1992 AMC 8 Solutions

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1. What is the value of the following expression?

$$\frac{10-9+8-7+6-5+4-3+2-1}{1-2+3-4+5-6+7-8+9}$$

A -1

B 1

C 5

D 9

E 10

Solution:

Grouping the numerator in pairs gives $(10-9)+(8-7)+(6-5)+(4-3)+(2-1)=5.$

Grouping the denominator as $(1-2)+(3-4)+(5-6)+(7-8)+9$ gives four pairs of -1 plus 9, which is $5.$

So the expression is $\frac{5}{5}=1.$

Thus, the correct answer is **B** .

2. Which of the following is **not** equal to $\frac{5}{4}$?

A $\frac{10}{8}$

B $1\frac{1}{4}$

C $1\frac{3}{12}$

D $1\frac{1}{5}$

E $1\frac{10}{40}$

Solution:

Note $\frac{5}{4} = 1\frac{1}{4}$. Now check each choice: $\frac{10}{8} = \frac{5}{4}, 1\frac{1}{4} = \frac{5}{4}, 1\frac{3}{12} = 1\frac{1}{4} = \frac{5}{4}$, and $1\frac{10}{40} = 1\frac{1}{4} = \frac{5}{4}$.

But $1\frac{1}{5} = \frac{6}{5} \neq \frac{5}{4}$.

Thus, the correct answer is **D** .

3. What is the largest difference that can be formed by subtracting two numbers chosen from the set $\{-16, -4, 0, 2, 4, 12\}$?

A 10

B 12

C 16

D 28

E 48

Solution:

The largest difference uses the largest number, 12, minus the smallest number, -16.

This gives $12 - (-16) = 28$.

Thus, the correct answer is **D** .

4. During the softball season, Judy had 35 hits. Among her hits were 1 home run, 1 triple, and 5 doubles. The rest of her hits were singles. What percent of her hits were singles?

A 28%

B 35%

C 70%

D 75%

E 80%

Solution:

The non-single hits number $1 + 1 + 5 = 7$, so the singles number $35 - 7 = 28$.

The fraction of singles is $\frac{28}{35} = \frac{4}{5} = 80\%$.

Thus, the correct answer is **E**.

5. A circle of diameter 1 is removed from a 2×3 rectangle. Which whole number is closest to the area of the shaded region that remains?

A 1

B 2

C 3

D 4

E 5

Solution:

The rectangle has area 6, and the removed circle has area $\pi \left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \approx 0.79.$

So the shaded region has area $6 - 0.79 \approx 5.2$, whose closest whole number is $5.$

Thus, the correct answer is **E**.

6. Define an operation on three numbers—a top number, a lower-left number, and a lower-right number—whose value is (top number) $+$ (lower-left number) $-$ (lower-right number). For example, top 5, lower-left 4, lower-right 6 gives $5 + 4 - 6 = 3$. What is the sum of the operation applied to top 1, lower-left 3, lower-right 4 and the operation applied to top 2, lower-left 5, lower-right 6?

A -2

B -1

C 0

D 1

E 2

Solution:

The first triple gives $1 + 3 - 4 = 0$, and the second gives $2 + 5 - 6 = 1$.

Their sum is $0 + 1 = 1$.

Thus, the correct answer is **D** .

7. The digit-sum of 998 is $9 + 9 + 8 = 26$. How many 3-digit whole numbers, whose digit-sum is 26, are even?

A 1

B 2

C 3

D 4

E 5

Solution:

A digit-sum of 26 with three digits requires the digits $9, 9, 8$. The 3-digit numbers using them are $899, 989,$ and 998.

Of these, only 998 is even, so there is exactly 1 such number.

Thus, the correct answer is **A** .

8. A store owner bought 1500 pencils at \$0.10 each. If he sells them for \$0.25 each, how many of them must he sell to make a profit of exactly \$100.00?

