1995 AMC 8 Solutions

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1. Walter has exactly one penny, one nickel, one dime and one quarter in his pocket. What percent of one dollar is in his pocket?

A 4%

B 25%

C 40%

D 41%

E 59%

Solution:

The coins total $1 + 5 + 10 + 25 = 41$ cents.

Since one dollar is 100 cents, this is 41% of a dollar.

Thus, the correct answer is **D** .

2. Jose is 4 years younger than Zack. Zack is 3 years older than Inez. Inez is 15 years old. How old is Jose?

A 8

B 11

C 14

D 16

E 22

Solution:

Zack is $15 + 3 = 18$ years old.

Jose is 4 years younger, so Jose is $18 - 4 = 14$.

Thus, the correct answer is **C** .

3. Which of the following operations has the same effect on a number as multiplying by $\frac{3}{4}$ and then dividing by $\frac{3}{5}$?

A dividing by $\frac{4}{3}$

B dividing by $\frac{9}{20}$

C multiplying by $\frac{9}{20}$

D dividing by $\frac{5}{4}$

E multiplying by $\frac{5}{4}$

Solution:

Dividing by $\frac{3}{5}$ is the same as multiplying by $\frac{5}{3}$, so the two operations together multiply by

$$\frac{3}{4} \times \frac{5}{3} = \frac{5}{4}.$$

So the combined effect is multiplying by $\frac{5}{4}$.

Thus, the correct answer is **E**.

4. A teacher tells the class, "Think of a number, add 1 to it, and double the result. Give the answer to your partner. Partner, subtract 1 from the number you are given and double the result to get your answer." Ben thinks of 6, and gives his answer to Sue. What should Sue's answer be?

A 18

B 24

C 26

D 27

E 30

Solution:

Ben computes $(6+1) \times 2 = 14$ and gives 14 to Sue.

Sue computes $(14-1) \times 2 = 26$.

Thus, the correct answer is **C** .

5. Find the smallest whole number that is larger than the sum

$$2\tfrac{1}{2} + 3\tfrac{1}{3} + 4\tfrac{1}{4} + 5\tfrac{1}{5}.$$

A 14

B 15

C 16

D 17

E 18

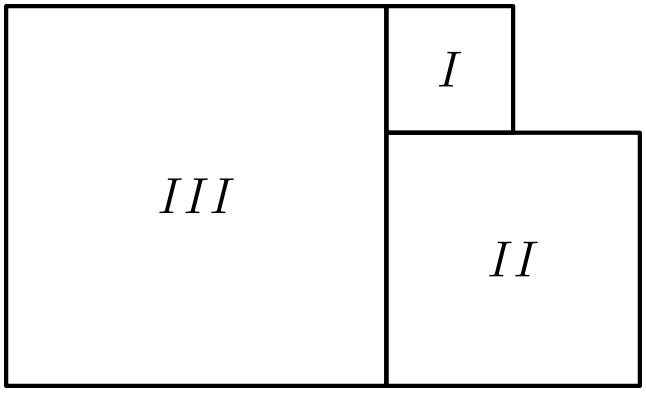
Solution:

The whole-number parts sum to $2 + 3 + 4 + 5 = 14$.

The fractions $\dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5}$ add to about 1.28, which is between 1 and 2. So the total is between 15 and 16, and the smallest whole number larger than it is 16.

Thus, the correct answer is **C** .

6. Figures I, II and III are squares. The perimeter of I is 12 and the perimeter of II is 24. The perimeter of III is



A 9

B 18

C 36

D 72

E 81

Solution:

Square I has side $12 \div 4 = 3$, and square II has side $24 \div 4 = 6$.

From the figure, the side of III is $3 + 6 = 9$, so its perimeter is $4 \times 9 = 36$.

Thus, the correct answer is **C** .

7. At Clover View Junior High, one half of the students go home on the school bus. One fourth go home by automobile. One tenth go home on their bicycles. The rest walk home. What fractional part of the students walk home?

A $\frac{1}{16}$

B $\frac{3}{20}$

C $\frac{1}{3}$

D $\frac{17}{20}$

E $\frac{9}{10}$

Solution:

The students who ride make up

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{10}=\frac{10}{20}+\frac{5}{20}+\frac{2}{20}=\frac{17}{20}.$$

So the fraction who walk is $1-\frac{17}{20}=\frac{3}{20}$.

Thus, the correct answer is **B** .

8. An American traveling in Italy wishes to exchange American money (dollars) for Italian money (lire). If 3000 lire $= \$1.60$, how many lire will the traveler receive in exchange for $\$1.00$?

