 5650.

Thus, **E** is the correct answer.

5. There are many two-digit multiples of 7, but only two of the multiples have a digit sum of 10. The sum of these two multiples of 7 is

- A 119
- B 126
- C 140
- D 175
- E 189

Solution(s):

Listing out all the two-digit multiples of 7, we get

$$14, 21, 28, 35, 42, 49, 56, 63, 70,$$

$$77, 84, 91, \text{ and } 98.$$

We have that 28 and 91 are the two multiples whose digits add to 10.

The sum of these numbers is $28 + 91 = 119$.

Thus, **A** is the correct answer.

6. In the number 74982.1035 the value of the *place* occupied by the digit 9 is how many times as great as the value of the *place* occupied by the digit 3?

A $1,000$

B $10,000$

C $100,000$

D $1,000,000$

E $10,000,000$

Solution(s):

Note that each digit has ten times the value of the digit to its right.

The *place* occupied by 9 is 5 spaces to the right of the *place* occupied by 3.

This means that it is

$$10^5 = 100,000$$

times as a great.

Thus, **C** is the correct answer.

7. The area of the smallest square that will contain a circle of radius 4 is

- A 8
- B 16
- C 32
- D 64
- E 128

Solution(s):

We can inscribe the circle inside the square so that it is tangent to the midpoints of each side of the square.

This means that the side length of the square is two times the radius of the circle, making it $2 \cdot 4 = 8$.

Then the area of the square is

$$8^2 = 64.$$

Thus, **D** is the correct answer.

8. Walter gets up at $6:30$ a.m., catches the school bus at $7:30$ a.m., has 6 classes that last 50 minutes each, has 30 minutes for lunch, and has 2 hours additional time at school. He takes the bus home and arrives at $4:00$ p.m. How many minutes has he spent on the bus?

A 30

B 60

C 75

D 90

E 120

Solution(s):

There are 8.5 hours between the time Walter catches the school bus and arrives at home.

This is a total of

$$60 \cdot 8.5 = 510$$

minutes.

The total time Walter spends at school is

$$6 \cdot 50 + 30 + 2 \cdot 60 =$$

$$300 + 30 + 120 = 450$$

minutes.

This means that Walter spends

$$510 - 450 = 60$$

minutes on the bus.

Thus, **B** is the correct answer.

9. Three students, with different names, line up single file. What is the probability that they are in alphabetical order from front-to-back?

A $\frac{1}{12}$

B $\frac{1}{9}$

C $\frac{1}{6}$

D $\frac{1}{3}$

E $\frac{2}{3}$

Solution(s):

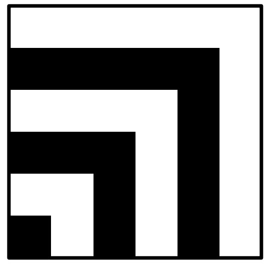
There are 3 options for the person in front. Then, there are 2 options for the person in the middle.

This leaves 1 choice for the person at the end. There are $3 \cdot 2 \cdot 1 = 6$ ways for the people to line up.

Only one of these lines is correct, and the probability it occurs is $\frac{1}{6}$.

Thus, **C** is the correct answer.

10. What fraction of this square region is shaded? Stripes are equal in width, and the figure is drawn to scale.



A $\frac{5}{12}$

B $\frac{1}{2}$

C $\frac{7}{12}$

D $\frac{2}{3}$

E $\frac{5}{6}$

Solution(s):

We can split the square into $6^2 = 36$ unit squares. The number of black squares is

$$3 + 7 + 11 = 21.$$

The fraction of the square that is shaded is

$$\frac{21}{36} = \frac{7}{12}.$$

Thus, **C** is the correct answer.

11. Let $\boxed{N}$ mean the number of whole number divisors of N. For example, $\boxed{3} = 2$ because 3 has two divisors, 1 and 3. Find the value of

$$\boxed{\boxed{11} \times \boxed{20}}.$$

A 6

B 8

C 12

D 16

E 24

Solution(s):

We know that 11 is prime, which means that it only has 2 divisors.