A 400

B 667

C 1000

D 1500

E 1900

Solution:

The pencils cost $1500 \times \$0.10 = \$150.$ To make a \$100 profit, the revenue must be $\$150 + \$100 = \$250.$

At \$0.25 each, he must sell $\dfrac{\$250}{\$0.25} = 1000$ pencils.

Thus, the correct answer is **C** .

9. The population of a small town is 480. A bar graph of the number of females and males, with the vertical scale omitted, shows that there are twice as many females as males. How many males live in the town?

A 120

B 160

C 200

D 240

E 360

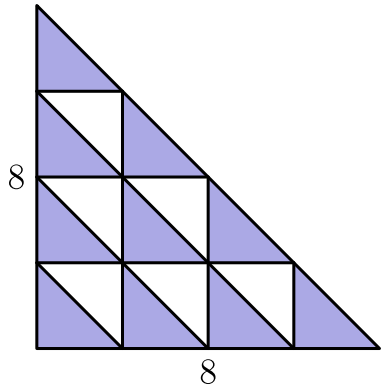
Solution:

If there are M males, then there are $2M$ females, and together $M + 2M = 480$.

So $3M = 480$, giving $M = 160$.

Thus, the correct answer is **B** .

10. An isosceles right triangle with legs of length 8 is partitioned into 16 congruent triangles as shown. The shaded area is



A 10

B 20

C 32

D 40

E 64

Solution:

The large triangle has area $\frac{1}{2} \times 8 \times 8 = 32$, so each of the 16 congruent small triangles has area $\frac{32}{16} = 2$.

Ten of the small triangles are shaded, so the shaded area is $10 \times 2 = 20$.

Thus, the correct answer is **B** .

11. A survey on color preferences gave these results: red 50, blue 60, brown 40, pink 60, green 40. What percent preferred blue?

A 20%

B 24%

C 30%

D 36%

E 42%

Solution:

The total number surveyed is $50 + 60 + 40 + 60 + 40 = 250$.

The percent preferring blue is $\frac{60}{250} = 24\%$.

Thus, the correct answer is **B** .

12. The five tires of a car (four road tires and a full-sized spare) were rotated so that each tire was used the same number of miles during the first $30{,}000$ miles the car traveled. For how many miles was each tire used?

A 6000

B 7500

C $24{,}000$

D $30{,}000$

E $37{,}500$

Solution:

During the $30{,}000$ miles, 4 tires are always in use, so the total tire-mileage is $4 \times 30{,}000 = 120{,}000$ tire-miles.

Split equally among 5 tires, each tire is used $\frac{120{,}000}{5} = 24{,}000$ miles.

Thus, the correct answer is **C** .

13. Five test scores have a mean (average score) of 90, a median (middle score) of 91, and a mode (most frequent score) of 94. The sum of the two lowest test scores is

A 170

B 171

C 176

D 177

E not determined by the information given

Solution:

The five scores sum to $5 \times 90 = 450$. The median is the third score, 91. Since 94 is the mode, it must appear at least twice, and both copies lie above the median, so the two highest scores are $94, 94$.

The three highest scores are $91, 94, 94$, summing to 279. So the two lowest sum to $450 - 279 = 171$.

Thus, the correct answer is **B** .

14. When four gallons are added to a tank that is one-third full, the tank is then one-half full. The capacity of the tank in gallons is

A 8

B 12

C 20

D 24

E 48

Solution:

The 4 gallons account for the change from $\frac{1}{3}$ full to $\frac{1}{2}$ full, which is $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ of the capacity.

If $\frac{1}{6}$ of the capacity is 4 gallons, the full capacity is $4 \times 6 = 24$ gallons.

Thus, the correct answer is **D** .

15. What is the 1992nd letter in this sequence?

ABCDEDCBAABCDEDCBAABCDEDCBAABCDEDC$\cdots$

- A A
- B B
- C C
- D D
- E E

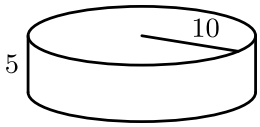
Solution:

The sequence is the 9-letter block $ABCDEDCBA$ repeated again and again.

