1990 AMC 8 Solutions

Typeset by: LIVE by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc8/1990/solutions

1. What is the smallest sum of two 3-digit numbers that can be obtained by placing each of the six digits $4, 5, 6, 7, 8, 9$ into one of the six boxes of a sum of two 3-digit numbers?

A 947

B 1037

C 1047

D 1056

E 1245

Solution:

To make the sum small, the two smallest digits go in the hundreds places, the next two in the tens places, and the two largest in the units places.

One such arrangement is $468 + 579 = 1047$, and every arrangement of this type gives the same sum.

Thus, the correct answer is **C** .

2. Which digit of $.12345$, when changed to 9, gives the largest number?

A 1

B 2

C 3

D 4

E 5

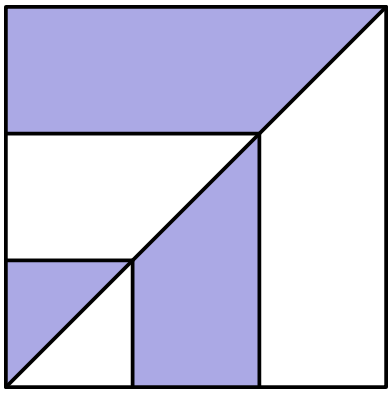
Solution:

Changing a digit in the tenths place changes the number more than changing any digit farther to the right. The digit in the tenths place is 1.

Changing it gives $.92345$, which is the largest possible result.

Thus, the correct answer is **A** .

3. What fraction of the square is shaded?



A $\frac{1}{3}$

B $\frac{2}{5}$

C $\frac{5}{12}$

D $\frac{3}{7}$

E $\frac{1}{2}$

Solution:

The diagonal from one corner to the opposite corner splits the square into two equal halves. Every shaded piece on one side of the diagonal is the mirror image of an equal unshaded piece on the other side.

So the shaded and unshaded areas are equal, and exactly $\frac{1}{2}$ of the square is shaded.

Thus, the correct answer is **E** .

4. Which of the following could **not** be the units digit [ones digit] of the square of a whole number?

A 1

B 4

C 5

D 6

E 8

Solution:

The units digit of a square is determined by the units digit of the number squared. Squaring 0 through 9 gives units digits $0, 1, 4, 9, 6, 5, 6, 9, 4, 1$.

So a square can only end in $0, 1, 4, 5, 6,$ or 9; it can never end in $2, 3, 7,$ or 8. Among the choices, only 8 is impossible.

Thus, the correct answer is **E**.

5. Which of the following is closest to the product $(.48017)(.48017)(.48017)$?

A 0.011

B 0.110

C 1.10

D 11.0

E 110

Solution:

Since $.48017$ is close to $\frac{1}{2}$, the product is close to

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.125.$$

The choice closest to 0.125 is 0.110.

Thus, the correct answer is **B** .

6. Which of these five numbers is the largest?

A $13579 + \frac{1}{2468}$

B $13579 - \frac{1}{2468}$

C $13579 \times \frac{1}{2468}$

D $13579 \div \frac{1}{2468}$

E 13579.2468

Solution:

Choices A, B, and E are all very close to 13579, and choice C multiplies 13579 by a tiny number, making it much smaller.

Choice D divides by $\frac{1}{2468}$, which is the same as multiplying by 2468. This makes it thousands of times larger than 13579, so it is the largest.

Thus, the correct answer is **D** .

7. When three different numbers from the set $\{-3, -2, -1, 4, 5\}$ are multiplied, the largest possible product is

A 10

B 20

C 30

D 40

E 60

Solution:

For the product of three numbers to be positive, either all three are positive or exactly two are negative. There are only two positive numbers, so we must use two negatives and one positive.

To maximize, take the two most negative numbers and the largest positive: $(-3)(-2)(5) = 30$.

Thus, the correct answer is **C** .

8. A dress originally priced at $\$80$ was put on sale at 25% off. If 10% tax was added to the sale price, then the total selling price of the dress was

A $\$45$

B $\$52$

C $\$54$

D $\$66$

E $\$68$

Solution:

The sale price is $\frac{3}{4} \cdot \$80 = \$60.$

The tax is 10% of $\$60,$ which is $\$6,$ so the total is $\$60 + \$6 = \$66.$

Thus, the correct answer is **D** .

