2019 AMC 12B Solutions

Typeset by: LIVE by Po-Shen Loh
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1. Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the first container to the volume of the second container?

A $\frac{5}{8}$

B $\frac{4}{5}$

C $\frac{7}{8}$

[✓ D] $\frac{9}{10}$

E $\frac{11}{12}$

Solution:

The volume of water is the same before and after, so $\frac{5}{6}V_1 = \frac{3}{4}V_2$.

Then

$$\frac{V_1}{V_2} = \frac{3/4}{5/6} = \frac{3}{4} \cdot \frac{6}{5} = \frac{9}{10}.$$

Thus, **D** is the correct answer.

2. Consider the statement, "If n is not prime, then $n-2$ is prime." Which of the following values of n is a counterexample to this statement?

A 11

B 15

C 19

D 21

[✓ E] 27

Solution:

A counterexample needs n not prime (so the hypothesis holds) and $n-2$ not prime (so the conclusion fails).

Among the choices, 27 is not prime and $27-2=25$ is not prime. The primes 11 and 19 fail the hypothesis, and $15-2=13, 21-2=19$ are prime.

Thus, **E** is the correct answer.

3. Which one of the following rigid transformations (isometries) maps the line segment $\overline{AB}$ onto the line segment $\overline{A'B'}$ so that the image of $A(-2, 1)$ is $A'(2, -1)$ and the image of $B(-1, 4)$ is $B'(1, -4)$?

A reflection in the y-axis

B counterclockwise rotation around the origin by 90°

C translation by 3 units to the right and 5 units down

D reflection in the x-axis

[✓ E] clockwise rotation about the origin by 180°

Solution:

Each point maps by $(x, y) \to (-x, -y)$: indeed $(-2, 1) \to (2, -1)$ and $(-1, 4) \to (1, -4)$.

The map $(x, y) \to (-x, -y)$ is a 180° rotation about the origin.

Thus, **E** is the correct answer.

4. A positive integer n satisfies the equation $(n+1)! + (n+2)! = 440 \cdot n!$. What is the sum of the digits of n?

A 2

B 5

[✓ C] 10

D 12

E 15

Solution:

Factor the left side:

$$(n+1)! + (n+2)! = (n+1)!\,[1 + (n+2)] = (n+1)!\,(n+3).$$

Dividing both sides by $n!$ and using $(n+1)! = (n+1)\,n!$ gives

$$(n+1)(n+3) = 440.$$

So $n^2 + 4n - 437 = 0$, which factors as $(n-19)(n+23) = 0$, giving $n = 19$. Its digit sum is $1 + 9 = 10$.

Thus, **C** is the correct answer.

5. Each piece of candy in a store costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the smallest possible value of n?

A 18

[✓ B] 21

C 24

D 25

E 28

Solution:

Let M be Casper's money in cents. Since he can exactly buy $12, 14,$ or 15 whole-cent pieces, M is a multiple of $\operatorname{lcm}(12, 14, 15) = 420.$

Purple candy costs 20 cents, so $n = \frac{M}{20}$. The smallest M is $420,$ giving $n = \frac{420}{20} = 21.$

Thus, **B** is the correct answer.

6. In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

[✓ A] 0

B 2

C 4

D 8

E infinitely many

Solution:

The perimeter condition gives $CA + CB = 50 - 10 = 40$, so C lies on an ellipse with foci A, B and major axis $2a = 40$. Thus $a = 20$ and $c = 5$, so the semi-minor axis is

$$b = \sqrt{a^2 - c^2} = \sqrt{375} \approx 19.36.$$

For area 100 with base $AB = 10$, the height from C must be $\dfrac{2 \cdot 100}{10} = 20$. But the greatest possible height on the ellipse is $b \approx 19.36 < 20$, so no such C exists.

Thus, **A** is the correct answer.

7. What is the sum of all real numbers x for which the median of the numbers $4, 6, 8, 17,$ and x is equal to the mean of those five numbers?

A -5

B 0

C 5

D $\frac{15}{4}$

E $\frac{35}{4}$

Solution:

The mean is $\dfrac{4+6+8+17+x}{5} = \dfrac{35+x}{5}$.

If $x \leq 6$, the median is 6, so $\dfrac{35+x}{5} = 6$ gives $x = -5$, which is consistent.