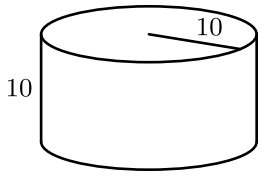
Since $1992 = 9 \times 221 + 3$, the 1992nd letter is the 3rd letter of a block, which is C.

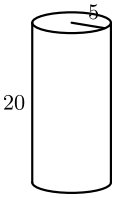
Thus, the correct answer is **C** .

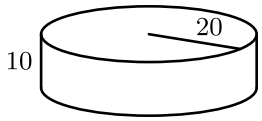
16. Which cylinder has twice the volume of the cylinder shown at right?



A 

B 

C 

D 

E None of the above

Solution:

The given cylinder has volume $\pi \times 10^2 \times 5 = 500\pi$, so a cylinder of twice the volume has volume 1000π.

The choices have volumes $\pi \times 20^2 \times 5 = 2000\pi$ (A), $\pi \times 10^2 \times 10 = 1000\pi$ (B), $\pi \times 5^2 \times 20 = 500\pi$ (C), and $\pi \times 20^2 \times 10 = 4000\pi$ (D). Only (B) equals 1000π.

Thus, the correct answer is **B** .

17. The sides of a triangle have lengths 6.5, 10, and s, where s is a whole number. What is the smallest possible value of s?

A 3

B 4

C 5

D 6

E 7

Solution:

By the triangle inequality, $6.5 + s$ must exceed the longest side 10, so $s > 3.5$.

The smallest whole number greater than 3.5 is 4, and it does form a valid triangle.

Thus, the correct answer is **B** .

18. On a trip, a car traveled 80 miles in an hour and a half, then was stopped in traffic for 30 minutes, then traveled 100 miles during the next 2 hours. What was the car's average speed in miles per hour for the 4-hour trip?

A 45

B 50

C 60

D 75

E 90

Solution:

The car covered $80 + 100 = 180$ miles in total, over $1.5 + 0.5 + 2 = 4$ hours (the traffic stop still counts as time).

The average speed is $\frac{180}{4} = 45$ miles per hour.

Thus, the correct answer is **A** .

19. The distance between the 5th and 26th exits on an interstate highway is 118 miles. If any two exits are at least 5 miles apart, then what is the largest number of miles there can be between two consecutive exits that are between the 5th and 26th exits?

A 8

B 13

C 18

D 47

E 98

Solution:

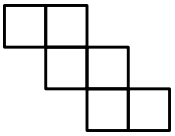
From the 5th exit to the 26th exit there are $26 - 5 = 21$ gaps between consecutive exits, each at least 5 miles.

To maximize one gap, make the other 20 gaps exactly 5 miles, using $20 \times 5 = 100$ miles. The remaining gap is $118 - 100 = 18$ miles.

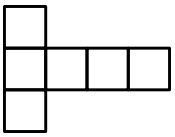
Thus, the correct answer is **C** .

20. Which pattern of identical squares could **not** be folded along the lines shown to form a cube?

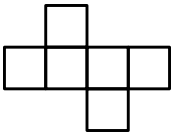
A



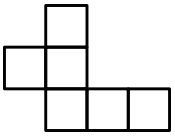
B



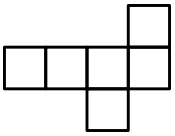
C



D



E



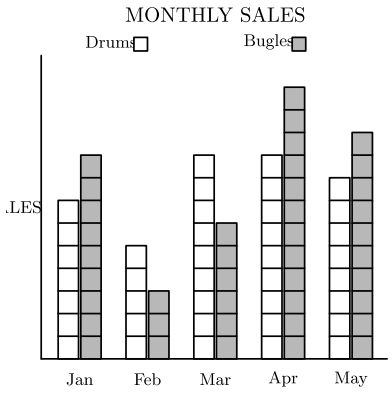
Solution:

Each of the five patterns has 6 squares. Folding patterns (A), (B), (C), and (E) wraps the squares neatly onto the six faces of a cube.

