1991 AMC 8 Solutions

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1. What is the value of

$$1{,}000{,}000{,}000{,}000 - 777{,}777{,}777{,}777?$$

A $222{,}222{,}222{,}222$

B $222{,}222{,}222{,}223$

C $233{,}333{,}333{,}333$

D $322{,}222{,}222{,}223$

E $333{,}333{,}333{,}333$

Solution:

Lining up the subtraction gives

$$1{,}000{,}000{,}000{,}000 - 777{,}777{,}777{,}777 = 222{,}222{,}222{,}223.$$

Equivalently, to climb from $777{,}777{,}777{,}777$ up to $1{,}000{,}000{,}000{,}000$ you add 3 in the units place and 2 in each of the other eleven places.

Thus, the correct answer is **B** .

2. What is the value of

$$\frac{16+8}{4-2}?$$

- A 4
- B 8
- C 12
- D 16
- E 20

Solution:

$$\frac{16+8}{4-2}=\frac{24}{2}=12.$$

Thus, the correct answer is **C** .

3. Two hundred thousand times two hundred thousand equals

- A four hundred thousand
- B four million
- C forty thousand
- D four hundred million
- E forty billion

Solution:

$200{,}000 \times 200{,}000 = 4 \times 10^{10} = 40{,}000{,}000{,}000$, which is forty billion.

Thus, the correct answer is **E** .

4. If $991 + 993 + 995 + 997 + 999 = 5000 - N$, then $N =$

- A 5
- B 10
- C 15
- D 20
- E 25

Solution:

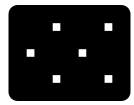
Since $991 + 993 + 995 + 997 + 999 = (1000 - 9) + (1000 - 7) + (1000 - 5) + (1000 - 3) + (1000 - 1) = 5000 - (9 + 7 + 5 + 3 + 1) = 5000 - 25$, we get $N = 25$.

Thus, the correct answer is **E** .

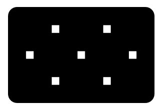
5. A "domino" is made up of two small squares:

. Which of the "checkerboards" illustrated below CANNOT be covered exactly and completely by a whole number of non-overlapping dominoes?

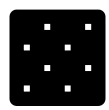
A 3×4



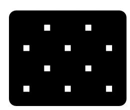
B 3×5



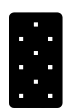
C 4×4



D 4×5



E 6×3



Solution:

Every domino covers exactly 2 squares, so any board that is completely covered by non-overlapping dominoes must contain an even number of small squares.

Counting squares: $3 \times 4 = 12, 3 \times 5 = 15, 4 \times 4 = 16, 4 \times 5 = 20$, and $6 \times 3 = 18$. Only $3 \times 5 = 15$ is odd, so that board cannot be covered. (Each of the even boards has a side of even length and is easily tiled with dominoes.)

Thus, the correct answer is **B** .

6. Which number in the array below is both the largest in its column and the smallest in its row? (Columns go up and down, rows go right and left.)

$$\begin{array}{ccccc} 10 & 6 & 4 & 3 & 2 \\ 11 & 7 & 14 & 10 & 8 \\ 8 & 3 & 4 & 5 & 9 \\ 13 & 4 & 15 & 12 & 1 \\ 8 & 2 & 5 & 9 & 3 \end{array}$$

- A 1
- B 6
- C 7
- D 12
- E 15

Solution:

The largest entry in each column is 13 (column 1), 7 (column 2), 15 (column 3), 12 (column 4), and 9 (column 5).

Of these, only 7 is the smallest number in its own row (row 2 is $11, 7, 14, 10, 8$).

Thus, the correct answer is **C** .

7. The value of

$$\frac{(487{,}000)(12{,}027{,}300) + (9{,}621{,}001)(487{,}000)}{(19{,}367)(.05)}$$

is closest to

- A $10{,}000{,}000$
- B $100{,}000{,}000$
- C $1{,}000{,}000{,}000$
- D $10{,}000{,}000{,}000$
- E $100{,}000{,}000{,}000$

Solution:

Factor the numerator: $487{,}000\,(12{,}027{,}300 + 9{,}621{,}001)$.