A 180

B 480

C 1800

D 1875

E 4875

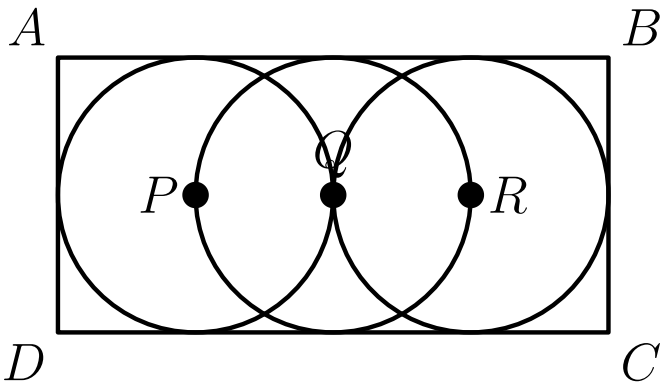
Solution:

Since $\$1.00$ is $\frac{1.00}{1.60} = \frac{5}{8}$ of $\$1.60$, the traveler gets $\frac{5}{8}$ of 3000 lire.

That is $\frac{5}{8} \times 3000 = 1875$ lire.

Thus, the correct answer is **D** .

9. Three congruent circles with centers P, Q and R are tangent to the sides of rectangle $ABCD$ as shown. The circle centered at Q has diameter 4 and passes through points P and R. The area of the rectangle is



A 16

B 24

C 32

D 64

E 128

Solution:

Each circle has diameter 4. The short side of the rectangle equals one diameter, so it is 4.

Since the circle at Q passes through P and R, all three circles have radius 2, and the long side spans two full diameters: $4 + 4 = 8$. The area is $8 \times 4 = 32$.

Thus, the correct answer is **C** .

10. A jacket and a shirt originally sold for \$80 and \$40, respectively. During a sale Chris bought the \$80 jacket at a 40% discount and the \$40 shirt at a 55% discount. The total amount saved was what percent of the total of the original prices?

A 45%

B $47\frac{1}{2}\%$

C 50%

D $79\frac{1}{6}\%$

E 95%

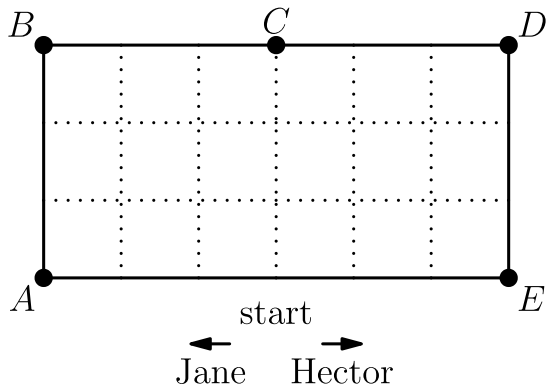
Solution:

The jacket discount saves 40% of $\$80 = \$32$, and the shirt discount saves 55% of $\$40 = \$22$. The total saved is $\$32 + \$22 = \$54$.

The original total is $\$80 + \$40 = \$120$, so the percent saved is $\frac{54}{120} = 0.45 = 45\%$.

Thus, the correct answer is **A** .

11. Jane can walk any distance in half the time it takes Hector to walk the same distance. They set off in opposite directions around the outside of the 18-block area as shown. When they meet for the first time, they will be closest to



A A

B B

C C

D D

E E

Solution:

The perimeter of the region is 18 blocks, so when Jane and Hector meet they have together walked 18 blocks. Since Jane walks twice as fast, she covers 12 blocks and Hector covers 6.

Starting from the middle of the bottom edge, Hector walks 6 blocks (to E, then up to D), and Jane walks 12 blocks (to A, up to B, then across the top to D). They meet at D.

Thus, the correct answer is **D** .

12. A *lucky* year is one in which at least one date, when written in the form month/day/year, has the following property: the product of the month times the day equals the last two digits of the year. For example, 1956 is a lucky year because it has the date $7/8/56$ and $7 \times 8 = 56$. Which of the following is NOT a lucky year?

