2025 AMC 12B Solutions

Typeset by: LIVE by Po-Shen Loh
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1. The instructions on a 350-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 20 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

A 16

B 17

C 18

D 19

E 20

Solution:

Each mug uses 20 grams, and $\dfrac{350}{20} = 17.5$. Only complete mugs can be brewed, so the greatest number is 17.

Thus, the correct answer is **B**.

2. Jerry wrote down the ones digit of each of the first 2025 positive squares: $1, 4, 9, 6, 5, 6, \ldots$. What is the sum of all the numbers Jerry wrote down?

- A 9025
- B 9070
- C 9090
- D 9115
- E 9160

Solution:

The ones digits of $1^2, 2^2, \ldots, 10^2$ are $1, 4, 9, 6, 5, 6, 9, 4, 1, 0$, which sum to 45. The 2025 terms contain 202 full blocks (2020 terms) plus 5 more with digits $1, 4, 9, 6, 5$ summing to 25. The total is $202 \cdot 45 + 25 = 9090 + 25 = 9115$.

Thus, the correct answer is **D**.

3. What is the value of $i(i-1)(i-2)(i-3)$, where $i=\sqrt{-1}$?

A $6-5i$

B $-10i$

C $10i$

D -10

E 10

Solution:

$i(i-1)=i^2-i=-1-i$, and $(i-2)(i-3)=i^2-5i+6=5-5i$. Then $(-1-i)(5-5i)=-5+5i-5i+5i^2=-5-5=-10.$

Thus, the correct answer is **D**.

4. The value of the two-digit number $\underline{a}\,\underline{b}$ in base seven equals the value of the two-digit number $\underline{b}\,\underline{a}$ in base nine. What is $a+b$?

A 7

B 9

C 10

D 11

E 14

Solution:

Setting $7a+b=9b+a$ gives $6a=8b$, so $3a=4b$. The digits are $a=4, b=3$, which check out since $43_7=31=34_9$. Hence $a+b=7$.

Thus, the correct answer is **A**.

5. Positive integers x and y satisfy the equation $57x + 22y = 400$. What is the least possible value of $x + y$?

A 10

B 11

C 13

D 14

E 15

Solution:

Modulo 22, the equation gives $13x \equiv 4$, so $x \equiv 2 \pmod{22}$. With $57x < 400$, the only option is $x = 2$, which gives $22y = 286$, so $y = 13$. Then $x + y = 15$.

Thus, the correct answer is **E**.

6. Emmy says to Max, "I ordered 36 math club sweatshirts today." Max asks, "How much did each shirt cost?" Emmy responds, "I'll give you a hint. The total cost was $\$\underline{A}\,\underline{B}\,\underline{B}.\underline{B}\,\underline{A}$, where A and B are digits and $A \neq 0$." After a pause, Max says, "That was a good price." What is $A + B$?

- A 7
- B 8
- C 11
- D 14
- E 15

Solution:

The total in cents is $10000A + 1110B + A = 10001A + 1110B$, which must be a multiple of 36. Since $10001 \equiv 29$ and $1110 \equiv 30 \pmod{36}$, the condition is $29A + 30B \equiv 0$, i.e. $7A + 6B \equiv 0 \pmod{36}$. The only digit solution with $A \neq 0$ is $A = 6, B = 5$ ($7 \cdot 6 + 6 \cdot 5 = 72$), giving $\$655.56 = 36 \times \$18.21$. So $A + B = 11$.

Thus, the correct answer is **C**.

7. What is the value of

$$\sum_{n=2}^{255} \frac{\log_2\left(1+\frac{1}{n}\right)}{(\log_2 n)(\log_2(n+1))}?$$

A $\frac{3}{4}$

B $1 - \frac{1}{\log_2 255}$

C $\frac{7}{8}$

D $\frac{15}{16}$

E 1

Solution:

Let $a_n = \log_2 n$. The numerator equals $a_{n+1} - a_n$, so each term is $\frac{a_{n+1} - a_n}{a_n a_{n+1}} = \frac{1}{a_n} - \frac{1}{a_{n+1}}$. Telescoping from $n = 2$ to 255 leaves $\frac{1}{\log_2 2} - \frac{1}{\log_2 256} = 1 - \frac{1}{8} = \frac{7}{8}$.