For pattern (D), any attempt to fold forces two of the squares to land on the same face, so they overlap and no cube can be formed.

Thus, the correct answer is **D** .

21. Northside's Drum and Bugle Corps raised money for a trip. The drummers and bugle players kept separate sales records. According to the double bar graph of monthly sales below, in what month did one group's sales exceed the other's by the greatest percent?



A Jan

B Feb

C Mar

D Apr

E May

Solution:

Reading the graph gives (drums, bugles): January $(7, 9)$, February $(5, 3)$, March $(9, 6)$, April $(9, 12)$, and May $(8, 10)$.

The percent excess of the larger over the smaller is $\frac{2}{7} \approx 29\%$ in January, $\frac{2}{3} \approx 67\%$ in February, $\frac{3}{6} = 50\%$ in March, $\frac{3}{9} \approx 33\%$ in April, and $\frac{2}{8} = 25\%$ in May. The greatest is February, where 5 exceeds 3 by about 67%.

Thus, the correct answer is **B** .

22. Eight 1×1 square tiles are arranged so their outside edges form a polygon with a perimeter of 14 units. Two additional tiles of the same size are added to the figure so that at least one side of each added tile is shared with a side of one of the original squares. Which of the following could be the perimeter of the new figure?

A 15

B 17

C 18

D 19

E 20

Solution:

A tile that shares exactly one side adds 2 to the perimeter (four new edges minus two hidden), while a tile sharing two sides adds 0.

With two added tiles the perimeter can change by $0, 2,$ or $4,$ giving new perimeters of $14, 16,$ or $18.$ Of the choices, only 18 is possible.

Thus, the correct answer is **C** .

23. If two dice are tossed, the probability that the product of the numbers showing on the tops of the dice is greater than 10 is

A $\frac{3}{7}$

B $\frac{17}{36}$

C $\frac{1}{2}$

D $\frac{5}{8}$

E $\frac{11}{12}$

Solution:

There are 36 equally likely outcomes. Counting the ordered pairs whose product exceeds 10: with a first die of 2 there is 1 (namely 2×6); of 3, there are 3; of 4, there are 4; of 5, there are 4; of 6, there are 5.

That is $1 + 3 + 4 + 4 + 5 = 17$ favorable outcomes, so the probability is $\frac{17}{36}$.

Thus, the correct answer is **B** .

24. Four circles of radius 3 are arranged so that their centers are the vertices of a square and each circle is tangent to its two neighbors. The shaded region is the part of the square that lies outside all four circles. The area of the shaded region is closest to

A 7.7

B 12.1

C 17.2

D 18

E 27

Solution:

Because adjacent circles are tangent, the square through the centers has side $2 \times 3 = 6$ and area $36.$

Inside the square, each circle contributes a quarter-circle, and the four quarters make one full circle of area $9\pi \approx 28.3.$ The shaded region is $36 - 9\pi \approx 7.7.$

Thus, the correct answer is **A** .

25. One half of the water is poured out of a full container. Then one third of the remainder is poured out. Continue the process: one fourth of the remainder for the third pouring, one fifth of the remainder for the fourth pouring, and so on. After how many pourings does exactly one tenth of the original water remain?

A 6

B 7

C 8

D 9

E 10

Solution:

After the nth pouring, the fraction remaining is $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \cdots \times \frac{n}{n+1}$, which telescopes to $\frac{1}{n+1}$.

Setting $\frac{1}{n+1} = \frac{1}{10}$ gives $n = 9$.

Thus, the correct answer is **D** .

Problems: https://live.poshenloh.com/past-contests/amc8/1992