The prime factorization of 20 is

$$2^2 \cdot 5.$$

Recall that the number of divisors a number has is the product of all the exponents plus one in the prime factorization.

Here, that product would be

$$(2+1)(1+1) = 3 \cdot 2 = 6.$$

Then $2 \cdot 6 = 12$. We have the prime factorization of 12 is

$$2^2 \cdot 3.$$

This also has

$$(2+1)(1+1) = 3 \cdot 2 = 6$$

divisors.

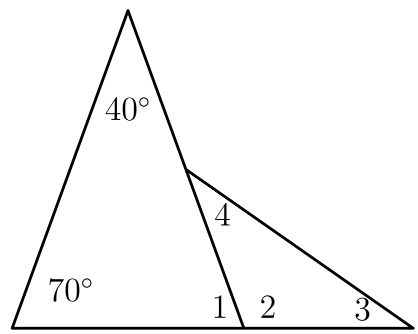
Thus, **A** is the correct answer.

12.

$$\angle 1 + \angle 2 = 180^\circ$$

$$\angle 3 = \angle 4$$

Find $\angle 4$



A 20°

B 25°

C 30°

D 35°

E 40°

Solution(s):

We have that

$$\angle 1 = 180^\circ - 70^\circ - 40^\circ = 70^\circ$$

using the fact that the interior angles of a triangle add to 180°.

This tells us that

$$\angle 2 = 180^\circ - \angle 1 = 110^\circ$$

since $\angle 1$ and $\angle 2$ are supplementary.

Finally,

$$\angle 3 + \angle 4 + 110^\circ = 180^\circ \Rightarrow$$

$$2\angle 4 = 70^\circ \Rightarrow \angle 4 = 35^\circ.$$

Thus, **D** is the correct answer.

13. Three bags of jelly beans contain $26, 28$, and 30 beans. The ratios of yellow beans to all beans in each of these bags are 50%, 25%, and 20%, respectively. All three bags of candy are dumped into one bowl. Which of the following is closest to the ratio of yellow jelly beans to all beans in the bowl?

A 31%

B 32%

C 33%

D 35%

E 95%

Solution(s):

There are

$$26 \cdot .5 = 26 \div 2 = 13$$

yellow jelly beans in the first bag,

$$28 \cdot .25 = 28 \div 4 = 7$$

yellow jelly beans in the second bag, and

$$30 \cdot .2 = 30 \div 5 = 6$$

yellow jelly beans in the third bag.

The total number of yellow jelly beans is

$$13 + 7 + 6 = 26$$

and the total number of jelly beans is

$$26 + 28 + 30 = 84.$$

The ratio of yellow jelly beans to all the beans is

$$\frac{26}{84} \cdot 100\% = \frac{13}{42} \cdot 100\%$$

$$\approx 30.9\%.$$

Thus, **A** is the correct answer.

14. There is a set of five positive integers whose average (mean) is 5, whose median is 5, and whose only mode is 8. What is the difference between the largest and smallest integers in the set?

A 3

B 5

C 6

D 7

E 8

Solution(s):

The sum of all the numbers in the list is $5 \cdot 5 = 25$.

The only mode is 8, which means that there are guaranteed two 8 s.

The sum of the numbers is up to

$$2 \cdot 8 + 5 = 16 + 5 = 21.$$

The other two numbers must then add to 4. They are both positive and not equal, leaving the only possible two numbers as

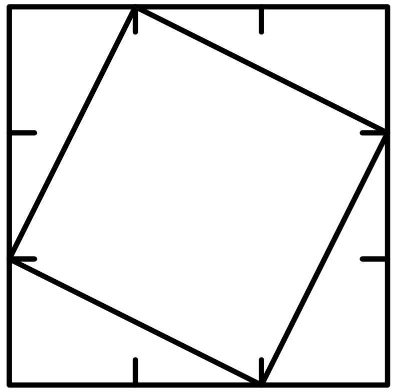
$$1 \text{ and } 3.$$

The desired difference is then

$$8 - 1 = 7.$$

Thus, **D** is the correct answer.

