1985 AMC 8 Solutions

Typeset by: LIVE by Po-Shen Loh
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1. What is the value of the following product?

$$\frac{3 \times 5}{9 \times 11} \times \frac{7 \times 9 \times 11}{3 \times 5 \times 7}$$

A 1

B 0

C 49

D $\frac{1}{49}$

E 50

Solution:

Combining the two fractions, the numerator is $3 \cdot 5 \cdot 7 \cdot 9 \cdot 11$ and the denominator is $9 \cdot 11 \cdot 3 \cdot 5 \cdot 7$. These are the same product.

So the value is 1.

Thus, the correct answer is **A** .

2. What is the value of $90 + 91 + 92 + \cdots + 98 + 99$?

A 845

B 945

C 1005

D 1025

E 1045

Solution:

There are 10 terms. The average of the first and last is $\frac{90 + 99}{2} = 94.5.$

So the sum is $10 \cdot 94.5 = 945.$

Thus, the correct answer is **B** .

3. What is the value of the following expression?

$$\frac{10^7}{5 \times 10^4}$$

A 0.002

B 0.2

C 20

D 200

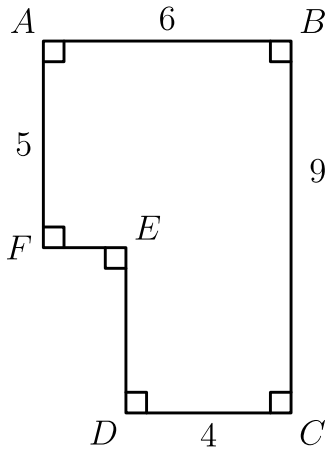
E 2000

Solution:

Since $\frac{10^7}{10^4} = 10^3 = 1000$, the expression is $\frac{1000}{5} = 200$.

Thus, the correct answer is **D** .

4. The area of polygon $ABCDEF$, in square units, is



A 24

B 30

C 46

D 66

E 74

Solution:

Completing the figure to the full 6×9 rectangle gives an area of 54. The piece removed from the lower-left corner is a rectangle measuring 2 by 4, with area 8.

So the polygon's area is $54 - 8 = 46$.

Thus, the correct answer is **C**.

5. The grades in a mathematics class for the last grading period were: 5 students earned an A, 4 earned a B, 3 earned a C, 3 earned a D, and 5 earned an F. If A, B, C, and D are satisfactory grades, what fraction of the grades are satisfactory?

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{4}{5}$

E $\frac{9}{10}$

Solution:

The satisfactory grades number $5+4+3+3=15$, and the total number of grades is $15+5=20$.

So the fraction that is satisfactory is $\frac{15}{20}=\frac{3}{4}$.

Thus, the correct answer is **C**.

6. A ream of paper containing 500 sheets is 5 cm thick. Approximately how many sheets of this type of paper would there be in a stack 7.5 cm high?

A 250

B 550

C 667

D 750

E 1250

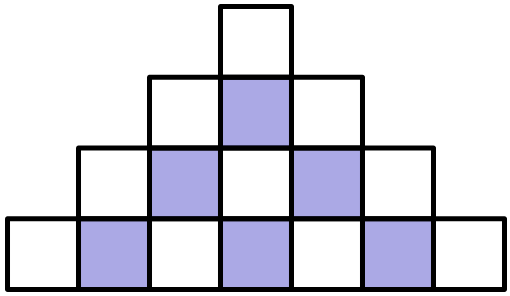
Solution:

Since 7.5 cm is 1.5 times 5 cm, the stack holds 1.5 times as many sheets.

That is $1.5 \times 500 = 750$ sheets.

Thus, the correct answer is **D** .

7. A "stair-step" figure is made up of alternating black and white squares in each row. Rows 1 through 4 are shown. All rows begin and end with a white square. The number of black squares in the 37th row is



A 34

B 35

C 36

D 37

E 38

Solution:

Row n contains $2n - 1$ squares. Because both ends are white and the colors alternate, each row has one more white square than black square, so the number of black squares is $n - 1$.

For the 37th row, this is $37 - 1 = 36$.

Thus, the correct answer is **C** .

8. If $a = -2$, the largest number in the set

$$\left\{-3a,\ 4a,\ \frac{24}{a},\ a^2,\ 1\right\}$$

is

A $-3a$

B $4a$

C $\frac{24}{a}$

D a^2

E 1

Solution:

Substituting $a = -2$ gives the values $-3a = 6, 4a = -8, \frac{24}{a} = -12, a^2 = 4$, and 1.

The largest of these is 6, which is $-3a$.

Thus, the correct answer is **A** .

