1996 AMC 8 Solutions

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1. How many positive factors of 36 are also multiples of 4?

- A 2
- B 3
- C 4
- D 5
- E 6

Solution:

The positive factors of 36 are $1, 2, 3, 4, 6, 9, 12, 18, 36$. Of these, only $4, 12$, and 36 are multiples of 4.

Thus, the correct answer is **B** .

2. José, Thuy, and Kareem each start with the number 10. José subtracts 1 from the number 10, doubles his answer, and then adds 2. Thuy doubles the number 10, subtracts 1 from her answer, and then adds 2. Kareem subtracts 1 from the number 10, adds 2 to his answer, and then doubles the result. Who gets the largest final answer?

- A José
- B Thuy
- C Kareem
- D José and Thuy
- E Thuy and Kareem

Solution:

Starting from 10: José computes $9, 18, 20$; Thuy computes $20, 19, 21$; Kareem computes $9, 11, 22$.

Kareem doubles last, so the 2 he adds is doubled too, giving the largest result.

Thus, the correct answer is **C** .

3. The 64 whole numbers from 1 through 64 are written, one per square, on a checkerboard (an 8 by 8 array of 64 squares). The first 8 numbers are written in order across the first row, the next 8 across the second row, and so on. After all 64 numbers are written, the sum of the numbers in the four corners will be

- A 130
- B 131
- C 132
- D 133
- E 134

Solution:

The first row is $1, 2, \ldots, 8$ and the last row is $57, 58, \ldots, 64$. The four corners are $1, 8, 57,$ and 64.

Their sum is $1 + 8 + 57 + 64 = 130$.

Thus, the correct answer is **A** .

4. What is the value of the following expression?

$$\frac{2+4+6+\cdots+34}{3+6+9+\cdots+51}$$

A $\frac{1}{3}$

B $\frac{2}{3}$

C $\frac{3}{2}$

D $\frac{17}{3}$

E $\frac{34}{3}$

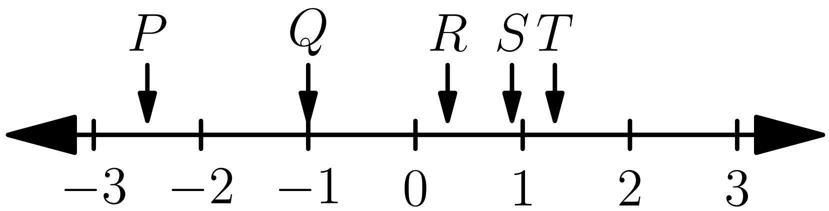
Solution:

The numerator is $2(1+2+\cdots+17)$ and the denominator is $3(1+2+\cdots+17)$.

The common factor cancels, leaving $\frac{2}{3}$.

Thus, the correct answer is **B** .

5. The letters $P, Q, R, S,$ and T represent numbers located on the number line as shown.



Which of the following expressions represents a negative number?

A $P - Q$

B $P \cdot Q$

C $\frac{S}{Q} \cdot P$

D $\frac{R}{P \cdot Q}$

E $\frac{S+T}{R}$

Solution:

From the number line, P and Q are negative and R, S, T are positive.

Then $P \cdot Q$ is positive; $\frac{S}{Q} \cdot P$ has two negative factors, so it is positive; $\frac{R}{P \cdot Q}$ is positive; and $\frac{S+T}{R}$ is positive. Since P is to the left of Q, $P - Q$ is negative.

Thus, the correct answer is **A** .

6. What is the smallest result that can be obtained by the following process? Choose three different numbers from the set $\{3, 5, 7, 11, 13, 17\}$, add two of them, then multiply their sum by the third number.

A 15

B 30

C 36

D 50

E 56

Solution:

Use the three smallest numbers $3, 5, 7$. The choices are $3(5+7) = 36$, $5(3+7) = 50$, and $7(3+5) = 56$.

Making the smallest number the multiplier gives the least result, 36.

Thus, the correct answer is **C** .