15. Each side of the large square in the figure is trisected (divided into three equal parts). The corners of an inscribed square are at these trisection points, as shown. The ratio of the area of the inscribed square to the area of the large square is



A $\frac{\sqrt{3}}{3}$

B $\frac{5}{9}$

C $\frac{2}{3}$

D $\frac{\sqrt{5}}{3}$

E $\frac{7}{9}$

Solution(s):

Let $3x$ be the side length of the large square. Then we can find the side length of the inner square via

$$\sqrt{(2x)^2+x^2}=\sqrt{5x^2}=x\sqrt{5}$$

from the Pythagorean Theorem.

The area of the larger square is

$$(3x)^2=9x^2$$

and that of the inner square is

$$(x\sqrt{5})^2 = 5x^2.$$

The ratio of the areas is then $\dfrac{5}{9}$.

Thus, **B** is the correct answer.

16. Penni Precisely buys $\$100$ worth of stock in each of three companies: Alabama Almonds, Boston Beans, and California Cauliflower. After one year, AA was up 20%, BB was down 25%, and CC was unchanged. For the second year, AA was down 20% from the previous year, BB was up 25% from the previous year, and CC was unchanged. If $A, B,$ and C are the final values of the stock, then

- A $A = B = C$
- B $A = B < C$
- C $C < B = A$
- D $A < B < C$
- E $B < A < C$

Solution(s):

After the first year, AA's stock's worth goes up to

$$\$100 \cdot 1.2 = \$120$$

and BB's goes down to

$$\$100 \cdot .75 = \$75.$$

After the second year, AA's stock is worth

$$\$120 \cdot .8 = \$96$$

and BB's is worth

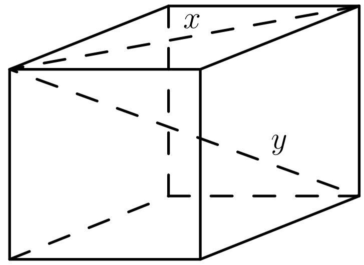
$$\$75 \cdot 1.25 = \$93.75.$$

CC's stock worth remains the same, so the ordering of the stock worths is now

$$B < A < C.$$

Thus, **E** is the correct answer.

17. A cube has eight vertices (corners) and twelve edges. A segment, such as x, which joins two vertices not joined by an edge is called a diagonal. Segment y is also a diagonal. How many diagonals does a cube have?



- A 6
- B 8
- C 12
- D 14
- E 16

Solution(s):

Each face has two diagonals connecting each of the two pairs of opposite vertices.

Also, for each vertex, there is one corresponding vertex that lies opposite it on the cube.

There are then $8 \div 2 = 4$ interior space diagonals in the cube.

The total number of diagonals is then

$$6 \cdot 2 + 4 = 12 + 4 = 16.$$

Thus, **E** is the correct answer.

18. At the grocery store last week, small boxes of facial tissue were priced at 4 boxes for $\$5$. This week they are on sale at 5 boxes for $\$4$. The percent decrease in the price per box during the sale was closest to

- A 30%
- B 35%
- C 40%
- D 45%
- E 65%

Solution(s):

Originally, each box is worth

$$\$5 \div 4 = \$1.25.$$

Now, each box is worth

$$\$4 \div 5 = \$.8.$$

The percent decrease is then

$$\frac{1.25 - .8}{1.25} \cdot 100\% = \frac{45}{125} \cdot 100\%$$
$$= \frac{9}{25} \cdot 100\% = 36\%.$$

Thus, **B** is the correct answer.

19. If the product

$$\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \frac{6}{5} \cdot \ldots \cdot \frac{a}{b} = 9,$$

what is the sum of a and b?