If $6 < x < 8$, the median is x, so $\dfrac{35+x}{5} = x$ gives $x = 8.75$, not in range. If $x \geq 8$, the median is 8, so $\dfrac{35+x}{5} = 8$ gives $x = 5$, not in range.

The only solution is $x = -5$, so the sum is -5.

Thus, **A** is the correct answer.

8. Let $f(x) = x^2(1-x)^2$. What is the value of the sum

$$f\left(\tfrac{1}{2019}\right) - f\left(\tfrac{2}{2019}\right) + f\left(\tfrac{3}{2019}\right) - f\left(\tfrac{4}{2019}\right) + \cdots + f\left(\tfrac{2017}{2019}\right) - f\left(\tfrac{2018}{2019}\right)?$$

[✓ A] 0

B $\dfrac{1}{2019^4}$

C $\dfrac{2018^2}{2019^4}$

D $\dfrac{2020^2}{2019^4}$

E 1

Solution:

Since $f(1-x) = (1-x)^2x^2 = f(x)$, we have $f\left(\frac{k}{2019}\right) = f\left(\frac{2019-k}{2019}\right)$.

In the sum, the term with index k has sign $(-1)^{k+1}$, while the term with index $2019-k$ equals it in value but has sign $(-1)^{2019-k+1} = (-1)^k$, the opposite.

Every term cancels with its partner, so the total is 0.

Thus, **A** is the correct answer.

9. For how many integral values of x can a triangle of positive area be formed having side lengths $\log_2 x, \log_4 x$, and 3?

A 57

[✓ B] 59

C 61

D 62

E 63

Solution:

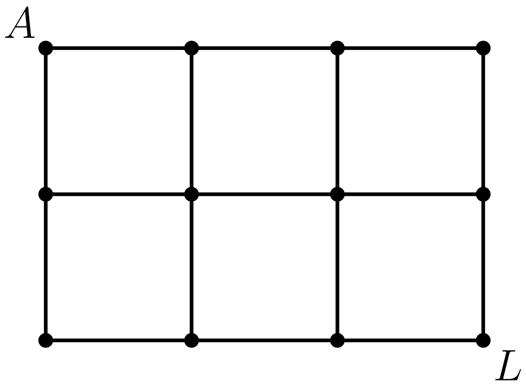
Let $t = \log_2 x$. Then $\log_4 x = \frac{t}{2}$, and the sides are $t,\ \frac{t}{2},\ 3$.

The triangle inequalities give $t + \frac{t}{2} > 3$ (so $t > 2$) and $\frac{t}{2} + 3 > t$ (so $t < 6$); the third inequality is automatic.

Thus $2 < \log_2 x < 6$, i.e. $4 < x < 64$. The integers $5, 6, \ldots, 63$ number 59.

Thus, **B** is the correct answer.

10. The figure below is a map showing 12 cities and 17 roads connecting certain pairs of cities. Paula wishes to travel along exactly 13 of those roads, starting at city A and ending at city L, without traveling along any portion of a road more than once. (Paula is allowed to visit a city more than once.) How many different routes can Paula take?



A 0

B 1

C 2

D 3

[✓ E] 4

Solution:

A route uses 13 roads as an open trail from A to L, so on the used roads exactly A and L have odd degree and every other city has even degree.

In the full map the corner cities A and L already have even degree 2, and six edge-cities have odd degree 3. Removing 4 roads must flip the parity of A, L, and those six cities, and of no others. This forces the four removed roads to pair up those eight cities in the only possible way, so the set of 13 used roads is uniquely determined.

Counting the Eulerian trails from A to L on that graph gives exactly 4 routes.

Thus, **E** is the correct answer.

11. How many unordered pairs of edges of a given cube determine a plane?

A 12

B 28

C 36

[✓ D] 42

E 66

Solution:

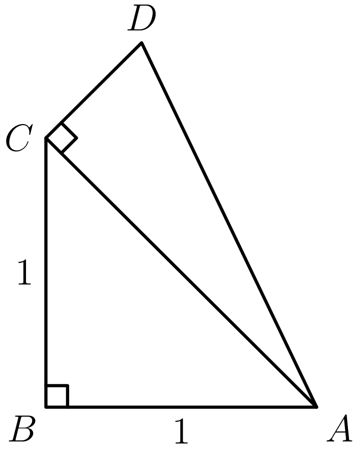
Two edges determine a plane exactly when they are coplanar, that is, parallel or intersecting.