7. Brent has goldfish that quadruple (become four times as many) every month, and Gretel has goldfish that double every month. If Brent has 4 goldfish at the same time that Gretel has 128 goldfish, then in how many months from that time will they have the same number of goldfish?

A 4

B 5

C 6

D 7

E 8

Solution:

Brent's counts are $4, 16, 64, 256, 1024, 4096$, and Gretel's are $128, 256, 512, 1024, 2048, 4096$.

They are equal after 5 months, when both have 4096.

Thus, the correct answer is **B** .

8. Points A and B are 10 units apart. Points B and C are 4 units apart. Points C and D are 3 units apart. If A and D are as close as possible, then the number of units between them is

- A 0
- B 3
- C 9
- D 11
- E 17

Solution:

The distance is smallest when the points are collinear with C and D toward A: take $A = 0, B = 10, C = 6, D = 3$.

Then $AD = 10 - 4 - 3 = 3$.

Thus, the correct answer is **B** .

9. If 5 times a number is 2, then 100 times the reciprocal of the number is

- A 2.5
- B 40
- C 50
- D 250
- E 500

Solution:

The number is $\frac{2}{5}$, whose reciprocal is $\frac{5}{2}$.

Then $100 \cdot \frac{5}{2} = 250$.

Thus, the correct answer is **D** .

10. When Walter drove up to the gasoline pump, he noticed that his gasoline tank was $\frac{1}{8}$ full. He purchased 7.5 gallons of gasoline for $10. With this additional gasoline, his gasoline tank was then $\frac{5}{8}$ full. The number of gallons of gasoline his tank holds when it is full is

- A 8.75
- B 10
- C 11.5
- D 15
- E 22.5

Solution:

The increase is $\frac{5}{8} - \frac{1}{8} = \frac{1}{2}$ of a tank, which equals 7.5 gallons.

So a full tank holds $2 \cdot 7.5 = 15$ gallons.

Thus, the correct answer is **D** .

11. Let x be the number

$$0.\underbrace{0000\ldots00001}_{1996 \text{ zeros}},$$

where there are 1996 zeros after the decimal point. Which of the following expressions represents the largest number?

- A $3+x$
- B $3-x$
- C $3\cdot x$
- D $3/x$
- E $x/3$

Solution:

Since x is a very small positive number, $3+x$ and $3-x$ are near 3, while $3\cdot x$ and $x/3$ are near 0.

But $3/x$ is 3 followed by 1997 zeros, far larger than any other choice.

Thus, the correct answer is **D** .

12. What number should be removed from the list

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$$

so that the average of the remaining numbers is 6.1?

- A 4
- B 5
- C 6
- D 7
- E 8

Solution:

The sum of 1 through 11 is 66. For ten numbers to average 6.1, their sum must be $10 \cdot 6.1 = 61$.

So the removed number is $66 - 61 = 5$.

Thus, the correct answer is **B** .

13. In the fall of 1996, a total of 800 students participated in an annual school clean-up day. The organizers of the event expect that in each of the years $1997, 1998,$ and 1999, participation will increase by 50% over the previous year. The number of participants the organizers expect in the fall of 1999 is

- A 1200
- B 1500
- C 2000
- D 2400
- E 2700

Solution:

Each year multiplies the count by 1.5: $800 \to 1200 \to 1800 \to 2700$.

So $800 \cdot 1.5^3 = 2700$ participants are expected in 1999.

Thus, the correct answer is **E** .

14. Six different digits from the set $\{1,2,3,4,5,6,7,8,9\}$ are placed in a figure made of a vertical column of three squares and a horizontal row of four squares that overlap in one shared square, so that the sum of the three entries in the vertical column is 23 and the sum of the four entries in the horizontal row is 12. The sum of the six digits used is

- A 27
- B 29
- C 31
- D 33
- E 35

Solution:

Three distinct digits from 1 through 9 summing to 23 must be $6, 8, 9$. The row's other three digits are at least $1+2+3=6$, so the shared square (belonging to both the column and the row) is at most $12-6=6$. Hence the shared digit is 6.