A 11

B 13

C 17

D 35

E 37

Solution(s):

Note that the numerator of each fraction cancels with the denominator of the fraction to its right.

We can then cancel out all these terms to get a final equation of

$$\frac{a}{2} = 9 \Rightarrow a = 18 \Rightarrow b = 17.$$

The desired sum is then

$$18 + 17 = 35.$$

Thus, **D** is the correct answer.

20. A pair of 8-sided dice have sides numbered 1 through 8. Each side has the same probability (chance) of landing face up. The probability that the product of the two numbers that land face-up exceeds 36 is

- **A** $\frac{5}{32}$
- **B** $\frac{11}{64}$
- **C** $\frac{3}{16}$
- **D** $\frac{1}{4}$
- **E** $\frac{1}{2}$

Solution(s):

We can case on the value of the first die. If its value is $1-4$, then it is impossible for the product be greater than 36.

If it is a 5, then the other dice has to roll an 8, otherwise the product is less than 36.

If the first die is a 6 or 7, then other die has to be at least a 6, giving $2 \cdot 3 = 6$ possibilities.

Finally, if the first roll is an 8, the other die must roll at least a 5, giving us 3 more possibilities.

The total number of working pairs is

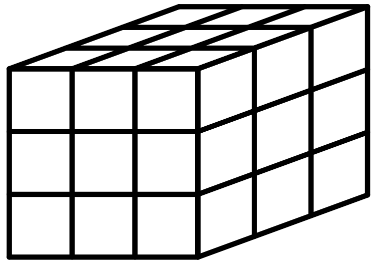
$$1 + 6 + 3 = 10,$$

and the total number of pairs is $8^2 = 64$. The desired probability is then

$$\frac{10}{64} = \frac{5}{32}.$$

Thus, **A** is the correct answer.

21. Each corner cube is removed from this $3\text{ cm} \times 3\text{ cm} \times 3\text{ cm}$ cube. The surface area of the remaining figure is



A 19 sq.cm

B 24 sq.cm

C 30 sq.cm

D 54 sq.cm

E 72 sq.cm

Solution(s):

Note that there is one unit cube between any pair of corner cubes, so the removal of each does not affect the others.

When we remove a corner, we are losing three unit squares. We, however, gain these back from the three faces that get uncovered.

This means that removing a corner cube does not change the surface area. The surface area of the original cube is

$$6 \cdot 3^2 = 6 \cdot 9 = 54$$

square centimeters.

Thus, **D** is the correct answer.

22. A two-inch cube $(2 \times 2 \times 2)$ of silver weighs 3 pounds and is worth $\$200$. How much is a three-inch cube of silver worth?

A $\$300$

B $\$375$

C $\$450$

D $\$560$

E $\$675$

Solution(s):

The two-inch cube consists of $2^3 = 8$ unit cubes. Each of these unit cubes is worth

$$200 \div 8 = 25$$

dollars. To form a three-inch cube, you need $3^3 = 27$ unit cubes. This means that it is worth

$$25 \cdot 27 = 675$$

dollars.

Thus, **E** is the correct answer.

23. There are positive integers that have these properties:

I. the sum of the squares of their digits is 50, and

II. each digit is larger than the one to its left.

The product of the digits of the largest integer with both properties is

A 7

B 25

C 36

D 48

E 60

Solution(s):

Note that if the number has 5 digits, then the sum of the squares of the digits is at least

$$1^2+2^2+3^2+4^2+5^2=55,$$

which violates the first condition.

This means that we should aim to find the largest four digit number that satisfies the properties.

Let the four digit number be $abcd$ where

$$0<a<b<c<d.$$

We must have that $d<8$ since otherwise

$$d\geq 8 \Rightarrow d^2\geq 64>50.$$

If $d=7$, then

$$a^2+b^2+c^2\geq 1^2+2^2+3^2=14,$$

which means that the sum of the digits is greater than 50.

