1987 AMC 8 Solutions

Typeset by: LIVE by Po-Shen Loh
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1. What is $.4 + .02 + .006$?

A $.012$

B $.066$

C $.12$

D $.24$

E $.426$

Solution:

Adding place by place, $.4 + .02 + .006 = .426.$

Thus, the correct answer is **E**.

2. What is $\frac{2}{25}$ written as a decimal?

A $.008$

B $.08$

C $.8$

D 1.25

E 12.5

Solution:

Multiplying top and bottom by 4 gives $\frac{2}{25} = \frac{8}{100} = .08.$

Thus, the correct answer is **B** .

3. What is the value of

$$2(81+83+85+87+89+91+93+95+97+99)?$$

A 1600

B 1650

C 1700

D 1750

E 1800

Solution:

The ten evenly spaced numbers have average 90, so their sum is $10 \times 90 = 900$.

Doubling gives $2 \times 900 = 1800$.

Thus, the correct answer is **E**.

4. Martians measure angles in clerts. There are 500 clerts in a full circle. How many clerts are there in a right angle?

A 90

B 100

C 125

D 180

E 250

Solution:

A right angle is a quarter of a full circle, so it contains $\frac{1}{4} \times 500 = 125$ clerts.

Thus, the correct answer is **C**.

5. A rectangular region is 0.4 m long and 0.22 m wide. What is its area, in square meters?

A $0.088\ \text{m}^2$

B $0.62\ \text{m}^2$

C $0.88\ \text{m}^2$

D $1.24\ \text{m}^2$

E $4.22\ \text{m}^2$

Solution:

The area is $0.4 \times 0.22 = 0.088$ square meters.

Thus, the correct answer is **A**.

6. The smallest product one could obtain by multiplying two numbers in the set $\{-7, -5, -1, 1, 3\}$ is

A -35

B -21

C -15

D -1

E 3

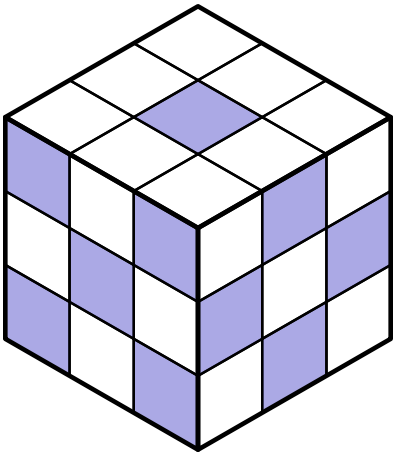
Solution:

A negative product comes from multiplying a negative number by a positive number, and it is most negative when both factors are largest in size.

The most negative product is $(-7)(3) = -21.$

Thus, the correct answer is **B** .

7. The large cube shown is made up of 27 identical sized smaller cubes. For each face of the large cube, the opposite face is shaded the same way. The total number of smaller cubes that must have at least one face shaded is



A 10

B 16

C 20

D 22

E 24

Solution:

Count the cubes with no shaded face and subtract from 27. A small cube is unshaded exactly when every one of its exposed squares is blank.

The three patterns are: the top and bottom faces show only their center square shaded; one pair of opposite side faces shows the four corners and the center shaded; the remaining pair shows the four edge-midpoints shaded.

Exactly 7 small cubes avoid every shaded square: the one hidden cube at the very center of the block; the two cubes at the centers of the edge-midpoints faces, whose only exposed square is that blank center; and the four cubes at the midpoints of the edges where a corners-and-center face meets the top or bottom face, since there each exposed square is a blank edge cell.

Hence $27 - 7 = 20$ cubes have at least one shaded face.

Thus, the correct answer is **C** .

8. In the addition problem below, A and B are nonzero digits:

$$\begin{array}{r} 9876 \\ A32 \\ +B1 \\ \hline \end{array}$$

How many digits (not necessarily different) are in the sum of the three whole numbers?

A 4

B 5

C 6

D 9

E depends on the values of A and B

Solution:

Since A and B are at least 1, the sum is at least $9876 + 132 + 11 = 10{,}019$, which has 5 digits.

At the other extreme, with $A = B = 9$ the sum is $9876 + 932 + 91 = 10{,}899$, still 5 digits. So the sum always has exactly 5 digits.

Thus, the correct answer is **B** .

9. When finding the sum

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7},$$

what is the least common denominator used?

A 120

B 210

C 420

D 840

E 5040

Solution:

The least common multiple must include $4 = 2^2, 3, 5,$ and 7 (the factor $6 = 2 \times 3$ is already covered).

So it is $4 \times 3 \times 5 \times 7 = 420.$

Thus, the correct answer is **C** .

10. What is the value of

$$4(299) + 3(299) + 2(299) + 298?$$

A 2889

B 2989

C 2991

D 2999

E 3009

Solution:

The first three terms combine to $(4 + 3 + 2)(299) = 9 \times 299 = 2691.$

Adding the last term gives $2691 + 298 = 2989.$

Thus, the correct answer is **B** .