9. The grading scale shown is used at Jones Junior High. The fifteen scores in Mr. Freeman's class were: $89, 72, 54, 97, 77, 92, 85, 74, 75, 63, 84, 78, 71, 80, 90$.

The grading scale is A: 93–100, B: 85–92, C: 75–84, D: 70–74, F: 0–69.

In Mr. Freeman's class, what percent of the students received a grade of C?

A 20%

B 25%

C 30%

D $33\frac{1}{3}\%$

E 40%

Solution:

A grade of C corresponds to scores from 75 to 84. The qualifying scores are $77, 75, 84, 78, 80$, which is 5 students.

So the percent is $\frac{5}{15} = \frac{1}{3} = 33\frac{1}{3}\%$.

Thus, the correct answer is **D** .

10. On this monthly calendar, the date behind one of the letters is added to the date behind C. If this sum equals the sum of the dates behind A and B, then the letter is

Tues.	Wed.	Thurs.	Fri.	Sat.
		C	A	
		Q		
S	B	P	T	R

A P

B Q

C R

D S

E T

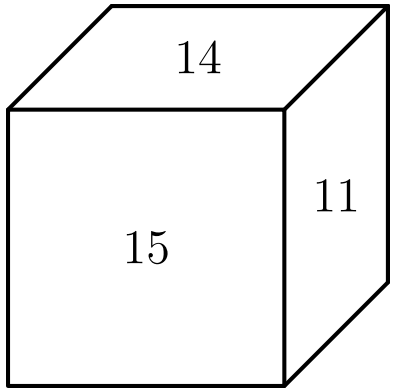
Solution:

Moving one column right adds 1 to the date, and moving one row down adds 7. Let C have date n. Then A, one column to the right, is $n+1$. The letter P sits two rows directly below C, so $P = n+14$, and B, one column left of P in the bottom row, is $n+13$.

We need a letter whose date d satisfies $d+n = (n+1)+(n+13)$, so $d = n+14$. That date is behind P.

Thus, the correct answer is **A** .

11. The numbers on the faces of this cube are consecutive whole numbers. The sums of the two numbers on each of the three pairs of opposite faces are equal. The sum of the six numbers on this cube is



A 75

B 76

C 78

D 80

E 81

Solution:

The six consecutive numbers include $11, 14, 15$, so five of the faces are $11, 12, 13, 14, 15$ and the sixth is 10 or 16.

If the sixth were 10, equal opposite sums would force the pairs $(10, 15), (11, 14), (12, 13)$, making 11 and 14 opposite. But the figure shows $11, 14, 15$ meeting at one corner, so no two of them are opposite. Hence the sixth number is 16, with pairs $(11, 16), (12, 15), (13, 14)$, each summing to 27.

The total is $3 \cdot 27 = 81$.

Thus, the correct answer is **E**.

12. There are twenty-four 4-digit whole numbers that use each of the four digits 2, 4, 5, and 7 exactly once. Listed in numerical order from smallest to largest, the number in the 17th position in the list is

A 4527

B 5724

C 5742

D 7245

E 7524

Solution:

Each leading digit accounts for 6 of the 24 numbers. Positions 1–6 begin with 2, positions 7–12 begin with 4, and positions 13–18 begin with 5.

So the 17th number is the 5th one beginning with 5: 5247, 5274, 5427, 5472, 5724. The fifth is 5724.

Thus, the correct answer is **B** .

13. One proposal for new postage rates for a letter was $30¢$ for the first ounce and $22¢$ for each additional ounce (or fraction of an ounce). The postage for a letter weighing 4.5 ounces was

A $96¢$

B $1.07

C $1.18

D $1.20

E $1.40

Solution:

The first ounce costs $30¢$. The remaining 3.5 ounces are charged as 4 additional ounces, since any fraction rounds up to a full ounce.

That is $4 \cdot 22¢ = 88¢$, so the total is $30¢ + 88¢ = 118¢ = \$1.18.$

Thus, the correct answer is **C** .