The six digits are then $6, 8, 9$ and $1, 2, 3$, whose sum is 29. Equivalently, $23+12-6=29$.

Thus, the correct answer is **B** .

15. The remainder when the product $1492 \cdot 1776 \cdot 1812 \cdot 1996$ is divided by 5 is

- A 0
- B 1
- C 2
- D 3
- E 4

Solution:

The units digit of the product equals the units digit of $2 \cdot 6 \cdot 2 \cdot 6 = 144$, which is 4.

A number ending in 4 leaves remainder 4 when divided by 5.

Thus, the correct answer is **E** .

16. What is the value of the following expression?

$$1-2-3+4+5-6-7+8+9-10-11+12+13-\cdots+1992+1993-1994-1995+1996$$

A -998

B -1

C 0

D 1

E 998

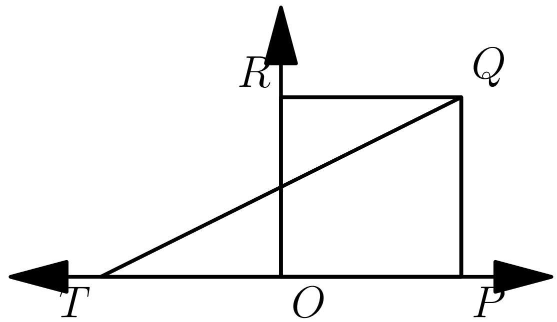
Solution:

Grouping in blocks of four gives $1-2-3+4=0, 5-6-7+8=0$, and so on.

There are $1996/4=499$ such blocks, each equal to 0, so the total is 0.

Thus, the correct answer is **C** .

17. Figure $OPQR$ is a square. Point O is the origin, and point Q has coordinates $(2, 2)$. What are the coordinates for T so that the area of triangle PQT equals the area of square $OPQR$?



A $(-6, 0)$

B $(-4, 0)$

C $(-2, 0)$

D $(2, 0)$

E $(4, 0)$

Solution:

Since $OPQR$ is a square with $O = (0, 0)$ and $Q = (2, 2)$, we have $P = (2, 0)$ and $R = (0, 2)$, so the area is $2^2 = 4$.

Triangle PQT has vertical base PQ of length 2, and $T = (t, 0)$ lies on the x-axis. Its area is $\frac{1}{2} \cdot 2 \cdot (2 - t) = 2 - t$. Setting $2 - t = 4$ gives $t = -2$, so $T = (-2, 0)$.

Thus, the correct answer is **C** .

18. Ana's monthly salary was \$2000 in May. In June she received a 20% raise. In July she received a 20% pay cut. After the two changes in June and July, Ana's monthly salary was

- A \$1920
- B \$1980
- C \$2000
- D \$2020
- E \$2040

Solution:

After the raise, the salary is $2000 \cdot 1.2 = 2400$.

After the cut, it is $2400 \cdot 0.8 = 1920$.

Thus, the correct answer is **A** .

19. The percent of students who prefer golf, bowling, or tennis at East Junior High School and West Middle School is as follows. At East (2000 students): golf 30%, bowling 48%, tennis 22%. At West (2500 students): golf 24%, bowling 36%, tennis 40%. In the two schools combined, the percent of students who prefer tennis is

- A 30%
- B 31%
- C 32%
- D 33%
- E 34%

Solution:

East has 22% of $2000 = 440$ tennis fans, and West has 40% of $2500 = 1000$.

Together 1440 of the 4500 students prefer tennis, which is $\frac{1440}{4500} = 32\%$.

Thus, the correct answer is **C** .