11. The sum $2\frac{1}{7} + 3\frac{1}{2} + 5\frac{1}{19}$ is between which two values?

A 10 and $10\frac{1}{2}$

B $10\frac{1}{2}$ and 11

C 11 and $11\frac{1}{2}$

D $11\frac{1}{2}$ and 12

E 12 and $12\frac{1}{2}$

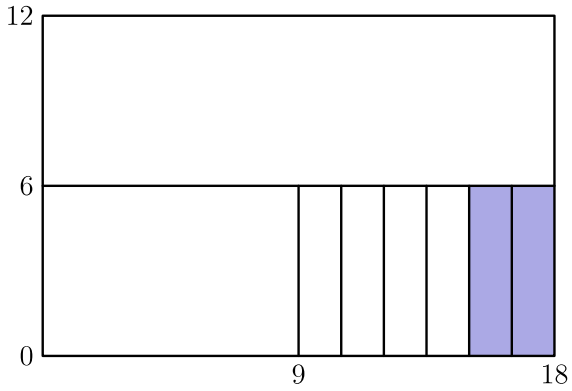
Solution:

The whole-number parts sum to 10. The fractional parts are $\frac{1}{2} + \frac{1}{7} + \frac{1}{19}$.

Since $\frac{1}{7} + \frac{1}{19}$ is a small positive amount, the fractional total is more than $\frac{1}{2}$ but well under 1. So the sum lies between $10\frac{1}{2}$ and 11.

Thus, the correct answer is **B** .

12. What fraction of the large 12 by 18 rectangular region is shaded?



A $\frac{1}{108}$

B $\frac{1}{18}$

C $\frac{1}{12}$

D $\frac{2}{9}$

E $\frac{1}{3}$

Solution:

The whole region has area $12 \times 18 = 216.$ The shaded region is 3 wide and 6 tall, with area $3 \times 6 = 18.$

So the shaded fraction is $\frac{18}{216} = \frac{1}{12}.$ Equivalently, it is $\frac{1}{3}$ of $\frac{1}{4}$ of the whole.

Thus, the correct answer is **C** .

13. Which of the following fractions has the largest value?

A $\frac{3}{7}$

B $\frac{4}{9}$

C $\frac{17}{35}$

D $\frac{100}{201}$

E $\frac{151}{301}$

Solution:

For $\frac{3}{7}$, $\frac{4}{9}$, $\frac{17}{35}$, and $\frac{100}{201}$, each numerator is less than half its denominator, so each is less than $\frac{1}{2}$.

For $\frac{151}{301}$, the numerator 151 is more than half of 301, so this fraction exceeds $\frac{1}{2}$ and is the largest.

Thus, the correct answer is **E**.

14. A computer can do $10{,}000$ additions per second. How many additions can it do in one hour?

A 6 million

B 36 million

C 60 million

D 216 million

E 360 million

Solution:

One hour has 3600 seconds, so the computer does $10{,}000 \times 3600 = 36{,}000{,}000$ additions.

That is 36 million.

Thus, the correct answer is **B** .

15. A sale ad read: "Buy three tires at the regular price and get the fourth tire for $3." Sam paid $240 for a set of four tires at the sale. What was the regular price of one tire?

A $59.25

B $60

C $70

D $79

E $80

Solution:

The three regular-price tires cost $\$240 - \$3 = \$237.$

So one tire costs $\dfrac{\$237}{3} = \$79.$

Thus, the correct answer is **D** .

16. Joyce made 12 of her first 30 shots in the first three games of the basketball season, so her seasonal shooting average was 40%. In her next game, she took 10 shots and raised her seasonal shooting average to 50%. How many of these 10 shots did she make?

A 2

B 3

C 5

D 6

E 8

Solution:

After the fourth game she has taken 40 shots. A 50% average means she made $\frac{1}{2} \times 40 = 20$ shots in total.

She had already made 12, so she made $20 - 12 = 8$ of the last 10.

Thus, the correct answer is **E**.

17. Abby, Bret, Carl, and Dana are seated in a row of four seats numbered 1 to 4. Joe looks at them and says: "Bret is next to Carl." "Abby is between Bret and Carl." However, each one of Joe's statements is false. Bret is actually sitting in seat 3. Who is sitting in seat 2?

A Abby

B Bret

C Carl

D Dana

E There is not enough information to be sure

Solution:

Bret is in seat 3. Since "Bret is next to Carl" is false, Carl is not in seat 2 or seat 4, so Carl must be in seat 1.

The seat between Bret (seat 3) and Carl (seat 1) is seat 2. Since "Abby is between Bret and Carl" is false, Abby is not in seat 2, so Abby must be in seat 4.

That leaves Dana in seat 2.

Thus, the correct answer is **D** .

18. Half the people in a room left. One third of those remaining started to dance. There were then 12 people who were not dancing. What was the original number of people in the room?

A 24

B 30

C 36

D 42

E 72

Solution:

Of those remaining, $\frac{1}{3}$ danced, so $\frac{2}{3}$ did not. Thus $\frac{2}{3}$ of the remaining people equals 12, giving 18 remaining.

Those 18 are half the original group, so the room started with $2 \times 18 = 36$ people.

Thus, the correct answer is **C** .