14. A bag contains only blue balls and green balls. There are 6 blue balls. If the probability of drawing a blue ball at random from this bag is $\frac{1}{4}$, then the number of green balls in the bag is

A 12

B 18

C 24

D 30

E 36

Solution:

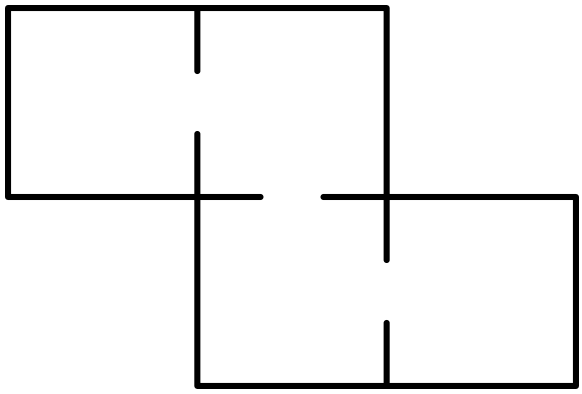
Since blue balls make up $\frac{1}{4}$ of the bag and there are 6 of them, the total is $6 \cdot 4 = 24$ balls.

So the number of green balls is $24 - 6 = 18$.

Thus, the correct answer is **B** .

15. The area of this figure is $100\ \text{cm}^2$. Its perimeter is

(The figure consists of four identical squares.)



A $20\ \text{cm}$

B $25\ \text{cm}$

C $30\ \text{cm}$

D $40\ \text{cm}$

E $50\ \text{cm}$

Solution:

Each of the four squares has area $\frac{100}{4} = 25\ \text{cm}^2$, so each side is $5\ \text{cm}$.

The outline of this staircase shape is made up of 10 such sides, so the perimeter is $10 \cdot 5 = 50\ \text{cm}$.

Thus, the correct answer is **E**.

16. What is the value of the following expression?

$$1990 - 1980 + 1970 - 1960 + \cdots - 20 + 10$$

A -990

B -10

C 990

D 1000

E 1990

Solution:

Group the terms as $(1990 - 1980) + (1970 - 1960) + \cdots + (30 - 20) + 10$. Each parenthesized pair equals 10.

The first elements $1990, 1970, \ldots, 30$ number 99, so there are 99 pairs, giving 990, plus the leftover $+10$ at the end for a total of 1000.

Thus, the correct answer is **D** .

17. A straight concrete sidewalk is to be 3 feet wide, 60 feet long, and 3 **inches** thick. How many cubic yards of concrete must a contractor order for the sidewalk if concrete must be ordered in a whole number of cubic yards?

A 2

B 5

C 12

D 20

E more than 20

Solution:

The thickness is 3 inches $= \frac{1}{4}$ foot, so the volume is $3 \cdot 60 \cdot \frac{1}{4} = 45$ cubic feet.

Since 1 cubic yard $= 27$ cubic feet, this is $\frac{45}{27} = 1\frac{2}{3}$ cubic yards. Rounding up to a whole number, the contractor must order 2 cubic yards.

Thus, the correct answer is **A** .

18. Every corner of a rectangular prism is cut off by a straight slice through the three edges meeting at that corner, removing a small triangular piece at each of the eight corners. How many edges does the new figure have?

A 24

B 30

C 36

D 42

E 48

Solution:

A rectangular prism starts with 12 edges, and cutting corners only shortens them without removing any.

Each of the 8 corner cuts creates a small triangular face with 3 new edges, adding $8 \cdot 3 = 24$ edges. The total is $12 + 24 = 36$.

Thus, the correct answer is **C** .

19. There are 120 seats in a row. What is the fewest number of seats that must be occupied so the next person to be seated must sit next to someone?

A 30

B 40

C 41

D 60

E 119

Solution:

To force the next person next to someone, every empty seat must be adjacent to an occupied one. The most efficient way is to seat people in a repeating pattern of occupied-empty-empty, filling the middle seat of every group of three.

With 120 seats, this uses $\frac{120}{3} = 40$ occupied seats.

Thus, the correct answer is **B** .

20. The annual incomes of $1,000$ families range from \$8200 to \$98,000. In error, the largest income was entered on the computer as \$980,000. The difference between the mean of the incorrect data and the mean of the actual data is

A \$882

B \$980

C \$1078

D \$482,000

E \$882,000

Solution:

Only one entry changed. The incorrect total exceeds the actual total by $\$980{,}000 - \$98{,}000 = \$882{,}000.$

Since this extra amount is spread over 1000 families, the means differ by $\dfrac{\$882{,}000}{1000} = \$882.$

Thus, the correct answer is **A** .

