1986 AMC 8 Solutions

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1. In July $1861, 366$ inches of rain fell in Cherrapunji, India. What was the average rainfall in inches per hour during that month?

A $\frac{366}{31 \times 24}$

B $\frac{366 \times 31}{24}$

C $\frac{366 \times 24}{31}$

D $\frac{31 \times 24}{366}$

E $366 \times 31 \times 24$

Solution:

The average rainfall per hour equals the total rainfall divided by the total number of hours. July has 31 days, or 31×24 hours, so the average is $\frac{366}{31 \times 24}$ inches per hour.

Thus, the correct answer is **A** .

2. Which of the following numbers has the largest reciprocal?

A $\frac{1}{3}$

B $\frac{2}{5}$

C 1

D 5

E 1986

Solution:

A large positive number has a small reciprocal, and a small positive number has a large reciprocal. The smallest number listed is $\frac{1}{3}$, whose reciprocal 3 is the largest.

Thus, the correct answer is **A** .

3. The smallest sum one could get by adding three different numbers from the set $\{7, 25, -1, 12, -3\}$ is

A -3

B -1

C 3

D 5

E 21

Solution:

The three smallest numbers in the set are -3, -1, and 7. Their sum is $-3 + (-1) + 7 = 3$.

Thus, the correct answer is **C** .

4. The product $(1.8)(40.3 + .07)$ is closest to

A 7

B 42

C 74

D 84

E 737

Solution:

The second factor $40.3 + .07 = 40.37$ is about 40, and 1.8 is about 2. A quick estimate is $2(40) - .2(40) = 80 - 8 = 72$, so the product is closest to 74.

Thus, the correct answer is **C** .

5. A contest began at noon one day and ended 1000 minutes later. At what time did the contest end?

A $10:00$ p.m.

B midnight

C $2:30$ a.m.

D $4:40$ a.m.

E $6:40$ a.m.

Solution:

Since 1000 minutes $= \dfrac{1000}{60}$ hours $= 16\dfrac{2}{3}$ hours $= 16$ hours 40 minutes, the contest ended 16 hours 40 minutes past noon, which is $4:40$ a.m.

Thus, the correct answer is **D** .

6. What is the value of the following expression?

$$\frac{2}{1-\frac{2}{3}}$$

A -3

B $-\frac{4}{3}$

C $\frac{2}{3}$

D 2

E 6

Solution:

The denominator is $1-\frac{2}{3}=\frac{1}{3}$, so the expression is $\frac{2}{1/3}=2\times 3=6.$

Thus, the correct answer is **E**.

7. How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$?

A 5

B 6

C 7

D 8

E 9

Solution:

Since $\sqrt{8} < \sqrt{9} = 3$ and $\sqrt{80} < \sqrt{81} = 9$, the whole numbers strictly between $\sqrt{8}$ and $\sqrt{80}$ are $3, 4, 5, 6, 7, 8$. There are six of them.

Thus, the correct answer is **B** .

8. In the multiplication shown, B represents a digit. What is the value of B?

$$\begin{array}{r} B2 \\ \times\, 7B \\ \hline 6396 \end{array}$$

A 3

B 5

C 6

D 7

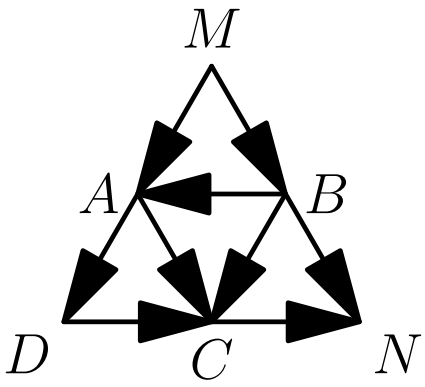
E 8

Solution:

The units digit of the product comes from $B \times 2$, which ends in 6 only when $B = 3$ or $B = 8$. Since the product 6396 exceeds 6000, B must be 8 : indeed $82 \times 78 = 6396$.

Thus, the correct answer is **E** .

9. Using only the paths and the directions shown, how many different routes are there from M to N?



A 2

B 3

C 4

D 5

E 6

Solution:

Following the arrows, the possible routes are $MADCN$, $MACN$, $MBADCN$, $MBACN$, $MBCN$, and MBN. There are six of them.

Thus, the correct answer is **E**.

10. A picture 3 feet across is hung in the center of a wall that is 19 feet wide. How many feet from the end of the wall is the nearest edge of the picture?

A $1\frac{1}{2}$

B 8

C $9\frac{1}{2}$

D 16

E 22

Solution:

The picture leaves $19 - 3 = 16$ feet of wall, split equally into two gaps of 8 feet each. So the nearest edge of the picture is 8 feet from the end of the wall.

Thus, the correct answer is **B** .

11. If $A * B$ means $\frac{A+B}{2}$, then $(3 * 5) * 8$ is

A 6

B 8

C 12

D 16

E 30

Solution:

First, $3 * 5 = \frac{3+5}{2} = 4$. Then $4 * 8 = \frac{4+8}{2} = 6$.

Thus, the correct answer is **A** .

