1994 AMC 8 Solutions

Typeset by: LIVE by Po-Shen Loh
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1. Which of the following is the largest?

A $\frac{1}{3}$

B $\frac{1}{4}$

C $\frac{3}{8}$

D $\frac{5}{12}$

E $\frac{7}{24}$

Solution:

Over the common denominator 24, the fractions are $\frac{8}{24}, \frac{6}{24}, \frac{9}{24}, \frac{10}{24}$, and $\frac{7}{24}$.

The largest numerator is 10, so $\frac{5}{12} = \frac{10}{24}$ is the largest.

Thus, the correct answer is **D** .

2. What is the value of the following expression?

$$\frac{1}{10}+\frac{2}{10}+\frac{3}{10}+\frac{4}{10}+\frac{5}{10}+\frac{6}{10}+\frac{7}{10}+\frac{8}{10}+\frac{9}{10}+\frac{55}{10}$$

A $4\frac{1}{2}$

B 6.4

C 9

D 10

E 11

Solution:

The numerators sum to $1+2+\cdots+9+55=45+55=100.$

So the total is $\frac{100}{10}=10.$

Thus, the correct answer is **D** .

3. Each day Maria must work 8 hours. This does not include the 45 minutes she takes for lunch. If she begins working at 7:25 A.M. and takes her lunch break at noon, then her working day will end at

A 3:40 P.M.

B 3:55 P.M.

C 4:10 P.M.

D 4:25 P.M.

E 4:40 P.M.

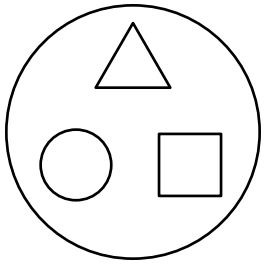
Solution:

Eight hours after 7:25 A.M. is 3:25 P.M.

Adding the 45-minute lunch break gives an ending time of 4:10 P.M.

Thus, the correct answer is **C** .

4. Which of the following represents the result when the figure shown at the right is rotated clockwise 120° about its center?



A 

B 

C 

D 

E 

Solution:

A clockwise turn of 120° sends each shape to the next position clockwise: the triangle at the top moves to the lower right, the square at the lower right moves to the lower left, and the circle at the lower left moves to the top.

The result therefore has a circle on top, a square at the lower left, and a triangle at the lower right, with each shape keeping its original form.

Thus, the correct answer is **B** .

5. Given that 1 mile $=$ 8 furlongs and 1 furlong $=$ 40 rods, the number of rods in one mile is

A 5

B 320

C 660

D 1760

E 5280

Solution:

One mile is 8 furlongs, and each furlong is 40 rods, so one mile is $8 \times 40 = 320$ rods.

Thus, the correct answer is **B** .

6. The unit's digit (one's digit) of the product of any six consecutive positive whole numbers is

A 0

B 2

C 4

D 6

E 8

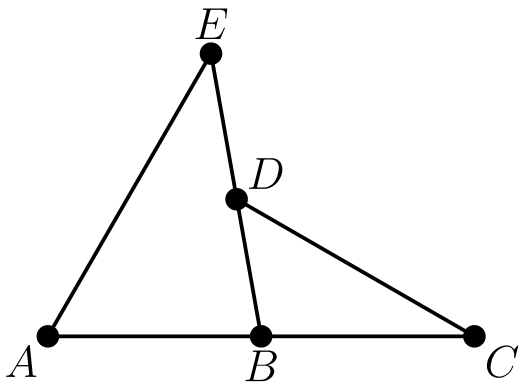
Solution:

Any six consecutive whole numbers include at least one multiple of 5 and at least one even number, so their product is a multiple of $5 \times 2 = 10$.

A multiple of 10 always ends in 0.

Thus, the correct answer is **A** .

7. If $\angle A = 60^\circ, \angle E = 40^\circ$, and $\angle C = 30^\circ$, then $\angle BDC =$



A 40°

B 50°

C 60°

D 70°

E 80°

Solution:

In triangle $ABE, \angle ABE = 180^\circ - (60^\circ + 40^\circ) = 80^\circ.$

Since A, B, C are collinear and D lies on segment EB, the angle $\angle DBC = 180^\circ - 80^\circ = 100^\circ.$

In triangle $BDC, \angle BDC = 180^\circ - (100^\circ + 30^\circ) = 50^\circ.$

Thus, the correct answer is **B**.

8. For how many three-digit whole numbers does the sum of the digits equal 25?

A 2

B 4

C 6

D 8

E 10

Solution:

Since the digits sum to 25 and the maximum is 27, at least one digit is 9. The possible digit sets are $\{9, 9, 7\}$ and $\{9, 8, 8\}$.