21. A list of 8 numbers is formed by beginning with two given numbers. Each new number in the list is the product of the two previous numbers. Find the first number if the last three are $16, 64, 1024$:

$$?, \_, \_, \_, \_, 16, 64, 1024$$

A $\frac{1}{64}$

B $\frac{1}{4}$

C 1

D 2

E 4

Solution:

Since each term is the product of the two before it, dividing any term by the term just before it recovers the term two positions earlier. Working backwards from the last three terms $16, 64, 1024$:

$$64 \div 16 = 4, \quad 16 \div 4 = 4, \quad 4 \div 4 = 1, \quad 4 \div 1 = 4, \quad 1 \div 4 = \frac{1}{4}.$$

These fill in the earlier terms, giving the full list $\frac{1}{4}, 4, 1, 4, 4, 16, 64, 1024$. The first number is $\frac{1}{4}$.

Thus, the correct answer is **A** .

22. Several students are seated at a large circular table. They pass around a bag containing 100 pieces of candy. Each person receives the bag, takes one piece of candy, and then passes the bag to the next person. If Chris takes the first and the last piece of candy, then the number of students at the table could be

A 10

B 11

C 19

D 20

E 25

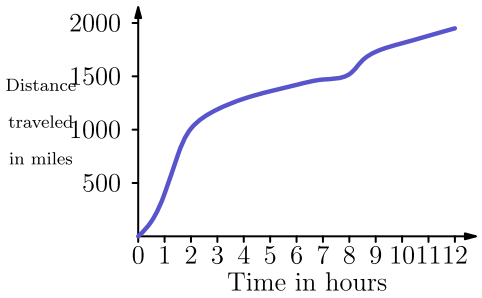
Solution:

Chris takes the first piece, and then the bag goes around until Chris takes the last (100 th) piece. So the 99 pieces after Chris's first must be exactly a whole number of trips around the table back to Chris.

This means the number of students divides 99. Among the choices, only 11 divides 99.

Thus, the correct answer is **B** .

23. The graph relates the distance traveled (in miles) to the time elapsed (in hours) on a trip taken by an experimental airplane. During which hour was the average speed of this airplane the largest?



A first $(0\text{-}1)$

B second $(1\text{-}2)$

C third $(2\text{-}3)$

D ninth $(8\text{-}9)$

E last $(11\text{-}12)$

Solution:

The average speed during any single hour equals the distance the airplane covers in that hour, which on the graph is the vertical rise of the curve over that one-hour interval. So the largest average speed happens during the hour where the curve is steepest.

From $t = 1$ to $t = 2$ the curve climbs from about 500 miles to about 1000 miles, a rise of roughly 500 miles, giving an average speed near 500 mph. During every other hour the curve rises by less than 350 miles, so those average speeds are all smaller. The steepest climb, and hence the largest average speed, is during the second hour.

Thus, the correct answer is **B** .

24. Three triangles and a diamond balance nine dots. Also, one triangle balances a diamond and a dot. How many dots will balance two diamonds?

A 1

B 2

C 3

D 4

E 5

Solution:

Let a triangle, diamond, and dot weigh t, d, and 1. The conditions give $3t + d = 9$ and $t = d + 1$.

Substituting, $3(d + 1) + d = 9$, so $4d + 3 = 9$ and $d = \frac{3}{2}$. Then two diamonds weigh $2d = 3$ dots.

Thus, the correct answer is **C** .

25. How many different patterns can be made by shading exactly two of the nine unit squares in a 3×3 grid? Patterns that can be matched by flips and/or turns are not considered different.

A 3

B 6

C 8

D 12

E 18

Solution:

Classify the two shaded squares up to rotations and reflections. Patterns that include a corner: two adjacent corners (same edge), two diagonally opposite corners, corner with the center, corner with an adjacent edge-middle, and corner with a far edge-middle. That is 5 patterns.

Patterns with no corner: two adjacent edge-middles, two opposite edge-middles, and an edge-middle with the center. That is 3 more.

In total there are $5 + 3 = 8$ distinct patterns.

Thus, the correct answer is **C** .

Problems: https://live.poshenloh.com/past-contests/amc8/1990