Then if $d=6$, we have that $a=1$, $b=2$, and $c=3$ work. If $d=5$, then

$$a^2 + b^2 + c^2 = 25.$$

The only option is $(0, 4, 5)$ which causes us to end up with a 3-digit number.

If $d = 4$, then

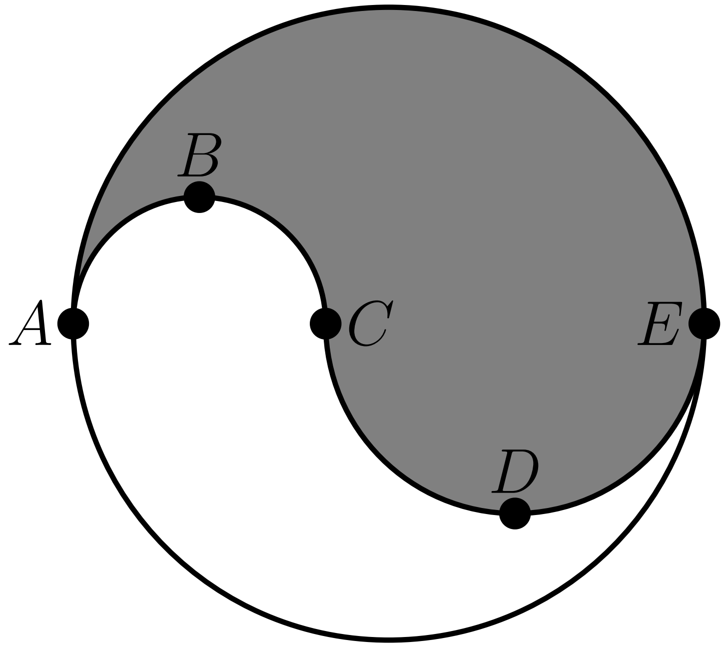
$$a^2 + b^2 + c^2 + d^2 = 14,$$

which does not satisfy the first condition. The only viable option is then 1236. The product of the digits is then

$$1 \cdot 2 \cdot 3 \cdot 6 = 36.$$

Thus, **C** is the correct answer.

24. Diameter ACE is divided at C in the ratio $2:3$. The two semicircles, ABC and CDE, divide the circular region into an upper (shaded) region and a lower region. The ratio of the area of the upper region to that of the lower region is



- A $2:3$
- B $1:1$
- C $3:2$
- D $9:4$
- E $5:2$

Solution(s):

WLOG, let $AE = 10$. Then

$$AC = \frac{2}{5} \cdot 10 = 4$$

and

$$CE = \frac{3}{5} \cdot 10 = 6.$$

Then the area of the semicircle ABC is

$$\frac{1}{2} \cdot \pi(4 \div 2)^2 = 2\pi.$$

We also have that the area of semicircle CDE is

$$\frac{1}{2} \cdot \pi(6 \div 2)^2 = \frac{9}{2}\pi.$$

Then the area of the upper shaded region is

$$\frac{1}{2} \cdot (10 \div 2)^2 - 2\pi + \frac{9}{2}\pi = 15\pi.$$

Subtracting this from the area of the total circle gives us that the area of the lower region is

$$\pi 5^2 - 15\pi = 10\pi.$$

The desired ratio is then

$$\frac{15\pi}{10\pi} = \frac{3}{2} = 3 : 2.$$

Thus, **C** is the correct answer.

25. All of the even numbers from 2 to 98 inclusive, excluding those ending in 0, are multiplied together. What is the rightmost digit (the units digit) of the product?

A 0

B 2

C 4

D 6

E 8

Solution(s):

We only care about the units digit, which means that the tens digits don't matter.

Then we have 10 groups of

$$2 \cdot 4 \cdot 6 \cdot 8 = 384.$$

The units digit of each group is 4. We now need to find the units digit of 4^{10}.

The units digit of $4^2 = 16$ is 6. This means we only need to find the units digit of 6^5.

Note that every power of 6 always ends in a 6 (e.g. $6, 36, 216, \cdots$).

Thus, **D** is the correct answer.

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