The 12 edges split into 3 directions of 4 parallel edges, giving $3\binom{4}{2} = 18$ parallel pairs. Edges sharing a vertex give $8\binom{3}{2} = 24$ intersecting pairs.

The total is $18 + 24 = 42.$

Thus, **D** is the correct answer.

12. Right triangle ACD with right angle at C is constructed outwards on the hypotenuse $\overline{AC}$ of isosceles right triangle ABC with leg length 1, as shown, so that the two triangles have equal perimeters. What is $\sin(2\angle BAD)$?



A $\frac{1}{3}$

B $\frac{\sqrt{2}}{2}$

C $\frac{3}{4}$

[✓ D] $\frac{7}{9}$

E $\frac{\sqrt{3}}{2}$

Solution:

Triangle ABC has perimeter $1 + 1 + \sqrt{2} = 2 + \sqrt{2}$ and $AC = \sqrt{2}$. In $\triangle ACD$ let $CD = d$, so $AD = \sqrt{2 + d^2}$ and equal perimeters give

$$\sqrt{2} + d + \sqrt{2 + d^2} = 2 + \sqrt{2}.$$

Then $\sqrt{2 + d^2} = 2 - d$, so $2 + d^2 = 4 - 4d + d^2$, giving $d = \frac{1}{2}$ and $AD = \frac{3}{2}$.

Since $\angle BAC = 45^\circ$, writing $\theta = \angle CAD$ gives $2\angle BAD = 90^\circ + 2\theta$, so $\sin(2\angle BAD) = \cos 2\theta$. With $\tan\theta = \dfrac{CD}{AC} = \dfrac{1}{2\sqrt{2}}$, we get

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{1}{8}}{1+\frac{1}{8}} = \frac{7}{9}.$$

Thus, **D** is the correct answer.

13. A red ball and a green ball are randomly and independently tossed into bins numbered with the positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3, \ldots$ What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?

A $\frac{1}{4}$

B $\frac{2}{7}$

[✓ C] $\frac{1}{3}$

D $\frac{3}{8}$

E $\frac{3}{7}$

Solution:

The probability the balls land in the same bin is

$$\sum_{k=1}^{\infty} \left(2^{-k}\right)^2 = \sum_{k=1}^{\infty} 4^{-k} = \frac{1/4}{1 - 1/4} = \frac{1}{3}.$$

By symmetry, the red ball being higher and the green ball being higher are equally likely, so each has probability

$$\frac{1 - \frac{1}{3}}{2} = \frac{1}{3}.$$

Thus, **C** is the correct answer.

14. Let S be the set of all positive integer divisors of $100{,}000$. How many numbers are the product of two distinct elements of S?

A 98

B 100

[✓ C] 117

D 119

E 121

Solution:

Since $100{,}000 = 2^5 \cdot 5^5$, every divisor is $2^a 5^b$ with $0 \leq a, b \leq 5$. A product of two divisors is $2^x 5^y$ with $0 \leq x, y \leq 10$, and every such pair (x, y) is attainable, giving $11 \cdot 11 = 121$ values.

We need two \emph{distinct} divisors. A value $2^x 5^y$ is forced to be a divisor times itself only when both x and y have a unique split, which happens exactly when $x, y \in \{0, 10\}$. Those 4 corner values ($1,\ 2^{10},\ 5^{10},\ 2^{10}5^{10}$) cannot use two distinct divisors.

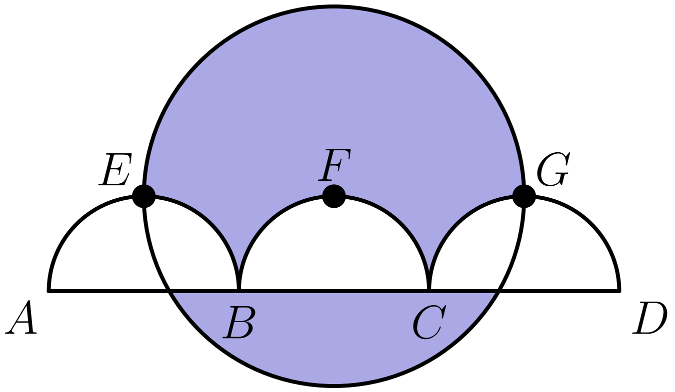
The count is $121 - 4 = 117$.