Each set has 3 distinct arrangements ($997, 979, 799$ and $988, 898, 889$), giving 6 numbers in all.

Thus, the correct answer is **C** .

9. A shopper buys a $\$100$ coat on sale for 20% off. An additional $\$5$ is taken off the sale price by using a discount coupon. A sales tax of 8% is paid on the final selling price. The total amount the shopper pays for the coat is

A $\$81.00$

B $\$81.40$

C $\$82.00$

D $\$82.08$

E $\$82.40$

Solution:

The 20% discount lowers the price to $\$80$, and the $\$5$ coupon reduces it to $\$75$.

Adding 8% tax gives $1.08 \times \$75 = \$81.00$.

Thus, the correct answer is **A** .

10. For how many positive integer values of N $(N > 0)$ is the expression

$$\frac{36}{N+2}$$

an integer?

A 7

B 8

C 9

D 10

E 12

Solution:

The expression is an integer when $N+2$ divides 36. The divisors of 36 are $1, 2, 3, 4, 6, 9, 12, 18, 36$.

Because $N > 0$, we need $N+2 > 2$, leaving the divisors $3, 4, 6, 9, 12, 18, 36$: that is 7 values.

Thus, the correct answer is **A** .

11. Last summer 100 students attended basketball camp. Of those attending, 52 were boys and 48 were girls. Also, 40 students were from Jones Middle School and 60 were from Clay Middle School. Twenty of the girls were from Jones Middle School. How many of the boys were from Clay Middle School?

A 20

B 32

C 40

D 48

E 52

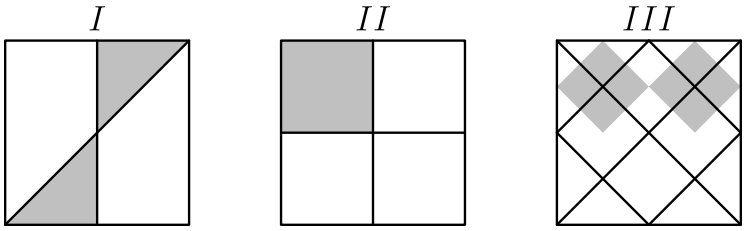
Solution:

Since 48 girls attended and 20 were from Jones, $48 - 20 = 28$ girls were from Clay.

Clay had 60 students total, so the number of Clay boys is $60 - 28 = 32.$

Thus, the correct answer is **B** .

12. Each of the three large squares shown is the same size. Segments that intersect the sides of the squares intersect at the midpoints of the sides. How do the shaded areas of these squares compare?



A The shaded areas in all three are equal.

B Only the shaded areas of I and II are equal.

C Only the shaded areas of I and III are equal.

D Only the shaded areas of II and III are equal.

E The shaded areas of I, II, and III are all different.

Solution:

In square II, 1 of the 4 equal cells is shaded, so $\frac{1}{4}$ of it is shaded.

Square I breaks into 8 equal triangles with 2 shaded, and square III breaks into 16 equal triangles with 4 shaded; each of these equals $\frac{2}{8} = \frac{4}{16} = \frac{1}{4}$.

Since every figure has exactly $\frac{1}{4}$ shaded, the shaded areas are all equal.

Thus, the correct answer is **A** .

13. The number halfway between $\frac{1}{6}$ and $\frac{1}{4}$ is

A $\frac{1}{10}$

B $\frac{1}{5}$

C $\frac{5}{24}$

D $\frac{7}{24}$

E $\frac{5}{12}$

Solution:

The number halfway between two values is their average: $\frac{\frac{1}{6}+\frac{1}{4}}{2}$.

Since $\frac{1}{6}+\frac{1}{4}=\frac{2}{12}+\frac{3}{12}=\frac{5}{12}$, the average is $\frac{5}{12}\div 2=\frac{5}{24}$.

Thus, the correct answer is **C**.

14. Two children at a time can play pairball. For 90 minutes, with only two children playing at one time, five children take turns so that each one plays the same amount of time. The number of minutes each child plays is

A 9

B 10

C 18

D 20

E 36

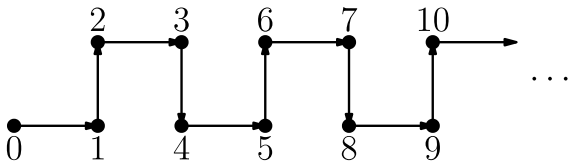
Solution:

Two children play at every moment for 90 minutes, so the total playing time is $2 \times 90 = 180$ child-minutes.

Split equally among 5 children, each plays $\frac{180}{5} = 36$ minutes.

Thus, the correct answer is **E**.

15. If this path is to continue in the same pattern, then which sequence of arrows goes from point 425 to point 427?



A 

B 

C 

D 

E 

Solution:

The pattern repeats every 4 points, so the arrows leaving a point depend only on its remainder upon division by 4.