19. A calculator has a squaring key that replaces the number currently displayed with its square. For example, if the display reads 3 and the squaring key is pressed, the display becomes 9. If the display reads 2, how many times must the squaring key be pressed to produce a displayed number greater than 500?

A 4

B 5

C 8

D 9

E 250

Solution:

Pressing the key repeatedly gives $2 \to 4 \to 16 \to 256 \to 65536$.

Since $256 < 500 < 65536$, the display first exceeds 500 on the fourth press.

Thus, the correct answer is **A** .

20. Consider the statement: "If a whole number n is not prime, then the whole number $n-2$ is not prime." Which of the following values of n shows this statement to be false?

A 9

B 12

C 13

D 16

E 23

Solution:

A counterexample needs n not prime but $n-2$ prime. For $n=9$, the number 9 is not prime while $9-2=7$ is prime, so it breaks the statement.

The others fail to be counterexamples: $12-2=10$ and $16-2=14$ are not prime, while 13 and 23 are themselves prime.

Thus, the correct answer is **A**.

21. Suppose n^* means $\frac{1}{n}$, the reciprocal of n. For example, $5^* = \frac{1}{5}$. How many of the following four statements are true?

(i) $3^* + 6^* = 9^*$; (ii) $6^* - 4^* = 2^*$; (iii) $2^* \cdot 6^* = 12^*$; (iv) $10^* \div 2^* = 5^*$

A 0

B 1

C 2

D 3

E 4

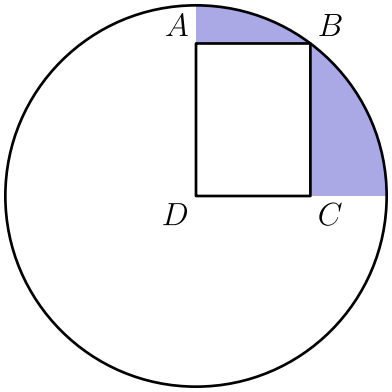
Solution:

(i) $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$, but $9^* = \frac{1}{9}$, so false. (ii) $\frac{1}{6} - \frac{1}{4} = -\frac{1}{12}$, but $2^* = \frac{1}{2}$, so false.
(iii) $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} = 12^*$, true. (iv) $\frac{1}{10} \div \frac{1}{2} = \frac{1}{5} = 5^*$, true.

So exactly 2 of the statements are true.

Thus, the correct answer is **C**.

22. $ABCD$ is a rectangle, D is the center of the circle, and B is on the circle. If $AD = 4$ and $CD = 3$, then the area of the shaded region is between which two values?



A 4 and 5

B 5 and 6

C 6 and 7

D 7 and 8

E 8 and 9

Solution:

Since $AD = 4$ and $CD = 3$, the diagonal $DB = \sqrt{3^2 + 4^2} = 5$, which is the radius.

The shaded region is the quarter circle at D minus the rectangle: $\frac{1}{4}\pi(5)^2 - (3)(4) = \frac{25\pi}{4} - 12 \approx 19.6 - 12 = 7.6.$

This lies between 7 and 8.

Thus, the correct answer is **D** .

23. In 1980, the U.S. Black population (in millions) was 5 in the Northeast, 5 in the Midwest, 15 in the South, and 2 in the West. To the nearest percent, what percent of the U.S. Black population lived in the South?

A 20%

B 25%

C 40%

D 56%

E 80%

Solution:

The total Black population is $5 + 5 + 15 + 2 = 27$ million.

The South's share is $\frac{15}{27} = \frac{5}{9} \approx 55.6\%$, which rounds to 56%.

Thus, the correct answer is **D** .

24. A multiple choice examination consists of 20 questions. The scoring is $+5$ for each correct answer, -2 for each incorrect answer, and 0 for each unanswered question. John's score on the examination is 48. What is the maximum number of questions he could have answered correctly?

A 9

B 10

C 11

D 12

E 16

Solution:

Let c be the number correct and w the number wrong, so $5c - 2w = 48$. Then $2w = 5c - 48$, which requires c to be even.

Trying $c = 14$ gives $w = 11$, but $c + w = 25 > 20$, too many. Trying $c = 12$ gives $w = 6$ and $c + w = 18 \leq 20$, which works. So the maximum is 12.

Thus, the correct answer is **D** .

25. Ten balls numbered 1 to 10 are in a jar. Jack reaches into the jar and randomly removes one of the balls. Then Jill reaches into the jar and randomly removes a different ball. What is the probability that the sum of the two numbers on the balls removed is even?

A $\frac{4}{9}$

B $\frac{9}{19}$

C $\frac{1}{2}$

D $\frac{10}{19}$

E $\frac{5}{9}$

Solution:

The sum is even when both balls are odd or both are even. There are $\binom{5}{2} = 10$ all-odd pairs and $\binom{5}{2} = 10$ all-even pairs, for 20 favorable pairs.

The total number of pairs is $\binom{10}{2} = 45$, so the probability is $\frac{20}{45} = \frac{4}{9}$.

Thus, the correct answer is **A**.

Problems: https://live.poshenloh.com/past-contests/amc8/1987