Thus, the correct answer is **C**.

8. There are integers a and b such that the polynomial $x^3 - 5x^2 + ax + b$ has $4 + \sqrt{5}$ as a root. What is $a + b$?

A 13

B 17

C 20

D 30

E 68

Solution:

The conjugate $4 - \sqrt{5}$ is also a root, and these two are the roots of $x^2 - 8x + 11$. The third root r satisfies $8 + r = 5$, so $r = -3$. Then $(x^2 - 8x + 11)(x + 3) = x^3 - 5x^2 - 13x + 33$, giving $a = -13$ and $b = 33$, so $a + b = 20$.

Thus, the correct answer is **C**.

9. What is the tens digit of 6^{6^6}?

A 1

B 3

C 5

D 7

E 9

Solution:

Here $6^6 = 46656$. For $n \geq 2$, the last two digits of 6^n cycle with period 5 through $36, 16, 96, 76, 56$. Since $46656 \equiv 1 \pmod 5$, 6^{46656} ends in 56, so the tens digit is 5.

Thus, the correct answer is **C**.

10. The altitude to the hypotenuse of a 30-60-90 right triangle is divided into two segments of lengths $x < y$ by the median to the shortest side of the triangle. What is the ratio $\dfrac{x}{x+y}$?

A $\dfrac{3}{7}$

B $\dfrac{\sqrt{3}}{4}$

C $\dfrac{4}{9}$

D $\dfrac{5}{11}$

E $\dfrac{4\sqrt{3}}{15}$

Solution:

Take $C = (0,0), A = (\sqrt{3}, 0), B = (0,1)$, so AB is the hypotenuse and BC is the shortest side. The altitude from C meets AB at $H = \left(\frac{\sqrt{3}}{4}, \frac{3}{4}\right)$. The median from A to $M = \left(0, \frac{1}{2}\right)$ crosses the altitude at $P = \left(\frac{\sqrt{3}}{7}, \frac{3}{7}\right)$. This splits the altitude into $CP = \frac{2\sqrt{3}}{7} = \frac{4\sqrt{3}}{14}$ and $PH = \frac{3\sqrt{3}}{14}$, so $x = \frac{3\sqrt{3}}{14}$ and $\dfrac{x}{x+y} = \dfrac{3}{7}$.

Thus, the correct answer is **A**.

11. Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

A $\frac{2}{9}$

B $\frac{2}{7}$

C $\frac{9}{28}$

D $\frac{1}{3}$

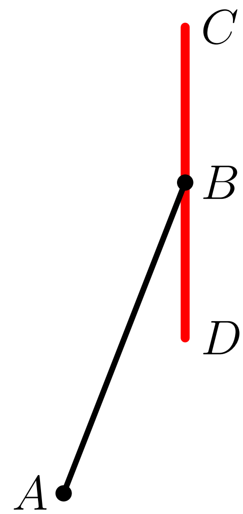
E $\frac{3}{8}$

Solution:

Each group has 3 slots. The three tallest athletes are the captains precisely when they fall into three different groups. Placing them one at a time into the 9 slots, the second must avoid the first's group (6 of the remaining 8 slots) and the third must avoid both used groups (3 of the remaining 7 slots). The probability is $\frac{6}{8} \cdot \frac{3}{7} = \frac{9}{28}$.

Thus, the correct answer is **C**.

12. The windshield wiper on the driver's side of a large bus is depicted below.



Arm $\overline{AB}$ pivots back and forth around point A, sweeping out an arc of 60°, symmetric about the vertical line through A. The wiper blade $\overline{CD}$ is attached to B at its midpoint and stays vertical as the arm moves. The arm is 3 feet long, and the wiper blade is 3.5 feet tall. What is the area of the windshield cleaned by the wiper, in square feet, to the nearest hundredth? (Assume that the windshield is a flat vertical surface.)