Rounding to leading digits, this is about $500{,}000 \times (10{,}000{,}000 + 10{,}000{,}000) = 500{,}000 \times 2 \times 10^7$. The denominator is about $20{,}000 \times .05 = 1000$.

So the value is roughly $\dfrac{500{,}000 \times 2 \times 10^7}{1000} = 10^{10} = 10{,}000{,}000{,}000$.

Thus, the correct answer is **D** .

8. What is the largest quotient that can be formed using two numbers chosen from the set $\{-24, -3, -2, 1, 2, 8\}$?

- A -24
- B -3
- C 8
- D 12
- E 24

Solution:

For a large quotient it should be positive, so use two positive numbers or two negative numbers.

Best positive pair: $\frac{8}{1} = 8$. Best negative pair: $\frac{-24}{-2} = 12$. The larger is 12.

Thus, the correct answer is **D** .

9. How many whole numbers from 1 through 46 are divisible by either 3 or 5 or both?

- A 18
- B 21
- C 24
- D 25
- E 27

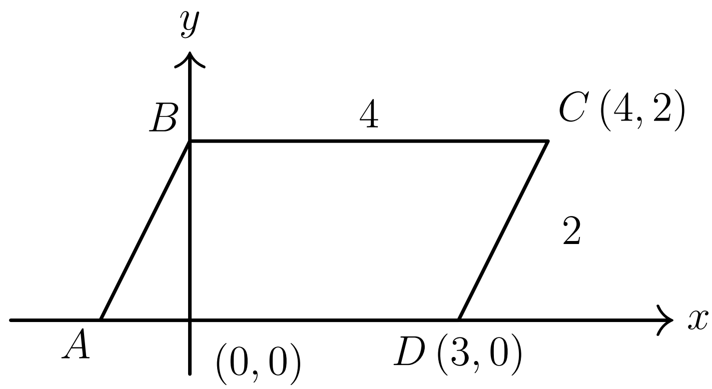
Solution:

There are 15 multiples of 3 and 9 multiples of 5 up to 46. The 3 multiples of 15 (namely $15, 30, 45$) were counted twice.

By inclusion-exclusion, the count is $15 + 9 - 3 = 21.$

Thus, the correct answer is **B** .

10. The area in square units of the region enclosed by parallelogram $ABCD$ is



A 6

B 8

C 12

D 15

E 18

Solution:

Side BC runs from $(0, 2)$ to $(4, 2)$, so the base is 4. The opposite side AD lies on the x-axis, so the height is $2.$

The area is $4 \times 2 = 8.$

Thus, the correct answer is **B** .

11. There are several sets of three different numbers whose sum is 15 which can be chosen from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many of these sets contain a 5?

- A 3
- B 4
- C 5
- D 6
- E 7

Solution:

With 5 chosen, the other two different numbers must sum to 10. The pairs are $1 + 9,\ 2 + 8,\ 3 + 7,\ 4 + 6,$ giving 4 sets.

Thus, the correct answer is **B** .

12. If

$$\frac{2+3+4}{3}=\frac{1990+1991+1992}{N},$$

then $N=$

A 3

B 6

C 1990

D 1991

E 1992

Solution:

The left side is $\frac{9}{3}=3$. The right side is $\frac{5973}{N}$, and setting it equal to 3 gives $N=1991$. Equivalently, $(k-1)+k+(k+1)=3k$, so dividing by 3 leaves the middle term. Here the middle term is 1991.

Thus, the correct answer is **D** .

13. How many zeros are at the end of the product

$$25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 8 \times 8 \times 8?$$

- A 3
- B 6
- C 9
- D 10
- E 12

Solution:

Since $25 = 5^2$, the seven 25's give 5^{14}. Since $8 = 2^3$, the three 8's give 2^9.

The number of trailing zeros is $\min(14, 9) = 9$.

