

1988 AMC 8 Solutions

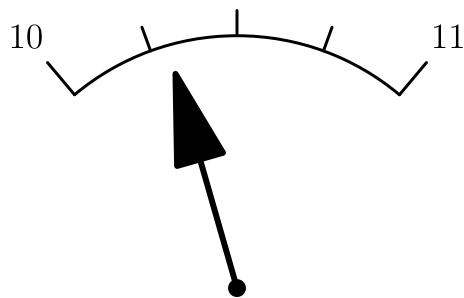
Typeset by: LIVE by Po-Shen Loh

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1. The diagram shows part of a scale of a measuring device. The arrow indicates an approximate reading of



- A 10.05
- B 10.15
- C 10.25
- D 10.3
- E 10.6

Solution:

The scale is divided into fourths between 10 and 11, so the interior tick marks fall at 10.25, 10.5, and 10.75.

The arrow points just past the 10.25 mark, so the reading is between 10.25 and 10.5. Of the choices, only 10.3 lies in that range.

Thus, the correct answer is **D**.

2. What is the product $8 \times .25 \times 2 \times .125$?

A $\frac{1}{8}$

B $\frac{1}{4}$

C $\frac{1}{2}$

D 1

E 2

Solution:

Write the decimals as fractions: $8 \times \frac{1}{4} \times 2 \times \frac{1}{8}$.

Grouping, $\left(8 \times \frac{1}{8}\right) \left(\frac{1}{4} \times 2\right) = 1 \times \frac{1}{2} = \frac{1}{2}$.

Thus, the correct answer is **C**.

3. What is the value of the following expression?

$$\frac{1}{10} + \frac{2}{20} + \frac{3}{30}$$

- A .1
- B .123
- C .2
- D .3
- E .6

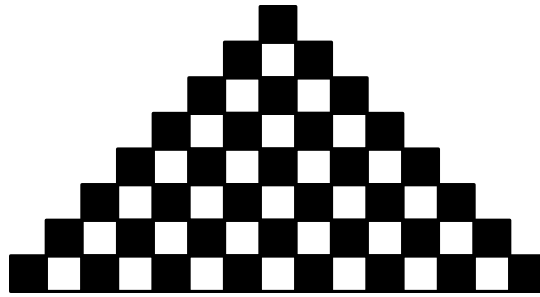
Solution:

Each fraction equals $\frac{1}{10} = .1$, since $\frac{2}{20} = \frac{3}{30} = \frac{1}{10}$.

So the sum is $.1 + .1 + .1 = .3$.

Thus, the correct answer is **D**.

4. The figure consists of alternating light and dark squares. The number of dark squares exceeds the number of light squares by



- A 7
- B 8
- C 9
- D 10
- E 11

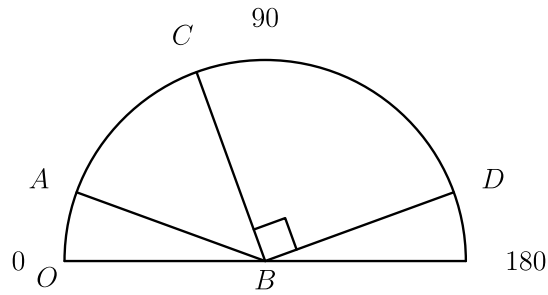
Solution:

In each horizontal row the squares alternate dark, light, dark, and so on, beginning and ending with a dark square. So every row has exactly one more dark square than light square.

The figure has 8 rows, so the dark squares exceed the light squares by 8.

Thus, the correct answer is **B**.

5. If $\angle CBD$ is a right angle, then this protractor indicates that the measure of $\angle ABC$ is approximately



- A 20°
- B 40°
- C 50°
- D 70°
- E 120°

Solution:

Reading the protractor, ray BA is at about 20° and ray BD is at about 160° .

Because $\angle CBD$ is a right angle, ray BC is at $160^\circ - 90^\circ = 70^\circ$.

Therefore $\angle ABC = 70^\circ - 20^\circ = 50^\circ$.

Thus, the correct answer is **C**.

6. What is the value of the following expression?

$$\frac{(.2)^3}{(.02)^2}$$

- A .2
- B 2
- C 10
- D 15
- E 20

Solution:

The numerator is $(.2)^3 = .008$ and the denominator is $(.02)^2 = .0004$.