9. What is the value of the product of the 9 factors below?

$$\left(1-\tfrac{1}{2}\right)\left(1-\tfrac{1}{3}\right)\left(1-\tfrac{1}{4}\right)\cdots\left(1-\tfrac{1}{10}\right)$$

A $\frac{1}{10}$

B $\frac{1}{9}$

C $\frac{1}{2}$

D $\frac{10}{11}$

E $\frac{11}{2}$

Solution:

Each factor $1-\frac{1}{k}$ equals $\frac{k-1}{k}$, so the product is

$$\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdots\frac{9}{10}.$$

Every numerator cancels the previous denominator, leaving $\frac{1}{10}$.

Thus, the correct answer is **A**.

10. What fraction lies halfway between $\frac{1}{5}$ and $\frac{1}{3}$ on the number line?

A $\frac{1}{4}$

B $\frac{2}{15}$

C $\frac{4}{15}$

D $\frac{53}{200}$

E $\frac{8}{15}$

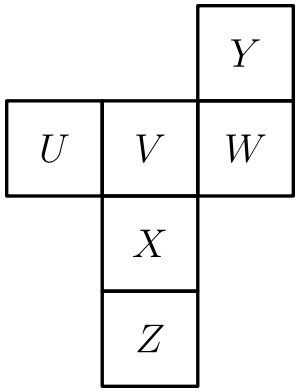
Solution:

The midpoint of $\frac{1}{5}$ and $\frac{1}{3}$ is their average.

$$\frac{\frac{1}{5}+\frac{1}{3}}{2}=\frac{\frac{8}{15}}{2}=\frac{4}{15}.$$

Thus, the correct answer is **C** .

11. A piece of paper containing six joined squares labeled as shown is folded along the edges of the squares to form a cube. The label of the face opposite the face labeled X is



A Z

B U

C V

D W

E Y

Solution:

Fold the net so that X is the bottom face. Then U, V, W, and Z wrap around to become the four side faces.

The only remaining square, Y, becomes the top face, which is opposite the bottom face X.

Thus, the correct answer is **E**.

12. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.2 cm, 8.3 cm, and 9.5 cm. The area of the square, in square centimeters, is

A $24\ \text{cm}^2$

B $36\ \text{cm}^2$

C $48\ \text{cm}^2$

D $64\ \text{cm}^2$

E $144\ \text{cm}^2$

Solution:

The triangle's perimeter is $6.2 + 8.3 + 9.5 = 24$ cm, so the square also has perimeter 24 cm.

The square's side is $\frac{24}{4} = 6$ cm, so its area is $6^2 = 36\ \text{cm}^2$.

Thus, the correct answer is **B**.

13. If you walk for 45 minutes at a rate of 4 mph and then run for 30 minutes at a rate of 10 mph, how many miles have you gone at the end of one hour and 15 minutes?

A 3.5 miles

B 8 miles

C 9 miles

D $25\frac{1}{3}$ miles

E 480 miles

Solution:

Walking: $4 \times \frac{3}{4} = 3$ miles. Running: $10 \times \frac{1}{2} = 5$ miles.

The total distance is $3 + 5 = 8$ miles.

Thus, the correct answer is **B**.

14. The difference between a 6.5% sales tax and a 6% sales tax on an item priced at $\$20$ before tax is

A $.01

B $.10

C $.50

D $1

E $10

Solution:

The two taxes differ by 0.5% of the price.

That is $0.005 \times \$20 = \$0.10.$

Thus, the correct answer is **B** .

15. How many whole numbers between 100 and 400 contain the digit 2?

A 100

B 120

C 138

D 140

E 148

Solution:

All 100 numbers from 200 to 299 contain a 2.

Among 100 to 199, there are 10 with a 2 in the tens place and 10 with a 2 in the units place, but 122 is counted twice, giving $10 + 10 - 1 = 19$. The range 300 to 399 similarly contributes 19.

The total is $100 + 19 + 19 = 138$.

Thus, the correct answer is **C** .

16. The ratio of boys to girls in Mr. Brown's math class is $2:3$. If there are 30 students in the class, how many more girls than boys are in the class?

A 1

B 3

C 5

D 6

E 10

Solution:

The 30 students split into 5 equal parts of 6. Boys make up 2 parts (12) and girls 3 parts (18).

So there are $18 - 12 = 6$ more girls than boys.

Thus, the correct answer is **D** .

17. If your average score on your first six mathematics tests was 84 and your average score on your first seven mathematics tests was 85, then your score on the seventh test was

A 86

B 88

C 90

D 91

E 92

Solution:

Seven tests averaging 85 total $7 \times 85 = 595$ points; six tests averaging 84 total $6 \times 84 = 504$ points.