Thus, the correct answer is **C** .

14. Several students are competing in a series of three races. A student earns 5 points for winning a race, 3 points for finishing second, and 1 point for finishing third. There are no ties. What is the smallest number of points that a student must earn in the three races to be guaranteed of earning more points than any other student?

A 9

B 10

C 11

D 13

E 15

Solution:

A total of 11 (for example $5 + 5 + 1$) does not guarantee first place, since another student could also reach 11.

But if one student scores $5 + 5 + 3 = 13$, the remaining places give every other student at most $3 + 3 + 5 = 11$. So 13 points guarantees the lead.

Thus, the correct answer is **D** .

15. All six faces of a rectangular solid are rectangles, and the solid measures 1 foot by 3 feet by 9 feet. A one-foot cube is cut out of the top of the solid to form a notch; the notch spans the full one-foot depth (from the front face to the back face) and lies partway along the nine-foot length. The total number of square feet in the surface of the new solid is how many more or less than that of the original solid?

A 2 less

B 1 less

C the same

D 1 more

E 2 more

Solution:

The removed cube had three faces on the surface of the solid (top, front, and back), so 3 square feet of surface are removed.

Cutting it out exposes three new faces (the floor of the notch and its two side walls), adding 3 square feet. The surface area is unchanged.

Thus, the correct answer is **C** .

16. The 16 squares on a piece of paper are numbered as shown in the diagram. While lying on a table, the paper is folded in half four times in the following sequence:

(1) fold the top half over the bottom half; (2) fold the bottom half over the top half; (3) fold the right half over the left half; (4) fold the left half over the right half.

Which numbered square is on top after step 4?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

A 1

B 9

C 10

D 14

E 16

Solution:

Fold 1 (top over bottom) leaves squares 9–16 on the bottom. Fold 2 (bottom over top) leaves 9–12 on the bottom. Fold 3 (right over left) leaves 9 and 10 on the bottom.

Fold 4 (left over right) puts 10 on the bottom and brings 9 to the top.

Thus, the correct answer is **B** .

17. An auditorium with 20 rows of seats has 10 seats in the first row. Each successive row has one more seat than the previous row. If students taking an exam are permitted to sit in any row, but not next to another student in that row, then the maximum number of students that can be seated for an exam is

A 150

B 180

C 200

D 400

E 460

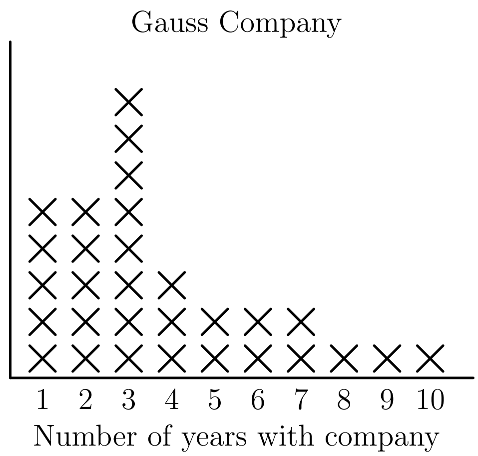
Solution:

Row k has $9+k$ seats, so it holds $\lceil (9+k)/2 \rceil$ students. For rows 1 through 20 (seats 10 through 29) the maxima are $5, 6, 6, 7, 7, 8, 8, \ldots, 14, 14, 15.$

These sum to $200.$

Thus, the correct answer is **C** .

18. The vertical axis indicates the number of employees, but the scale was accidentally omitted from this graph. What percent of the employees at the Gauss Company have worked there for 5 years or more?



A 9%

B $23\frac{1}{3}\%$

C 30%

D $42\frac{6}{7}\%$

E 50%

Solution:

No matter the missing scale, each X represents the same number of employees. There are 9 X's over years 5 through 10, and 30 X's in all.

So the fraction is $\frac{9}{30} = 30\%$.

Thus, the correct answer is **C** .