So the value is $\frac{.008}{.0004} = \frac{80}{4} = 20$.

Thus, the correct answer is **E**.

7. The expression $2.46 \times 8.163 \times (5.17 + 4.829)$ is closest to

- A 100
- B 200
- C 300
- D 400
- E 500

Solution:

Round each factor: $2.46 \approx 2.5$, $8.163 \approx 8$, and $5.17 + 4.829 \approx 10$.

The product is about $2.5 \times 8 \times 10 = 200$.

Thus, the correct answer is **B**.

8. Betty used a calculator to find the product 0.075×2.56 . She forgot to enter the decimal points. The calculator showed 19200. If Betty had entered the decimal points correctly, the answer would have been

- A .0192
- B .192**
- C 1.92
- D 19.2
- E 192

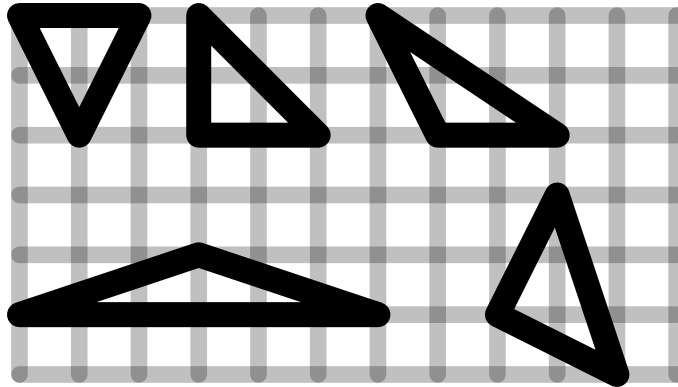
Solution:

The factors 0.075 and 2.56 have 3 and 2 decimal places, for a total of 5.

Placing the decimal point 5 digits from the right of 19200 gives $0.19200 = .192$.

Thus, the correct answer is **B**.

9. An isosceles triangle is a triangle with two sides of equal length. How many of the five triangles on the square grid below are isosceles?



- A 1
- B 2
- C 3
- D 4
- E 5

Solution:

Read each triangle's side lengths from the grid. A horizontal or vertical side is counted directly, and a slanted side spanning a units across and b units up has length $\sqrt{a^2 + b^2}$.

Top-left triangle: its two slanted sides each span 1 across and 2 up, so each has length $\sqrt{5}$, with a base of length 2. Two equal sides, so it is isosceles.

Top-middle triangle: it has a vertical side of length 2 and a horizontal side of length 2 (its third side is the slant $2\sqrt{2}$). Two equal sides, so it is isosceles.

Top-right triangle: its sides are 2, $\sqrt{5}$, and $\sqrt{13}$, which are all different, so it is **not** isosceles.

Bottom-left (wide, flat) triangle: its two slanted sides each span 3 across and 1 up, so each has length $\sqrt{10}$. Two equal sides, so it is isosceles.

Bottom-right triangle: two of its sides each span 2 and 1 (giving length $\sqrt{5}$), while the third is $\sqrt{10}$. Two equal sides, so it is isosceles.

Four of the five triangles are isosceles.

Thus, the correct answer is **D**.

10. Chris' birthday is on a Thursday this year. What day of the week will it be 60 days after her birthday?

- A Monday
- B Wednesday
- C Thursday
- D Friday
- E Saturday

Solution:

Days of the week repeat every 7 days. Since $60 = 8 \times 7 + 4$, the day 60 days later is 4 days after Thursday.

Counting forward from Thursday: Friday, Saturday, Sunday, Monday. So it is a Monday.

Thus, the correct answer is **A**.

11. The value of $\sqrt{164}$ is

A 42

B less than 10

C between 10 and 11

D between 11 and 12

E between 12 and 13

Solution:

Since $12^2 = 144$ and $13^2 = 169$, and $144 < 164 < 169$, the value $\sqrt{164}$ lies between 12 and 13.

Thus, the correct answer is **E**.

12. Suppose the estimated 20 billion dollar cost to send a person to the planet Mars is shared equally by the 250 million people in the U.S. Then each person's share is

- A \$40
- B \$50
- C \$80
- D \$100
- E \$125

Solution:

Each person's share is $\frac{20 \text{ billion}}{250 \text{ million}} = \frac{2 \times 10^{10}}{2.5 \times 10^8}$.

This equals $.8 \times 10^2 = 80$.