A 9.68

B 10.14

C 10.50

D 11.32

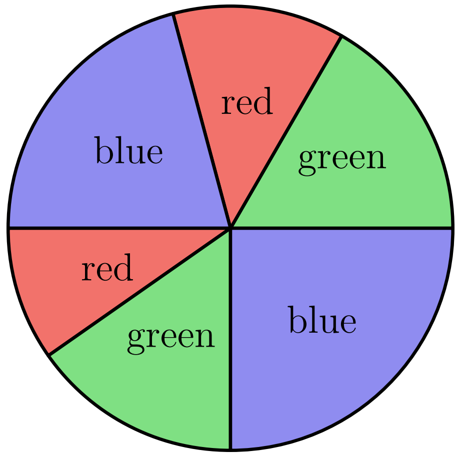
E 12.00

Solution:

Put A at the origin. Then $B = (3\sin\theta, 3\cos\theta)$ for $\theta \in [-30^\circ, 30^\circ]$, so the horizontal coordinate of B ranges over $[-1.5, 1.5]$, a width of 3. At each horizontal position exactly one vertical blade of height 3.5 passes through, so by Cavalieri's principle the cleaned area is $3.5 \times 3 = 10.5$ square feet.

Thus, the correct answer is **C**.

13. A circle has been divided into 6 sectors of different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below.



How many different colorings are possible?

A 12

B 16

C 18

D 24

E 28

Solution:

The two sectors of each color must be a non-adjacent pair, so a coloring is a way to split the 6 cyclic sectors into three non-adjacent pairs together with an assignment of the three colors. The non-adjacent pairs are the edges of the complement of the 6-cycle, the triangular prism, which has 4 perfect matchings. Assigning the three colors in $3!$ ways gives $4 \times 6 = 24$ colorings.

Thus, the correct answer is **D**.

14. Consider a decreasing sequence of n positive integers $x_1 > x_2 > \cdots > x_n$ that satisfies the following two conditions:

- The average (arithmetic mean) of the first 3 terms in the sequence is 2025.

- For all $4 \leq k \leq n$, the average of the first k terms in the sequence is 1 less than the average of the first $k - 1$ terms in the sequence.

What is the greatest possible value of n?

A 1013

B 1014

C 1016

D 2016

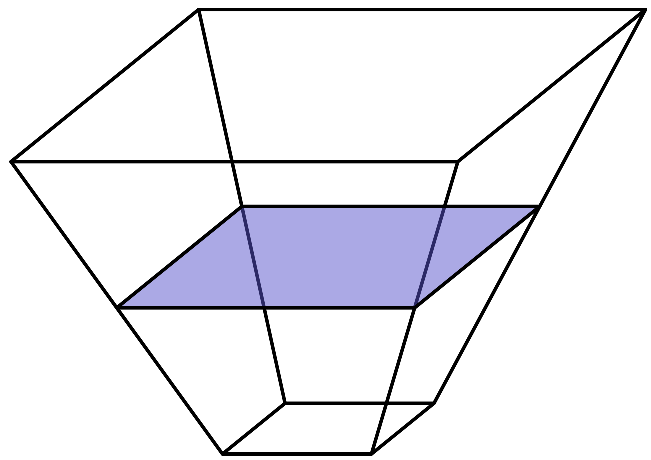
E 2025

Solution:

The average of the first k terms is $A_k = 2028 - k$ for $k \geq 3$, so the partial sum is $S_k = k(2028 - k)$. For $k \geq 4$, $x_k = S_k - S_{k-1} = 2029 - 2k$, which is positive exactly when $k \leq 1014$. A valid start such as $x_1, x_2, x_3 = 2030, 2023, 2022$ keeps the whole sequence strictly decreasing, so the greatest possible n is 1014.

Thus, the correct answer is **B**.

15. A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate takes 35 minutes to fill the container up to the midline of the trapezoids.



How many more minutes will it take to fill the remainder of the container?

A 70

B 85

C 90

D 95

E 105

Solution:

At height fraction t the square cross-section has side $1 + 2t$, so the volume filled up to height t is $\int_0^t (1+2u)^2\,du$. Up to the midline $\left(t = \frac{1}{2}\right)$ this is $\frac{7}{6}$, and the full volume is $\frac{13}{3}$. The remaining volume is $\frac{13}{3} - \frac{7}{6} = \frac{19}{6}$, which is $\frac{19}{7}$ times the first part. So the remainder takes $35 \cdot \frac{19}{7} = 95$ more minutes.