19. The average (arithmetic mean) of 10 different positive whole numbers is 10. The largest possible value of any of these numbers is

- A 10
- B 50
- C 55
- D 90
- E 91

Solution:

The ten numbers sum to 100. To maximize one of them, the other nine (all different positive whole numbers) should be as small as possible: $1 + 2 + \cdots + 9 = 45$.

The largest number is then $100 - 45 = 55$.

Thus, the correct answer is **C** .

20. In the addition problem shown, each digit has been replaced by a letter. If different letters represent different digits, then $C =$

$$\begin{array}{cccc} & A & B & C \\ & & A & B \\ + & & & A \\ \hline & 3 & 0 & 0 \end{array}$$

A 1

B 3

C 5

D 7

E 9

Solution:

The three numbers add to $111A + 11B + C = 300$. Since $A = 1$ is too small and $A \geq 3$ is too large, $A = 2$.

Then $11B + C = 78$, which forces $B = 7$ and $C = 1$. So $C = 1$.

Thus, the correct answer is **A** .

21. For every 3° rise in temperature, the volume of a certain gas expands by 4 cubic centimeters. If the volume of the gas is 24 cubic centimeters when the temperature is 32°, what was the volume of the gas in cubic centimeters when the temperature was 20°?

A 8

B 12

C 15

D 16

E 40

Solution:

From 32° to 20° is a 12° decrease, which is 4 steps of 3°.

The volume decreases by $4 \times 4 = 16$ cubic centimeters, from 24 down to $24 - 16 = 8$.

Thus, the correct answer is **A** .

22. One spinner is divided into three equal parts labeled $1, 2,$ and $3.$ A second spinner is divided into three equal parts labeled $4, 5,$ and $6.$ Each spinner is spun once and the two resulting numbers are multiplied. What is the probability that this product is an even number?

A $\frac{1}{3}$

B $\frac{1}{2}$

C $\frac{2}{3}$

D $\frac{7}{9}$

E 1

Solution:

The product is odd only when both numbers are odd. The first spinner is odd $(1$ or $3)$ with probability $\frac{2}{3}$, and the second is odd (5) with probability $\frac{1}{3}$.

So the product is odd with probability $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$, and even with probability $1 - \frac{2}{9} = \frac{7}{9}$.

Thus, the correct answer is **D** .

23. The Pythagoras High School band has 100 female and 80 male members. The Pythagoras High School orchestra has 80 female and 100 male members. There are 60 females who are members in both band and orchestra. Altogether, there are 230 students who are in either band or orchestra or both. The number of males in the band who are NOT in the orchestra is

A 10

B 20

C 30

D 50

E 70

Solution:

Females in band or orchestra: $100 + 80 - 60 = 120$. So males in at least one group: $230 - 120 = 110$.

With 80 males in band and 100 in orchestra, the males in both are $80 + 100 - 110 = 70$. Hence males in band but not orchestra: $80 - 70 = 10$.

Thus, the correct answer is **A** .

24. A cube of edge 3 cm is cut into N smaller cubes, not all the same size. If the edge of each of the smaller cubes is a whole number of centimeters, then $N =$

A 4

B 8

C 12

D 16

E 20

Solution:

The $3 \times 3 \times 3$ cube has volume $27.$ Since the cubes are not all the same size, at most one edge-2 cube (volume 8) fits.

The remaining $27 - 8 = 19$ of volume is filled by 19 unit cubes. That is $1 + 19 = 20$ cubes.

Thus, the correct answer is **E** .

25. An equilateral triangle is originally painted black. Each time the triangle is changed, the middle fourth of each black triangle turns white. After five changes, what fractional part of the original area of the black triangle remains black?

A $\frac{1}{1024}$

B $\frac{15}{64}$

C $\frac{243}{1024}$

D $\frac{1}{4}$

E $\frac{81}{256}$

Solution:

Each change leaves $\frac{3}{4}$ of the current black area black. After five changes the black fraction is

$$\left(\frac{3}{4}\right)^5 = \frac{243}{1024}.$$

Thus, the correct answer is **C** .

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