Thus, the correct answer is **C**.

13. If rose bushes are spaced about 1 foot apart, approximately how many bushes are needed to surround a circular patio whose radius is 12 feet?

- A 12
- B 38
- C 48
- D 75
- E 450

Solution:

The bushes surround the circular edge, so their number is about the circumference $2\pi r = 2\pi(12)$.

Using $\pi \approx 3.14$, this is about $2 \times 3.14 \times 12 \approx 75$ feet, so roughly 75 bushes are needed.

Thus, the correct answer is **D**.

14. If \diamond and \triangle are whole numbers and $\diamond \times \triangle = 36$, the largest possible value of $\diamond + \triangle$ is

- A 12
- B 13
- C 15
- D 20
- E 37**

Solution:

The factor pairs of 36 are 1×36 , 2×18 , 3×12 , 4×9 , and 6×6 .

The sum is largest for the most spread-out pair, $1 + 36 = 37$.

Thus, the correct answer is **E**.

15. What is the reciprocal of $\left(\frac{1}{2} + \frac{1}{3}\right)$?

A $\frac{1}{6}$

B $\frac{2}{5}$

C $\frac{6}{5}$

D $\frac{5}{2}$

E 5

Solution:

Adding, $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

Thus, the correct answer is **C**.

16. Placing no more than one X in each small square, what is the greatest number of X's that can be put on a 3×3 grid without getting three X's in a row vertically, horizontally, or diagonally?

- A 2
- B 3
- C 4
- D 5
- E 6**

Solution:

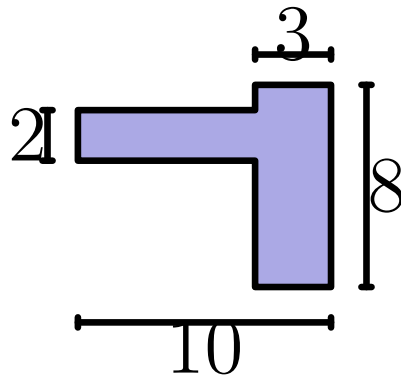
A placement of 6 X's works: leave the three squares along one main diagonal empty and fill the other six. Then each row and each column is missing one square, the used diagonal has an empty square, and the other diagonal passes through the empty center, so no line of three is complete.

Seven X's is impossible: only two squares would be empty, and two empty squares can lie in at most two of the three rows, forcing the remaining row to be completely filled with three X's.

So the greatest number is 6.

Thus, the correct answer is **E**.

17. The shaded area is formed by two intersecting perpendicular rectangles, with dimensions as shown. Its area, in square units, is



- A 23
- B 38
- C 44
- D 46
- E unable to be determined from the information given

Solution:

The horizontal rectangle has area $10 \times 2 = 20$, and the vertical rectangle has area $3 \times 8 = 24$. Their overlap, a 3×2 rectangle, has area 6.

Adding the two rectangles counts the overlap twice, so the shaded area is $20 + 24 - 6 = 38$.

Thus, the correct answer is **B**.

18. The average weight of 6 boys is 150 pounds and the average weight of 4 girls is 120 pounds. The average weight of the 10 children is

A 135 pounds

B 137 pounds

C 138 pounds

D 140 pounds

E 141 pounds

Solution:

The total weight is $6 \times 150 + 4 \times 120 = 900 + 480 = 1380$ pounds.

The average of the 10 children is $\frac{1380}{10} = 138$ pounds.

Thus, the correct answer is **C**.

19. What is the 100th number in the arithmetic sequence 1, 5, 9, 13, 17, 21, 25, ...?

A 397

B 399

C 401

D 403

E 405

Solution:

The sequence starts at 1 with common difference 4. The 100th term is reached by adding 4 to the first term 99 times.

So the 100th term is $1 + 99 \times 4 = 1 + 396 = 397$.

Thus, the correct answer is **A**.

20. The glass gauge on a cylindrical coffee maker shows there are 45 cups left when the coffee maker is 36% full. How many cups of coffee does it hold when it is full?

- A 80
- B 100
- C 125
- D 130
- E 262

Solution:

Let n be the full capacity. Then $\frac{45}{n} = \frac{36}{100}$.

Solving, $n = \frac{45 \times 100}{36} = 125$ cups.

Thus, the correct answer is **C**.