12. The table shown displays the grade distribution of the 30 students in a mathematics class on the last two tests. For example, exactly one student received a 'D' on Test 1 and a 'C' on Test 2 (the circled entry). What percent of the students received the same grade on both tests?

TEST 1 \ TEST 2	A	B	C	D	F
A	2	2	1	0	0
B	1	4	3	0	0
C	1	3	5	2	0
D	0	0	(1)	1	1
F	0	0	2	1	0

A 12%

B 25%

C $33\frac{1}{3}\%$

D 40%

E 50%

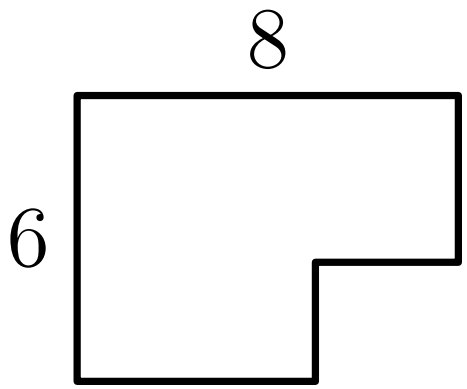
Solution:

A student received the same grade on both tests exactly when counted on the main diagonal. Those entries are $2+4+5+1+0=12.$

So the fraction is $\frac{12}{30}=\frac{4}{10}=40\%.$

Thus, the correct answer is **D** .

13. The perimeter of the polygon shown is



A 14

B 20

C 28

D 48

E cannot be determined from the information given

Solution:

Because every angle is a right angle, sliding the edges shows that the horizontal edges together traverse the width twice and the vertical edges traverse the height twice. So the perimeter equals that of the full 8 by 6 rectangle, $2(8+6)=28$.

The answer does not depend on exactly where the notch is cut.

Thus, the correct answer is **C** .

14. If $200 \leq a \leq 400$ and $600 \leq b \leq 1200$, then the largest value of the quotient $\frac{b}{a}$ is

A $\frac{3}{2}$

B 3

C 6

D 300

E 600

Solution:

The quotient $\frac{b}{a}$ is largest with the biggest b and smallest a, giving $\frac{1200}{200} = 6.$

Thus, the correct answer is **C** .

15. Sale prices at the Ajax Outlet Store are 50% below original prices. On Saturdays an additional discount of 20% off the sale price is given. What is the Saturday price of a coat whose original price is $180?

A $54

B $72

C $90

D $108

E $110

Solution:

The sale price is 50% of $\$180$, or $\$90$. The Saturday price takes another 20% off, leaving 80% of $\$90$, which is $\$72$.

Thus, the correct answer is **B** .

16. A fast food chain sold 4.5 million hamburgers in the spring, 5 million in the summer, and 4 million in the fall; the number sold in the winter is unknown. If exactly 25% of the chain's hamburgers are sold in the fall, how many million hamburgers are sold in the winter?

A 2.5

B 3

C 3.5

D 4

E 4.5

Solution:

If the fall sales of 4 million are 25% of the yearly total, then the yearly total is 16 million.

The winter sales are $16 - (4.5 + 5 + 4) = 2.5$ million.

Thus, the correct answer is **A** .

17. Let o be an odd whole number and let n be any whole number. Which of the following statements about the whole number $o^2 + no$ is always true?

A it is always odd

B it is always even

C it is even only if n is even

D it is odd only if n is odd

E it is odd only if n is even

Solution:

Factor $o^2 + no = o(o + n)$. Because o is odd, the product is odd exactly when $o + n$ is odd, which happens only when n is even. When n is odd, $o + n$ is even and the product is even.

So the number is odd only if n is even.

Thus, the correct answer is **E**.

18. A rectangular grazing area is to be fenced off on three sides using part of a 100 meter rock wall as the fourth side. Fence posts are to be placed every 12 meters along the fence, including the two posts where the fence meets the rock wall. What is the fewest number of posts required to fence an area 36 m by 60 m?

A 11

B 12

C 13

D 14

E 16

Solution:

The fewest posts are used when the wall serves as the longer 60 meter side, so the fence covers two 36 meter sides and one 60 meter side, a path of $36 + 60 + 36 = 132$ meters.

Placing a post every 12 meters, including both ends, uses $\frac{132}{12} + 1 = 12$ posts.

Thus, the correct answer is **B** .

19. At the beginning of a trip, the mileage odometer read $56{,}200$ miles. The driver filled the gas tank with 6 gallons of gasoline. During the trip, the driver filled the tank again with 12 gallons of gasoline when the odometer read $56{,}560$. At the end of the trip, the driver filled the tank again with 20 gallons of gasoline; the odometer read $57{,}060$. To the nearest tenth, what was the car's average miles-per-gallon for the entire trip?

A 22.5

B 22.6

C 24.0

D 26.9

E 27.5

Solution:

The trip was $57{,}060 - 56{,}200 = 860$ miles. The initial 6 gallons only topped off the tank before the trip; the gas actually used during the trip is the $12 + 20 = 32$ gallons later added to replace it.

So the average is $\frac{860}{32} \approx 26.9$ miles per gallon.

Thus, the correct answer is **D** .