A 1990

B 1991

C 1992

D 1993

E 1994

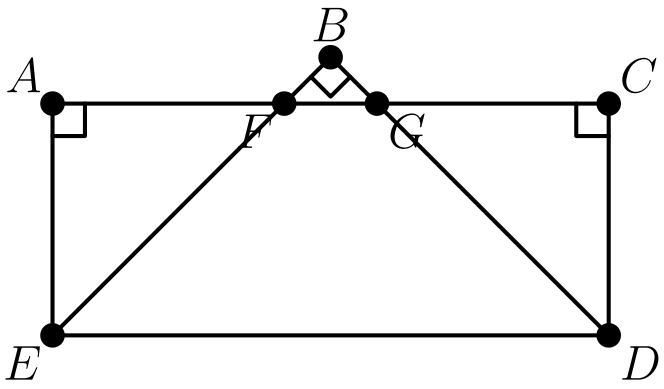
Solution:

Each of the other years works: $90 = 9 \times 10, 91 = 7 \times 13, 92 = 4 \times 23$, and $93 = 3 \times 31$, each a valid month/day.

For 1994, the last two digits factor only as $94 = 2 \times 47$, and 47 is too large for a day (and 94 or 47 is too large for a month). So 1994 has no lucky date.

Thus, the correct answer is **E**.

13. In the figure, $\angle A$, $\angle B$ and $\angle C$ are right angles. If $\angle AEB = 40°$ and $\angle BED = \angle BDE$, then $\angle CDE =$



A $75°$

B $80°$

C $85°$

D $90°$

E $95°$

Solution:

In triangle BDE, the angles at E and D are equal and $\angle B = 90°$, so $\angle BED = \angle BDE = 45°$.

Then $\angle AED = \angle AEB + \angle BED = 40° + 45° = 85°$. In quadrilateral $AEDC$, the angles at A and C are $90°$, so

$$\angle CDE = 360° - 90° - 90° - 85° = 95°.$$

Thus, the correct answer is **E**.

14. A team won 40 of its first 50 games. How many of the remaining 40 games must this team win so it will have won exactly 70% of its games for the season?

A 20

B 23

C 28

D 30

E 35

Solution:

The season has $50 + 40 = 90$ games, and 70% of 90 is 63 wins.

The team already has 40 wins, so it needs $63 - 40 = 23$ more.

Thus, the correct answer is **B** .

15. What is the 100th digit to the right of the decimal point in the decimal form of $\frac{4}{37}$?

A 0

B 1

C 2

D 7

E 8

Solution:

$\frac{4}{37} = 0.\overline{108}$, repeating with block length 3. The digits in positions $3, 6, 9, \ldots$ (multiples of 3) are 8.

Since 99 is a multiple of 3, the 99th digit is 8, so the 100th digit starts the next block: it is 1.

Thus, the correct answer is **B** .

16. Students from three middle schools worked on a summer project. Seven students from Allen School worked for 3 days. Four students from Balboa School worked for 5 days. Five students from Carver School worked for 9 days. The total amount paid for the students' work was $774. Assuming each student received the same amount for a day's work, how much did the students from Balboa School earn altogether?

A $9.00

B $48.38

C $180.00

D $193.50

E $258.00

Solution:

The total student-days are $7 \times 3 + 4 \times 5 + 5 \times 9 = 21 + 20 + 45 = 86.$

So each student-day pays $\$774 \div 86 = \$9.$ Balboa worked 20 student-days, earning $20 \times \$9 = \$180.$

Thus, the correct answer is **C** .

17. The table below gives the percent of students in each grade at Annville and Cleona elementary schools. The percentages for grades K, $1, 2, 3, 4, 5, 6$ are: Annville: $16\%, 15\%, 15\%, 14\%, 13\%, 16\%, 11\%$; Cleona: $12\%, 15\%, 14\%, 13\%, 15\%, 14\%, 17\%$.

Annville has 100 students and Cleona has 200 students. In the two schools combined, what percent of the students are in grade 6?

A 12%

B 13%

C 14%

D 15%

E 28%

Solution:

Annville has 11% of $100 = 11$ sixth graders, and Cleona has 17% of $200 = 34$ sixth graders.

Combined, that is $11 + 34 = 45$ out of 300 students, which is $\frac{45}{300} = 15\%$.

Thus, the correct answer is **D** .

18. The area of each of the four congruent L-shaped regions of this 100-inch by 100-inch square is $\frac{3}{16}$ of the total area. How many inches long is the side of the center square?

A 25

B 44

C 50

D 62

E 75

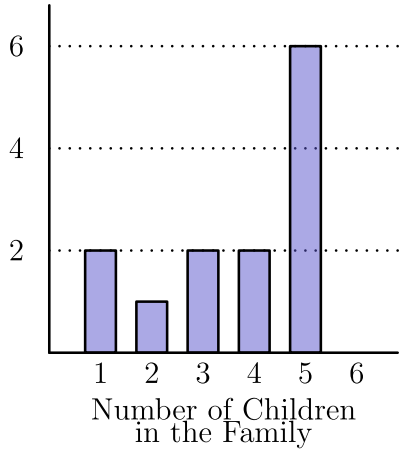
Solution:

The four L-shaped regions cover $4 \times \frac{3}{16} = \frac{3}{4}$ of the square, so the center square is the remaining $\frac{1}{4}$ of the total area.