20. Suppose there is a special key on a calculator that replaces the number x currently displayed with the number given by the formula $1/(1-x)$. For example, if the calculator is displaying 2 and the special key is pressed, then the calculator will display -1 since $1/(1-2)=-1$. Now suppose that the calculator is displaying 5. After the special key is pressed 100 times in a row, the calculator will display

- A -0.25
- B 0
- C 0.8
- D 1.25
- E 5

Solution:

Starting from 5: $1/(1-5)=-0.25$, then $1/(1+0.25)=0.8$, then $1/(1-0.8)=5$. The values repeat with period 3.

Since $100=3\cdot 33+1$, the 100th press gives the same result as the first press, -0.25.

Thus, the correct answer is **A** .

21. How many subsets containing three different numbers can be selected from the set $\{89,95,99,132,166,173\}$ so that the sum of the three numbers is even?

- A 6
- B 8
- C 10
- D 12
- E 24

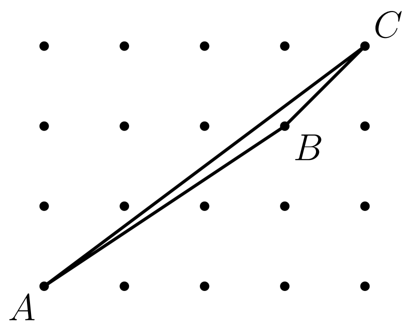
Solution:

The set has 4 odd numbers ($89,95,99,173$) and 2 even numbers ($132,166$). A sum of three is even only with two odds and one even, since three evens is impossible with just two available.

The count is $\binom{4}{2}\cdot\binom{2}{1}=6\cdot 2=12$.

Thus, the correct answer is **D** .

22. The horizontal and vertical distances between adjacent points equal 1 unit. The area of triangle ABC is



A $\frac{1}{4}$

B $\frac{1}{2}$

C $\frac{3}{4}$

D 1

E $\frac{5}{4}$

Solution:

Taking $A = (0, 0)$, $B = (3, 2)$, and $C = (4, 3)$, the enclosing 4×3 rectangle has area 12; subtracting the surrounding regions of areas $6, 3, 2,$ and $\frac{1}{2}$ leaves $\frac{1}{2}$.

Equivalently, by Pick's theorem with no interior lattice points and 3 boundary points, the area is $0 + \frac{3}{2} - 1 = \frac{1}{2}$.

Thus, the correct answer is **B** .

23. The manager of a company planned to distribute a \$50 bonus to each employee from the company fund, but the fund contained \$5 less than what was needed. Instead the manager gave each employee a \$45 bonus and kept the remaining \$95 in the company fund. The amount of money in the company fund before any bonuses were paid was

A \$945

B \$950

C \$955

D \$990

E \$995

Solution:

Let n be the number of employees. The fund is $50n - 5$ (five dollars short of 50 each) and also $45n + 95$.

Setting $50n - 5 = 45n + 95$ gives $5n = 100$, so $n = 20$. The fund is $45 \cdot 20 + 95 = 995$.

Thus, the correct answer is **E** .

24. The measure of angle ABC is $50°$. $\overline{AD}$ bisects angle BAC, and $\overline{DC}$ bisects angle BCA. The measure of angle ADC is

A $90°$

B $100°$

C $115°$

D $122.5°$

E $125°$

Solution:

In triangle ABC, $\angle BAC + \angle BCA = 180° - 50° = 130°$.

The bisectors give $\angle DAC + \angle DCA = \frac{130°}{2} = 65°$. In triangle ADC, $\angle ADC = 180° - 65° = 115°$.

Thus, the correct answer is **C** .

25. A point is chosen at random from within a circular region. What is the probability that the point is closer to the center of the region than it is to the boundary of the region?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

Solution:

Take the radius to be 1. A point at distance r from the center is closer to the center than to the boundary when $r < 1 - r$, i.e. $r < \frac{1}{2}$.

The favorable region is a circle of radius $\frac{1}{2}$, with area $\pi(\frac{1}{2})^2 = \frac{\pi}{4}$, out of the total area π. The probability is

$$\frac{\pi/4}{\pi} = \frac{1}{4}.$$

Thus, the correct answer is **A** .

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