1993 AMC 8 Solutions

Typeset by: LIVE by Po-Shen Loh
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1. Which pair of numbers does **not** have a product equal to 36?

A $\{-4, -9\}$

B $\{-3, -12\}$

C $\left\{\frac{1}{2}, -72\right\}$

D $\{1, 36\}$

E $\left\{\frac{3}{2}, 24\right\}$

Solution:

Checking each pair: $(-4)(-9) = 36, (-3)(-12) = 36, \frac{1}{2} \times (-72) = -36,$ $(1)(36) = 36,$ and $\frac{3}{2} \times 24 = 36.$

Only $\frac{1}{2} \times (-72) = -36$ fails to equal $36.$

Thus, the correct answer is **C** .

2. When the fraction $\frac{49}{84}$ is expressed in simplest form, then the sum of the numerator and the denominator will be

A 11

B 17

C 19

D 33

E 133

Solution:

Since $49 = 7 \times 7$ and $84 = 7 \times 12$, the fraction reduces to $\frac{7}{12}$.

The sum of numerator and denominator is $7 + 12 = 19$.

Thus, the correct answer is **C** .

3. Which of the following numbers has the largest prime factor?

A 39

B 51

C 77

D 91

E 121

Solution:

Factoring: $39 = 3 \times 13, 51 = 3 \times 17, 77 = 7 \times 11, 91 = 7 \times 13,$ and $121 = 11 \times 11.$

The largest prime factor among these is $17,$ which is a factor of $51.$

Thus, the correct answer is **B**.

4. $1000 \times 1993 \times 0.1993 \times 10 =$

A 1.993×10^3

B 1993.1993

C $(199.3)^2$

D $1{,}993{,}001.993$

E $(1993)^2$

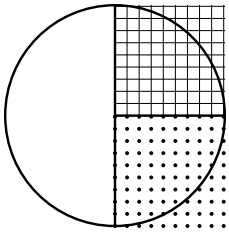
Solution:

Regroup as $(1000 \times 10) \times 0.1993 \times 1993 = 10000 \times 0.1993 \times 1993.$

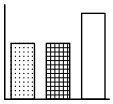
Since $10000 \times 0.1993 = 1993,$ the product is $1993 \times 1993 = (1993)^2.$

Thus, the correct answer is **E**.

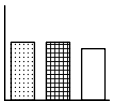
5. Which one of the following bar graphs could represent the data from the circle graph shown?



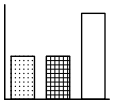
A



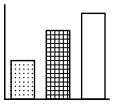
B



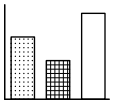
C



D



E



Solution:

The two shaded regions are each one quarter of the circle, and the unshaded region is one half. So the three quantities are in the ratio $\frac{1}{4} : \frac{1}{4} : \frac{1}{2}$, or $1 : 1 : 2$.

A matching bar graph must have the two shaded bars equal in height and the unshaded bar exactly twice as tall. Only one bar graph has two equal shaded bars with the white bar double their height.

Thus, the correct answer is **C** .

6. A can of soup can feed 3 adults or 5 children. If there are 5 cans of soup and 15 children are fed, then how many adults would the remaining soup feed?

A 5

B 6

C 7

D 8

E 10

Solution:

Feeding 15 children uses $15 \div 5 = 3$ cans, leaving $5 - 3 = 2$ cans.

Those 2 cans feed $2 \times 3 = 6$ adults.

Thus, the correct answer is **B** .

7. $3^3 + 3^3 + 3^3 =$

A 3^4

B 9^3

C 3^9

D 27^3

E 3^{27}

Solution:

Adding three equal terms, $3^3 + 3^3 + 3^3 = 3 \times 3^3 = 3^4 = 81.$

Thus, the correct answer is **A** .

8. To control her blood pressure, Jill's grandmother takes one half of a pill every other day. If one supply of medicine contains 60 pills, then the supply of medicine will last approximately

A 1 month

B 4 months

C 6 months

D 8 months

E 1 year

Solution:

She takes half a pill every two days, so one pill lasts 4 days. Then 60 pills last $60 \times 4 = 240$ days.