The total area is $100 \times 100 = 10000$ square inches, so the center square has area $\frac{1}{4} \times 10000 = 2500$, and its side is $\sqrt{2500} = 50$ inches.

Thus, the correct answer is **C**.

19. The graph shows the distribution of the number of children in the families of the students in Ms. Jordan's English class. The median number of children in the family for this distribution is



A 1

B 2

C 3

D 4

E 5

Solution:

The graph gives 2 families with 1 child, 1 with 2, 2 with 3, 2 with 4, and 6 with 5, for $2 + 1 + 2 + 2 + 6 = 13$ families.

The median is the 7th value in order. Listing the family sizes, the 7th value is 4.

Thus, the correct answer is **D** .

20. Diana and Apollo each roll a standard die obtaining a number at random from 1 to 6. What is the probability that Diana's number is larger than Apollo's number?

A $\frac{1}{3}$

B $\frac{5}{12}$

C $\frac{4}{9}$

D $\frac{17}{36}$

E $\frac{1}{2}$

Solution:

There are $6 \times 6 = 36$ equally likely outcomes, of which 6 are ties, leaving 30 outcomes with different numbers.

By symmetry, Diana is larger in exactly half of those, or 15, so the probability is $\frac{15}{36} = \frac{5}{12}$.

Thus, the correct answer is **B**.

21. A plastic snap-together cube has a protruding snap on one side and receptacle holes on the other five sides. What is the smallest number of these cubes that can be snapped together so that only receptacle holes are showing?

A 3

B 4

C 5

D 6

E 8

Solution:

Every cube's single snap must be plugged into another cube's hole to be hidden. With one, two, or three cubes, at least one snap is always left exposed.

Four cubes can be arranged in a square ring, each snap fitting into the neighbor's hole, so only receptacle holes show. The smallest number is 4.

Thus, the correct answer is **B** .

22. The number 6545 can be written as a product of a pair of positive two-digit numbers. What is the sum of this pair of numbers?

A 162

B 172

C 173

D 174

E 222

Solution:

The prime factorization is $6545 = 5 \times 7 \times 11 \times 17$. To split into two two-digit factors, pair the primes: $5 \times 17 = 85$ and $7 \times 11 = 77$.

These are the only two-digit pair, and their sum is $85 + 77 = 162$.

Thus, the correct answer is **A** .

23. How many four-digit whole numbers are there such that the leftmost digit is odd, the second digit is even, and all four digits are different?

A 1120

B 1400

C 1800

D 2025

E 2500

Solution:

The first digit is odd: 5 choices. The second is even: 5 choices (none of which repeats the odd first digit).

The third digit is any of the 8 unused digits, and the fourth is any of the 7 remaining. In total, $5 \times 5 \times 8 \times 7 = 1400$.

Thus, the correct answer is **B** .

24. In parallelogram $ABCD$, $\overline{DE}$ is the altitude to the base $\overline{AB}$ (with E on $\overline{AB}$) and $\overline{DF}$ is the altitude to the base $\overline{BC}$. If $DC = 12$, $EB = 4$, and $DE = 6$, then $DF =$

A 6.4

B 7

C 7.2

D 8

E 10

Solution:

Since $AB = DC = 12$, we get $AE = 12 - 4 = 8$. In right triangle ADE, $AD = \sqrt{8^2 + 6^2} = 10$, so $BC = AD = 10$.

The area is $AB \cdot DE = 12 \cdot 6 = 72$, and also $BC \cdot DF = 10 \cdot DF$. So $DF = \frac{72}{10} = 7.2$.

Thus, the correct answer is **C**.

25. Buses from Dallas to Houston leave every hour on the hour. Buses from Houston to Dallas leave every hour on the half hour. The trip from one city to the other takes 5 hours. Assuming the buses travel on the same highway, how many Dallas-bound buses does a Houston-bound bus pass on the highway (not in the station)?

A 5

B 6

C 9

D 10

E 11

Solution:

Consider a bus leaving Dallas at $6:00$, arriving in Houston at $11:00$. It meets every Dallas-bound bus that is on the highway during that window.

Dallas-bound buses leave Houston on the half hour and take 5 hours. The ones sharing the road (meeting away from a station) are those that left Houston at $1:30, 2:30, \ldots, 10:30$, which is 10 buses.

Thus, the correct answer is **D** .

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