Thus, the correct answer is **D**.

16. An analog clock starts at midnight and runs for 2025 minutes before stopping. What is the tangent of the acute angle between the hour hand and the minute hand when the clock stops?

A 0

B $\sqrt{2}-1$

C $2-\sqrt{2}$

D $\frac{\sqrt{2}}{2}$

E $3-\sqrt{2}$

Solution:

2025 minutes is 33 hours 45 minutes, which modulo 12 hours reads $9{:}45$. The minute hand points at 270° and the hour hand at $9.75 \times 30^\circ = 292.5^\circ$, so the acute angle between them is 22.5°. Using the half-angle value, $\tan 22.5^\circ = \sqrt{2}-1$.

Thus, the correct answer is **B**.

17. Each of the 9 squares in a 3×3 grid is to be colored red, blue, or yellow in such a way that each red square shares an edge with at least one blue square, each blue square shares an edge with at least one yellow square, and each yellow square shares an edge with at least one red square. Colorings that can be obtained from one another by rotations and/or reflections are to be considered the same. How many different colorings are possible?

A 3

B 9

C 12

D 18

E 27

Solution:

Each red square needs a blue neighbor, each blue a yellow, and each yellow a red, forcing all three colors to appear in an interlocking pattern. A systematic check gives 84 valid colorings of the labeled grid. Under the 8 symmetries of the square, only two diagonal reflections fix any colorings — 6 each — so Burnside's lemma gives $\frac{1}{8}(84 + 6 + 6) = 12$ distinct colorings.

Thus, the correct answer is **C**.

18. Awnik repeatedly plays a game that has a probability of winning of $\frac{1}{3}$. The outcomes of the games are independent. What is the expected value of the number of games he will play until he has both won and lost at least once?

A $\frac{5}{2}$

B 3

C $\frac{16}{5}$

D $\frac{7}{2}$

E $\frac{15}{4}$

Solution:

The first game produces one outcome. If it was a win (probability $\frac{1}{3}$), the expected wait for a loss is $\frac{1}{2/3} = \frac{3}{2}$; if it was a loss (probability $\frac{2}{3}$), the expected wait for a win is $\frac{1}{1/3} = 3$. So the expected total is $1 + \frac{1}{3} \cdot \frac{3}{2} + \frac{2}{3} \cdot 3 = 1 + \frac{1}{2} + 2 = \frac{7}{2}$.

Thus, the correct answer is **D**.

19. A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \times 91 = 12{,}831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

A 7

B 10

C 11

D 12

E 19

Solution:

In row r, column c, Horace writes $(r-1)\cdot 91 + c$ and Vera writes $(c-1)\cdot 141 + r$. Setting these equal gives $90r - 140c = -50$, i.e. $9r = 14c - 5$. This requires $c \equiv 1 \pmod 9$, so $c = 1, 10, 19, \ldots, 91$ — that is 11 values, and each yields a valid r between 1 and 141. So 11 squares match.

Thus, the correct answer is **C**.

20. A frog hops along the number line according to the following rules.

- It starts at 0.
- If it is at 0, then it moves to 1 with probability $\frac{1}{2}$ and it disappears with probability $\frac{1}{2}$.
- For $n = 1, 2,$ or $3,$ if it is at $n,$ then it moves to $n+1$ with probability $\frac{1}{4}$, it moves to $n-1$ with probability $\frac{1}{4}$, and it disappears with probability $\frac{1}{2}$.

What is the probability that the frog reaches 4?

A $\frac{1}{101}$

B $\frac{1}{100}$

C $\frac{1}{99}$

D $\frac{1}{98}$

E $\frac{1}{97}$

Solution:

Let $f(n)$ be the probability of reaching 4 from n. Then $f(0) = \frac{1}{2}f(1)$, and for $n = 1, 2, 3$, $f(n) = \frac{1}{4}f(n+1) + \frac{1}{4}f(n-1)$, with $f(4) = 1$. Solving upward gives $f(2) = \frac{7}{2}f(1)$ and $f(3) = 13f(1)$; then $4 \cdot 13f(1) = 1 + \frac{7}{2}f(1)$ yields $f(1) = \frac{2}{97}$, so $f(0) = \frac{1}{97}$.