At about 30 days per month, that is roughly $240 \div 30 = 8$ months.

Thus, the correct answer is **D** .

9. Consider the operation $*$ defined by the following table:

$*$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

For example, $3 * 2 = 1$. Then $(2 * 4) * (1 * 3) =$

A 1

B 2

C 3

D 4

E 5

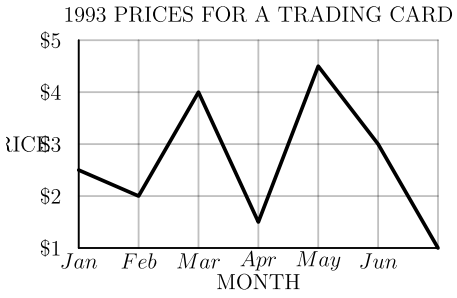
Solution:

From the table, $2 * 4 = 3$ and $1 * 3 = 3$.

Then $(2 * 4) * (1 * 3) = 3 * 3 = 4$.

Thus, the correct answer is **D** .

10. This line graph represents the price of a trading card during the first 6 months of 1993. The greatest monthly drop in price occurred during which month?



A January

B March

C April

D May

E June

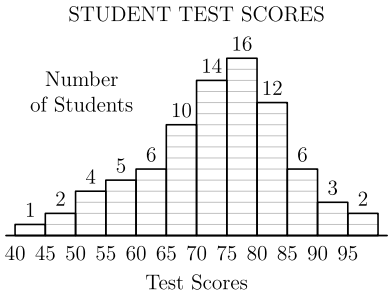
Solution:

The price changes month to month are: January $\$2.50 \to \$2.00$ (drop $\$0.50$), February $\$2.00 \to \$4.00$ (rise), March $\$4.00 \to \$1.50$ (drop $\$2.50$), April $\$1.50 \to \$4.50$ (rise), May $\$4.50 \to \$3.00$ (drop $\$1.50$), and June $\$3.00 \to \$1.00$ (drop $\$2.00$).

The largest drop is $\$2.50$, which occurred during March.

Thus, the correct answer is **B** .

11. Consider this histogram of the scores for 81 students taking a test. The median is in the interval labeled which value?



A 60

B 65

C 70

D 75

E 80

Solution:

Since 81 students took the test, the median is the 41st score counting up from the lowest.

Adding the bar heights from the left gives running totals $1, 3, 7, 12, 18, 28, 42, \ldots$ The total first passes 41 at the interval labeled 70, which contains the 29th through 42nd scores. So the 41st score lies in the interval labeled 70.

Thus, the correct answer is **C** .

12. If each of the three operation signs, $+, -, \times$, is used exactly **once** in one of the blanks in the expression

$$5 \_ 4 \_ 6 \_ 3$$

then the value of the result could equal

A 9

B 10

C 15

D 16

E 19

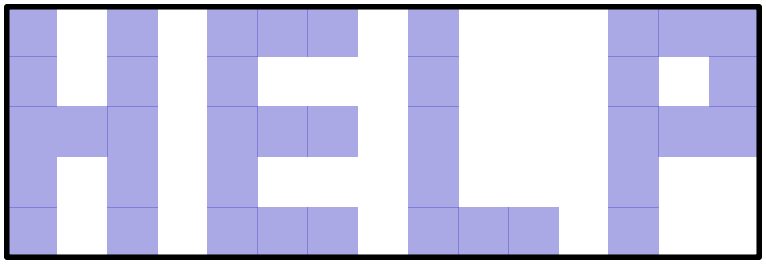
Solution:

The six arrangements give $5 \times 4 + 6 - 3 = 23, 5 \times 4 - 6 + 3 = 17, 5 + 4 \times 6 - 3 = 26, 5 - 4 \times 6 + 3 = -16, 5 + 4 - 6 \times 3 = -9$, and $5 - 4 + 6 \times 3 = 19$.

The only value among the choices is 19.

Thus, the correct answer is **E**.