21. A fifth number, n , is added to the set of numbers $\{3, 6, 9, 10\}$ to make the mean of the set of five numbers equal to its median. The number of possible values for n is

- A 1
- B 2
- C 3
- D 4
- E more than 4

Solution:

The five numbers have sum $28 + n$ and mean $\frac{28 + n}{5}$. The median is the middle value when sorted, which is 6, n , or 9 depending on the size of n .

If the median is 6: $\frac{28 + n}{5} = 6$ gives $n = 2$, which is indeed less than 6. If the median is n : $\frac{28 + n}{5} = n$ gives $n = 7$, which is between 6 and 9. If the median is 9: $\frac{28 + n}{5} = 9$ gives $n = 17$, which is indeed greater than 9.

Each case yields a valid value, so n can be 2, 7, or 17 — three possible values.

Thus, the correct answer is **C**.

22. Tom's Hat Shoppe increased all original prices by 25%. Now the shoppe is having a sale where all prices are 20% off these increased prices. Which statement best describes the sale price of an item?

- A The sale price is 5% higher than the original price.
- B The sale price is higher than the original price, but by less than 5%.
- C The sale price is higher than the original price, but by more than 5%.
- D The sale price is lower than the original price.
- E** The sale price is the same as the original price.

Solution:

Increasing by 25% multiplies the price by 1.25, and then taking 20% off multiplies by 0.80.

The overall factor is $1.25 \times 0.80 = 1.00$, so the sale price equals the original price.

Thus, the correct answer is **E**.

23. Maria buys computer disks at a price of 4 for \$5 and sells them at a price of 3 for \$5. How many computer disks must she sell in order to make a profit of \$100?

- A 100
- B 120
- C 200
- D 240
- E 1200

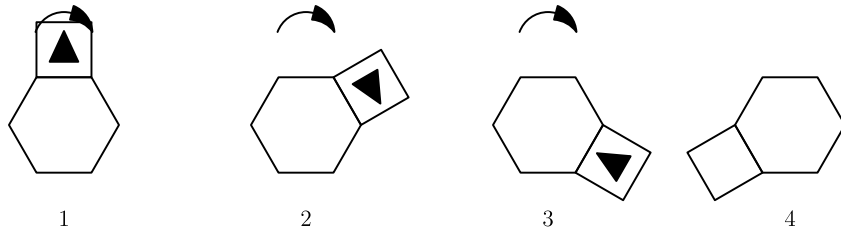
Solution:

Consider disks in dozens, since 12 is a multiple of both 4 and 3. A dozen disks costs $3 \times \$5 = \15 to buy and sells for $4 \times \$5 = \20 , for a profit of \$5 per dozen.

To make \$100 profit she needs $\frac{100}{5} = 20$ dozen, which is $20 \times 12 = 240$ disks.

Thus, the correct answer is **D**.

24. The square in the first diagram "rolls" clockwise around the fixed regular hexagon until it reaches the bottom. In which position will the solid triangle be in diagram 4?



A



B



C



D



E



Solution:

Each roll of the square over one edge of the hexagon turns it 150° clockwise. (At the pivot corner the square turns through 360° minus the square's own 90° corner minus the hexagon's 120° interior angle, which leaves 150° .)

From the top of the hexagon to the bottom is 3 rolls, for a total of $3 \times 150^\circ = 450^\circ$. That is one complete revolution (360°) plus an extra 90° clockwise.

The triangle starts pointing straight up, so after the extra quarter turn clockwise it points to the right.

Thus, the correct answer is **A**.

25. A palindrome is a whole number that reads the same forwards as backwards. If one neglects the colon, certain times displayed on a digital watch are palindromes. Three examples are 1:01, 4:44, and 12:21. How many times during a 12-hour period will be palindromes?

A 57

B 60

C 63

D 90

E 93

Solution:

For a single-digit hour h (from 1 to 9), the display $h:m_1m_2$ reads the same backwards when $m_2 = h$. The tens digit m_1 can be 0, 1, 2, 3, 4, or 5, giving 6 palindromes per hour, so $9 \times 6 = 54$.

For the two-digit hours 10, 11, 12, the display $h_1h_2:m_1m_2$ is a palindrome when $m_1 = h_2$ and $m_2 = h_1$, giving exactly 10:01, 11:11, and 12:21 — one each, for 3 more.

The total is $54 + 3 = 57$.

Thus, the correct answer is **A**.

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