Because $425 = 4(106) + 1$ and $427 = 4(106) + 3$, the path from 425 to 427 looks just like the path from point 1 to point 3: from 1 it goes up to 2, then right to 3.

Thus, the correct answer is **A** .

16. The perimeter of one square is 3 times the perimeter of another square. The area of the larger square is how many times the area of the smaller square?

A 2

B 3

C 4

D 6

E 9

Solution:

A square's perimeter is proportional to its side, so the larger square has side length 3 times the smaller one.

Area is the side squared, so the larger area is $3^2 = 9$ times the smaller.

Thus, the correct answer is **E**.

17. Pauline Bunyan can shovel snow at the rate of 20 cubic yards for the first hour, 19 cubic yards for the second, 18 for the third, etc., always shoveling one cubic yard less per hour than the previous hour. If her driveway is 4 yards wide, 10 yards long, and covered with snow 3 yards deep, then the number of hours it will take her to shovel it clean is closest to

A 4

B 5

C 6

D 7

E 12

Solution:

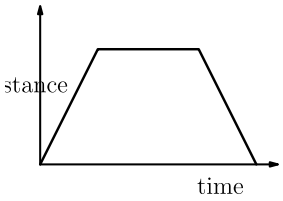
The volume of snow is $4 \times 10 \times 3 = 120$ cubic yards.

The amounts shoveled add up to $20 + 19 + 18 + 17 + 16 + 15 + 14 = 119$ after 7 hours, while 8 hours would give 132. Since 119 is much closer to 120 than 132 is, the 7-hour answer is closest.

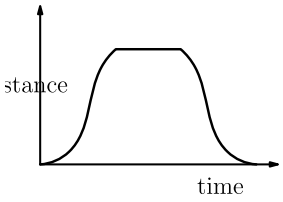
Thus, the correct answer is **D** .

18. Mike leaves home and drives slowly east through city traffic. When he reaches the highway he drives east more rapidly until he reaches the shopping mall where he stops. He shops at the mall for an hour. Mike returns home by the same route as he came, driving west rapidly along the highway and then slowly through city traffic. Each graph shows the distance from home on the vertical axis versus the time elapsed since leaving home on the horizontal axis. Which graph is the best representation of Mike's trip?

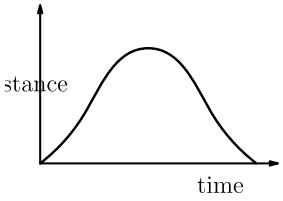
A



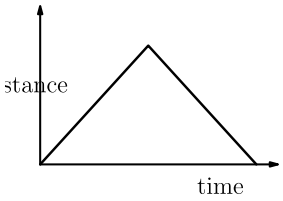
B



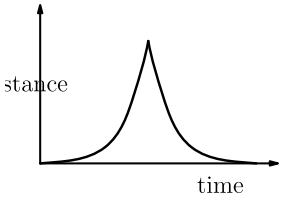
C



D



E



Solution:

On the way out Mike drives slowly (a shallow slope) then rapidly (a steep slope), so the distance rises slowly and then more steeply. During the hour at the mall the distance stays the same, a flat segment.

On the return he drives rapidly (steep) then slowly (shallow), so the distance falls steeply and then levels off. Only the graph with a slow-then-fast rise, a flat top, and a fast-then-slow fall fits.

Thus, the correct answer is **B** .

19. Around the outside of a 4 by 4 square, construct four semicircles with the four sides of the square as their diameters. Another square, $ABCD$, has its sides parallel to the corresponding sides of the original square, and each side of $ABCD$ is tangent to one of the semicircles. The area of the square $ABCD$ is

A 16

B 32

C 36

D 48

E 64

Solution:

Each semicircle is built on a side of length 4, so its radius is 2. A semicircle bulges out from the middle of each side by that radius.

Each side of $ABCD$ is the original side plus two radii: $4 + 2(2) = 8$. So the area of $ABCD$ is $8^2 = 64$.

Thus, the correct answer is **E** .

20. Let W, X, Y, and Z be four different digits selected from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

If the sum $\frac{W}{X} + \frac{Y}{Z}$ is to be as small as possible, then $\frac{W}{X} + \frac{Y}{Z}$ must equal

A $\frac{2}{17}$

B $\frac{3}{17}$

C $\frac{17}{72}$

D $\frac{25}{72}$

E $\frac{13}{36}$

Solution:

Small numerators and large denominators make small fractions, so use 1 and 2 as numerators and 8 and 9 as denominators.