13. The word "**HELP**" in block letters is painted as a shaded region with strokes 1 unit wide on a 5 by 15 rectangular sign. Each letter is 3 units wide with a 1-unit gap between letters, as shown. The area of the unshaded portion of the sign, in square units, is



A 30

B 32

C 34

D 36

E 38

Solution:

The full sign has area $5 \times 15 = 75$ square units. Counting the shaded unit squares in each letter gives $H = 11, E = 11, L = 7,$ and $P = 10,$ for a shaded total of $11 + 11 + 7 + 10 = 39.$

The unshaded area is $75 - 39 = 36.$

Thus, the correct answer is **D** .

14. The nine squares in the table shown are to be filled so that every row and every column contains each of the numbers $1, 2, 3$. Then $A + B =$

1		
	2	A
		B

A 2

B 3

C 4

D 5

E 6

Solution:

Filling the grid so each row and column has $1, 2, 3$, the top row becomes $1, 3, 2$, the middle row $3, 2, A$, and the bottom row $2, 1, B$. The middle row forces $A = 1$, and the last column $2, 1, B$ forces $B = 3$.

So $A + B = 1 + 3 = 4$.

Thus, the correct answer is **C**.

15. The arithmetic mean (average) of four numbers is 85. If the largest of these numbers is 97, then the mean of the remaining three numbers is

A 81.0

B 82.7

C 83.0

D 84.0

E 84.3

Solution:

The four numbers sum to $4 \times 85 = 340$, so the remaining three sum to $340 - 97 = 243$.

Their mean is $243 \div 3 = 81$.

Thus, the correct answer is **A** .

16. What is the value of the following expression?

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3}}}$$

A $\frac{1}{6}$

B $\frac{3}{10}$

C $\frac{7}{10}$

D $\frac{5}{6}$

E $\frac{10}{3}$

Solution:

Starting inside, $2+\frac{1}{3}=\frac{7}{3}$, so $\frac{1}{7/3}=\frac{3}{7}$.

Then $1+\frac{3}{7}=\frac{10}{7}$, and the whole expression is $\frac{1}{10/7}=\frac{7}{10}$.

Thus, the correct answer is **C** .

17. Square corners, 5 units on a side, are removed from a 20 unit by 30 unit rectangular sheet of cardboard. The sides are then folded to form an open box. The surface area, in square units, of the interior of the box is

A 300

B 500

C 550

D 600

E 1000

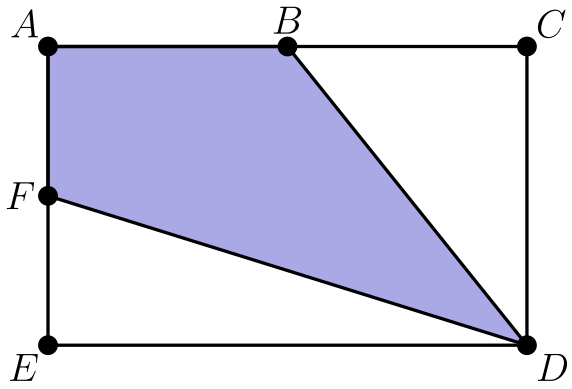
Solution:

The interior surface is exactly one face of the cardboard after the corners are removed. The sheet has area $20 \times 30 = 600$, and each removed corner has area $5 \times 5 = 25$.

So the interior surface area is $600 - 4 \times 25 = 500$.

Thus, the correct answer is **B** .

18. The rectangle shown has length $AC = 32$, width $AE = 20$, and B and F are midpoints of $\overline{AC}$ and $\overline{AE}$, respectively. The area of the quadrilateral $ABDF$ is



A 320

B 325

C 330

D 335

E 340

Solution:

Rectangle $ACDE$ has area $32 \times 20 = 640$. Triangle BCD has area $\frac{16 \times 20}{2} = 160$, and triangle DEF has area $\frac{10 \times 32}{2} = 160$.

The remaining region $ABDF$ has area $640 - (160 + 160) = 320$.

Thus, the correct answer is **A** .

19. What is the value of the following expression?

$$(1901 + 1902 + 1903 + \cdots + 1993) - (101 + 102 + 103 + \cdots + 193)$$

A $167{,}400$

B $172{,}050$

C $181{,}071$

D $199{,}300$

E $362{,}142$

Solution:

Each number in the first sum is exactly 1800 more than the matching number in the second sum, and there are 93 such pairs.