Thus, the correct answer is **E**.

21. Two non-congruent triangles have the same area. Each triangle has sides of length 8 and 9, and the third side of each triangle has integer length. What is the sum of the lengths of the third sides?

A 20

B 22

C 24

D 26

E 28

Solution:

The area with included angle θ is $36 \sin \theta$, so two triangles of equal area use angles θ and $180^\circ - \theta$, with cosines $\pm \cos \theta$. By the law of cosines the third sides satisfy $t^2 = 145 \mp 144 \cos \theta$, hence $t_1^2 + t_2^2 = 290$. The only integer values in the valid range $(1, 17)$ are 11 and 13 ($121 + 169 = 290$), so the sum is 24.

Thus, the correct answer is **C**.

22. What is the greatest possible area of the triangle in the complex plane with vertices $2z, (1+i)z$, and $(1-i)z$, where z is a complex number satisfying $|4z-2|=1$?

A $\frac{1}{4}$

B $\frac{1}{2}$

C $\frac{9}{16}$

D $\frac{3}{4}$

E 1

Solution:

The vertices are $z \cdot 2, z(1+i)$, and $z(1-i)$, so the triangle is the fixed triangle with vertices $2, 1+i, 1-i$ — which has area 1 — scaled by $|z|$, giving area $|z|^2$. The condition $|4z-2|=1$ is the circle $\left|z-\frac{1}{2}\right|=\frac{1}{4}$, on which $|z|$ is at most $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$. So the greatest area is $\left(\frac{3}{4}\right)^2=\frac{9}{16}$.

Thus, the correct answer is **C**.

23. Let S be the set of all integers $z > 1$ such that for all pairs of nonnegative integers (x, y) with $x < y < z$, the remainder when $2025x$ is divided by z is less than the remainder when $2025y$ is divided by z. What is the sum of the elements of S?

A 3041

B 3542

C 3750

D 4044

E 4319

Solution:

The condition requires $k \mapsto 2025k \bmod z$ to be strictly increasing on $\{0, 1, \ldots, z-1\}$. A strictly increasing list of z distinct values in $[0, z-1]$ must be $0, 1, \ldots, z-1$, so $2025 \equiv 1 \pmod{z}$, i.e. $z \mid 2024$. Since $2024 = 2^3 \cdot 11 \cdot 23$, the sum of all its divisors is $(1+2+4+8)(1+11)(1+23) = 15 \cdot 12 \cdot 24 = 4320$. Excluding $z = 1$ leaves 4319.

Thus, the correct answer is **E**.

24. How many real numbers satisfy the equation $\sin(20\pi x) = \log_{20}(x)$?

A 199

B 200

C 398

D 399

E 400

Solution:

Since $|\sin| \leq 1$, solutions need $x \in \left[\frac{1}{20}, 20\right]$, where $\log_{20} x$ rises from -1 to 1. The curve $\sin(20\pi x)$ has period $\frac{1}{10}$, and on every full monotonic branch inside this interval it crosses the slowly increasing log exactly once. There are 398 such full branches; the partial branch near $x = \frac{1}{20}$ adds one crossing, while the partial branch near $x = 20$ adds none (the sine cannot rise to the log's near-1 value there). This gives 399 solutions.

Thus, the correct answer is **D**.

25. Three concentric circles have radii $1, 2, 3$. An equilateral triangle with side length s has one vertex on each circle. What is s^2?

A 6

B $\frac{25}{4}$

C $\frac{13}{2}$

D $\frac{27}{4}$

E 7

Solution:

For the common center at distances $1, 2, 3$ from the vertices of an equilateral triangle of side s, the identity $3(1 + 16 + 81 + s^4) = (1 + 4 + 9 + s^2)^2$ holds. This simplifies to $2s^4 - 28s^2 + 98 = 0$, i.e. $(s^2 - 7)^2 = 0$, so $s^2 = 7$.

Thus, the correct answer is **E**.

Problems: https://live.poshenloh.com/past-contests/amc12/2025B