Pairing the larger numerator with the larger denominator gives $\frac{1}{8} + \frac{2}{9} = \frac{9+16}{72} = \frac{25}{72}$, which is smaller than $\frac{1}{9} + \frac{2}{8} = \frac{26}{72}$. So the minimum sum is $\frac{25}{72}$.

Thus, the correct answer is **D** .

21. A gumball machine contains 9 red, 7 white, and 8 blue gumballs. The least number of gumballs a person must buy to be sure of getting four gumballs of the same color is

A 8

B 9

C 10

D 12

E 18

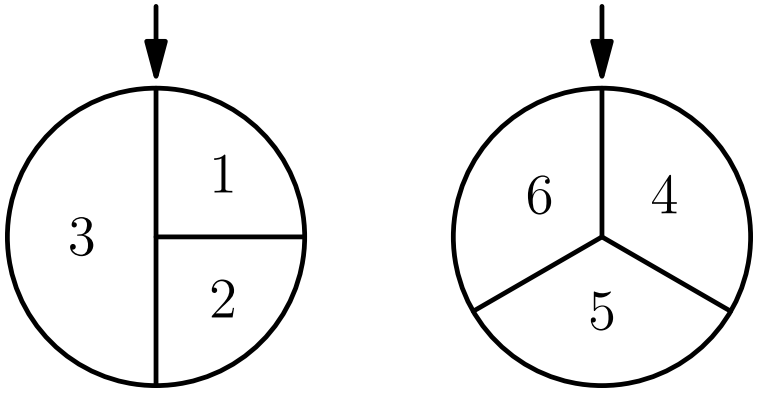
Solution:

In the worst case, a person could draw 3 red, 3 white, and 3 blue, which is 9 gumballs, without yet having four of any color.

The next (tenth) gumball must match one of these colors, giving four of that color. So 10 gumballs are needed.

Thus, the correct answer is **C** .

22. The two wheels shown at the right are spun and the two resulting numbers are added. The probability that the sum of the two numbers is even is



A $\frac{1}{6}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{5}{12}$

E $\frac{4}{9}$

Solution:

On the first wheel, $P(1) = \frac{1}{4}, P(2) = \frac{1}{4}$, and $P(3) = \frac{1}{2}$. On the second wheel, each of $4, 5, 6$ has probability $\frac{1}{3}$.

The sum is even when both numbers are odd or both are even. Both odd: the first is 1 or 3 $\left(\frac{3}{4}\right)$ and the second is 5 $\left(\frac{1}{3}\right)$, giving $\frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$. Both even: the first is 2 $\left(\frac{1}{4}\right)$ and the second is 4 or 6 $\left(\frac{2}{3}\right)$, giving $\frac{1}{4} \cdot \frac{2}{3} = \frac{1}{6}$.

The total probability is $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$.

Thus, the correct answer is **D**.

23. If X, Y, and Z are different digits, then the largest possible 3-digit sum for

$$\begin{array}{rr} & XXX \\ & YX \\ + & X \\ \hline \end{array}$$

has the form

A XXY

B XYZ

C YYX

D YYZ

E ZZY

Solution:

The hundreds digit of the sum comes from X plus any carry, so if $X = 9$ the sum would spill over into four digits. The largest allowed value is $X = 8$.

To make the sum as large as possible, take $Y = 9$: then $888 + 98 + 8 = 994$. Its digits are $9, 9, 4$; since $9 = Y$ and 4 is a new digit Z, the sum has the form YYZ.

Thus, the correct answer is **D** .

24. A 2 by 2 square is divided into four 1 by 1 squares. Each of the small squares is to be painted either green or red. In how many different ways can the painting be accomplished so that no green square shares its top or right side with any red square? There may be as few as zero or as many as four small green squares.

A 4

B 6

C 7

D 8

E 16

Solution:

The rule says a green square cannot have a red square on its top or right side, so any green square forces the squares above and to its right to be green as well. The green squares must therefore cluster toward the top-right corner.

The valid colorings are: all four red; only the top-right green; the whole top row green; the whole right column green; all green except the bottom-left; and all four green. That is 6 colorings.

Thus, the correct answer is **B** .

25. Find the sum of the digits in the answer to

$$\underbrace{9999\cdots99}_{94\text{ nines}}\times\underbrace{4444\cdots44}_{94\text{ fours}}$$

where a string of 94 nines is multiplied by a string of 94 fours.

A 846

B 855

C 945

D 954

E 1072

Solution:

Small cases show the pattern: $99\times44=4356$ and $999\times444=443556$. In general, a string of n nines times a string of n fours gives $(n-1)$ fours, then a 3, then $(n-1)$ fives, then a 6.

For $n=94$, the product has 93 fours, one 3, 93 fives, and one 6. The digit sum is $93(4)+3+93(5)+6=93(9)+9=94(9)=846$.

Thus, the correct answer is **A** .

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