So the difference is $93 \times 1800 = 167{,}400.$

Thus, the correct answer is **A** .

20. When $10^{93} - 93$ is expressed as a single whole number, the sum of the digits is

A 10

B 93

C 819

D 826

E 833

Solution:

Subtracting 93 from 10^{93} (a 1 followed by 93 zeros) gives a number that is 91 nines followed by 07.

The digit sum is $91 \times 9 + 0 + 7 = 819 + 7 = 826$.

Thus, the correct answer is **D** .

21. If the length of a rectangle is increased by 20% and its width is increased by 50%, then the area is increased by

A 10%

B 30%

C 70%

D 80%

E 100%

Solution:

The new length is 1.2 times the old and the new width is 1.5 times the old, so the new area is $1.2 \times 1.5 = 1.8$ times the old area.

That is an increase of 80%.

Thus, the correct answer is **D** .

22. Pat Peano has plenty of 0's, 1's, 3's, 4's, 5's, 6's, 7's, 8's and 9's, but he has only twenty-two 2's. How far can he number the pages of his scrapbook with these digits?

A 22

B 99

C 112

D 119

E 199

Solution:

Numbering 1 through 99 uses ten 2's in the units place and ten in the tens place, a total of twenty 2's. Pages 100 and 101 use none.

The remaining two 2's are used on pages 102 and 112. After that, pages 113 through 119 need no 2, but 120 would require another 2, so he can number up to 119.

Thus, the correct answer is **D** .

23. Five runners, P, Q, R, S, T, have a race, and P beats Q, P beats R, Q beats S, and T finishes after P and before Q. Who could **not** have finished third in the race?

A P and Q

B P and R

C P and S

D P and T

E P, S and T

Solution:

Since P beats Q, R, and T, and no one beats P, runner P finishes first and so cannot be third.

The clues give the chain P before T before Q before S. So P, T, and Q all finish ahead of S, meaning S is no better than fourth and cannot be third either.

Each of Q, R, T can finish third: for example P, T, Q, R, S puts Q third; P, R, T, Q, S puts T third; and P, T, R, Q, S puts R third. So only P and S cannot be third.

Thus, the correct answer is **C** .

24. What number is directly above 142 in this array of numbers?

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & 3 & 4 & & \\ & 5 & 6 & 7 & 8 & 9 & \\ 10 & 11 & 12 & \cdots & & & \end{array}$$

A 99

B 119

C 120

D 121

E 122

Solution:

Each row ends at a perfect square, so the row containing 142 ends at $144 = 12^2$, and the row above it ends at $121 = 11^2$.

Since the rows are aligned at their right edges, 121 sits directly above 143, and therefore 120 sits directly above 142.

Thus, the correct answer is **C** .

25. A checkerboard consists of one-inch squares. A square card, 1.5 inches on a side, is placed on the board so that it covers part or all of the area of each of n squares. The maximum possible value of n is

A 4 or 5

B 6 or 7

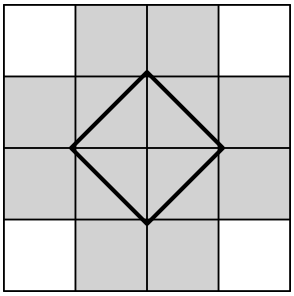
C 8 or 9

D 10 or 11

E 12 or more

Solution:

Tilt the card 45° and center it on a corner where four grid squares meet, as shown. Because the card's diagonal, $\sqrt{1.5^2 + 1.5^2} = \sqrt{4.5} \approx 2.1,$ is longer than $2,$ each of the four corners of the card reaches past a grid line into the next square.



The card covers the central 2×2 block of 4 squares and pokes into 2 more squares on each of its four sides, giving $4 + 4 \times 2 = 12$ squares.

This is also the most possible. The card is only 1.5 inches wide, so its overall width and height are each at most 2.1 inches; it therefore lies within a 4×4 block of 16 squares. Its four pointed corners are the only parts that reach the edge of that block, so it can never reach the four corner squares of the block, leaving at most $12.$ Since 12 is achievable, the maximum is $12,$ which falls in the range "12 or more."

Thus, the correct answer is **E**.

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