The seventh score is $595 - 504 = 91$.

Thus, the correct answer is **D** .

18. Nine copies of a certain pamphlet cost less than $\$10.00$ while ten copies of the same pamphlet (at the same price) cost more than $\$11.00$. How much does one copy of this pamphlet cost?

A $1.07

B $1.08

C $1.09

D $1.10

E $1.11

Solution:

From $9p < 10$ we get $p < 1.111\ldots$, and from $10p > 11$ we get $p > 1.10$.

The only price in whole cents between $\$1.10$ and $\$1.111\ldots$ is $\$1.11$.

Thus, the correct answer is **E**.

19. If the length and width of a rectangle are each increased by 10%, then the perimeter of the rectangle is increased by

A 1%

B 10%

C 20%

D 21%

E 40%

Solution:

The new perimeter is $2(1.1\ell + 1.1w) = 1.1 \cdot 2(\ell + w)$, which is 1.1 times the old perimeter.

That is a 10% increase.

Thus, the correct answer is **B** .

20. In a certain year, January had exactly four Tuesdays and four Saturdays. On what day did January 1 fall that year?

A Monday

B Tuesday

C Wednesday

D Friday

E Saturday

Solution:

Since $31 = 4 \cdot 7 + 3$, the weekdays falling on January $1, 2,$ and 3 each occur five times that month, and every other weekday occurs four times.

For Tuesday and Saturday to occur only four times, neither may be January $1, 2,$ or $3.$ The only starting day that works is Wednesday: then Wednesday, Thursday, and Friday occur five times, while Tuesday and Saturday each occur four times.

Thus, the correct answer is **C** .

21. Mr. Green receives a 10% raise every year. His salary after four such raises has gone up by what percent?

A less than 40%

B 40%

C 44%

D 45%

E more than 45%

Solution:

After four raises the salary is multiplied by $1.1^4 = 1.4641$.

That is an increase of 46.41%, which is more than 45%.

Thus, the correct answer is **E**.

22. Assume every 7-digit whole number is a possible telephone number except those that begin with 0 or 1. What fraction of telephone numbers begin with 9 and end with 0?

A $\frac{1}{63}$

B $\frac{1}{80}$

C $\frac{1}{81}$

D $\frac{1}{90}$

E $\frac{1}{100}$

Solution:

The first digit is one of 8 allowed digits (2 through 9), so $\frac{1}{8}$ of the numbers begin with 9.

The last digit is any of 10 digits, so $\frac{1}{10}$ end in 0.

These conditions are independent, so the fraction is $\frac{1}{8} \cdot \frac{1}{10} = \frac{1}{80}$.

Thus, the correct answer is **B**.

23. King Middle School has 1200 students. Each student takes 5 classes a day. Each teacher teaches 4 classes. Each class has 30 students and 1 teacher. How many teachers are there at King Middle School?

A 30

B 32

C 40

D 45

E 50

Solution:

Each day there are $1200 \times 5 = 6000$ student-class attendances. Since each class holds 30 students, there are $\frac{6000}{30} = 200$ classes.

Each teacher teaches 4 classes, so there are $\frac{200}{4} = 50$ teachers.

Thus, the correct answer is **E**.

24. The six whole numbers $10, 11, 12, 13, 14,$ and 15 are placed in six circles — one at each of the three corners of a triangle and one at the midpoint of each side — so that the sum S of the three numbers along each side of the triangle is the same. The largest possible value for S is

A 36

B 37

C 38

D 39

E 40

Solution:

Adding the three side sums gives $3S = (10 + 11 + \cdots + 15) + (\text{sum of corners}) = 75 + (\text{sum of corners})$. To maximize S, place the three largest numbers $13, 14, 15$ at the corners, giving corner sum 42.

Then $3S = 75 + 42 = 117$, so $S = 39$. This is achievable: with corners $13, 14, 15$ and midpoints $12, 10, 11$, each side sums to 39.

Thus, the correct answer is **D** .

25. Five cards are lying on a table. Each card has a letter on one side and a whole number on the other side. The visible faces show P, Q, 3, 4, and 6. Jane said, "If a vowel is on one side of any card, then an even number is on the other side." Mary showed Jane was wrong by turning over one card. Which card did Mary turn over?

A 3

B 4

C 6

D P

E Q

Solution:

To show the rule "a vowel forces an even number" is false, Mary needs a card with a vowel on one side and an odd number on the other. No card shows a vowel, since P and Q are consonants.

The only card showing an odd number is 3. Turning it over reveals a vowel, which contradicts Jane's claim.

Thus, the correct answer is **A** .

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