20. The value of the expression

$$\frac{(304)^5}{(29.7)(399)^4}$$

is closest to

A $.003$

B $.03$

C $.3$

D 3

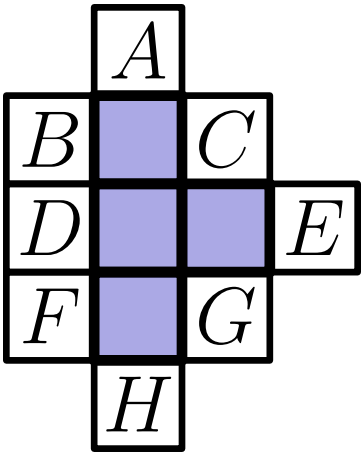
E 30

Solution:

Estimating, $\frac{300^5}{30 \cdot 400^4} = 10\left(\frac{300}{400}\right)^4 = 10\left(\frac{3}{4}\right)^4 = 10 \cdot \frac{81}{256} \approx 3.$

Thus, the correct answer is **D** .

21. Suppose one of the eight lettered identical squares is included with the four shaded squares in the T-shaped figure shown. How many of the resulting figures can be folded into a topless cubical box?



A 2

B 3

C 4

D 5

E 6

Solution:

The four shaded squares fold into four faces of an open box, and the added square supplies the fifth face. Picturing the folds, the squares A, E, H, B, D, and F each complete a valid topless box.

The squares C and G do not work, because folding would force four faces to meet at a single corner. That leaves 6 valid figures.

Thus, the correct answer is **E**.

22. Alan, Beth, Carlos, and Diana were discussing their possible grades in mathematics class this grading period. Alan said, "If I get an A, then Beth will get an A." Beth said, "If I get an A, then Carlos will get an A." Carlos said, "If I get an A, then Diana will get an A." All of these statements were true, but only two of the students received an A. Which two received A's?

A Alan, Beth

B Beth, Carlos

C Carlos, Diana

D Alan, Diana

E Beth, Diana

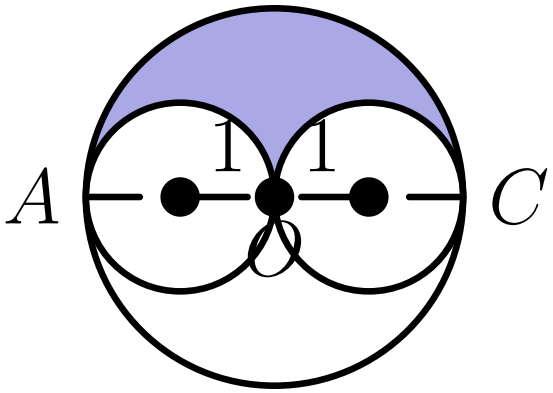
Solution:

The statements form a chain: Alan's A forces Beth's, Beth's forces Carlos's, and Carlos's forces Diana's. If Alan got an A, all four would; if Beth got an A, three would.

The only way to have exactly two A's is for them to be the last two in the chain, Carlos and Diana.

Thus, the correct answer is **C** .

23. The large circle has diameter AC. The two small circles have their centers on AC and just touch at O, the center of the large circle. If each small circle has radius 1, what is the value of the ratio of the area of the shaded region to the area of one of the small circles?



A between $\frac{1}{2}$ and 1

B 1

C between 1 and $\frac{3}{2}$

D between $\frac{3}{2}$ and 2

E cannot be determined from the information given

Solution:

The large circle has radius 2, so its area is $\pi(2)^2 = 4\pi$, and each small circle has area π.

By symmetry the shaded region is half the difference of the areas: $\frac{1}{2}(4\pi - 2\pi) = \pi$.

The ratio of this to one small circle's area π is 1.

Thus, the correct answer is **B** .

24. The 600 students at King Middle School are divided into three groups of equal size for lunch. Each group has lunch at a different time. A computer randomly assigns each student to one of the three lunch groups. The probability that three friends, Al, Bob, and Carol, will be assigned to the same lunch group is approximately

A $\frac{1}{27}$

B $\frac{1}{9}$

C $\frac{1}{8}$

D $\frac{1}{6}$

E $\frac{1}{3}$

Solution:

Whatever group Al is in, Bob joins that same group with probability about $\frac{1}{3}$, and Carol joins it with probability about $\frac{1}{3}$.

So all three share a group with probability about $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

Thus, the correct answer is **B** .

25. Which of the following sets of whole numbers has the largest average?

A multiples of 2 between 1 and 101

B multiples of 3 between 1 and 101

C multiples of 4 between 1 and 101

D multiples of 5 between 1 and 101

E multiples of 6 between 1 and 101

Solution:

For a set of evenly spaced whole numbers, the average is the average of the smallest and largest. The averages are: A: $\frac{2+100}{2} = 51$, B: $\frac{3+99}{2} = 51$, C: $\frac{4+100}{2} = 52$, D: $\frac{5+100}{2} = 52.5$, E: $\frac{6+96}{2} = 51$.

The largest average is 52.5, from the multiples of 5.

Thus, the correct answer is **D** .

Problems: https://live.poshenloh.com/past-contests/amc8/1986