Thus, **C** is the correct answer.

15. As shown in the figure, line segment $\overline{AD}$ is trisected by points B and C so that $AB = BC = CD = 2$. Three semicircles of radius 1, AEB, BFC, and CGD, have their diameters on $\overline{AD}$, and are tangent to line EG at E, F, and G, respectively. A circle of radius 2 has its center on F. The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form

$$\frac{a}{b} \cdot \pi - \sqrt{c} + d,$$

where $a, b, c,$ and d are positive integers and a and b are relatively prime. What is $a + b + c + d$?



A 13

B 14

C 15

D 16

[✓ E] 17

Solution:

Put $A = (0, 0),\ B = (2, 0),\ C = (4, 0),\ D = (6, 0)$, so the semicircles are centered at $(1, 0), (3, 0), (5, 0)$ and their tops are $E = (1, 1),\ F = (3, 1),\ G = (5, 1)$. The circle has center $F = (3, 1)$ and radius 2, so it passes through E and G, and has area 4π.

The middle semicircle BFC lies entirely inside the circle, removing area $\frac{\pi}{2}$. By symmetry the outer semicircles each contribute the same overlap $R = \frac{7\pi}{12} - 2 + \frac{\sqrt{3}}{2}$ inside the circle.

The shaded area is

$$4\pi - \frac{\pi}{2} - 2R = \frac{7}{3}\pi - \sqrt{3} + 4.$$

Hence $a = 7,\ b = 3,\ c = 3,\ d = 4,$ so $a + b + c + d = 17.$

Thus, **E** is the correct answer.

16. There are lily pads in a row numbered 0 to 11, in that order. There are predators on lily pads 3 and 6, and a morsel of food on lily pad 10. Fiona the frog starts on pad 0, and from any given lily pad, has a $\frac{1}{2}$ chance to hop to the next pad, and an equal chance to jump 2 pads. What is the probability that Fiona reaches pad 10 without landing on either pad 3 or pad 6?

[✓ A] $\frac{15}{256}$

B $\frac{1}{16}$

C $\frac{15}{128}$

D $\frac{1}{8}$

E $\frac{1}{4}$

Solution:

Let $p(n)$ be the probability of landing on pad n without first landing on pad 3 or 6. Each pad sends probability $\frac{1}{2}$ to the next pad and $\frac{1}{2}$ two pads ahead, and pads 3 and 6 pass nothing on.

Then $p(0) = 1,\ p(1) = \frac{1}{2},\ p(2) = \frac{3}{4}$, and (skipping 3) $p(4) = \frac{3}{8},\ p(5) = \frac{3}{16}$, then (skipping 6) $p(7) = \frac{3}{32},\ p(8) = \frac{3}{64},\ p(9) = \frac{9}{128}$.

Finally

$$p(10) = \frac{1}{2}p(8) + \frac{1}{2}p(9) = \frac{3}{128} + \frac{9}{256} = \frac{15}{256}.$$

Thus, **A** is the correct answer.

17. How many nonzero complex numbers z have the property that $0, z,$ and z^3, when represented by points in the complex plane, are the three distinct vertices of an equilateral triangle?

A 0

B 1

C 2

[✓ D] 4

E infinitely many

Solution:

The three points form an equilateral triangle iff $|z| = |z^3| = |z^3 - z|$. From $|z| = |z^3| = |z|^3$ we get $|z| = 1$.

Then $|z^3 - z| = |z|\,|z^2 - 1| = |z^2 - 1|$, so we need $|z^2 - 1| = 1$. Writing $z = e^{i\theta}$, $|z^2 - 1| = 2|\sin\theta| = 1$, so $|\sin\theta| = \frac{1}{2}$.

This gives $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$, four values of z, all yielding distinct vertices.

Thus, **D** is the correct answer.

18. Square pyramid $ABCDE$ has base $ABCD$, which measures 3 cm on a side, and altitude $\overline{AE}$ perpendicular to the base, which measures 6 cm. Point P lies on $\overline{BE}$, one third of the way from B to E; point Q lies on $\overline{DE}$, one third of the way from D to E; and point R lies on $\overline{CE}$, two thirds of the way from C to E. What is the area, in square centimeters, of $\triangle PQR$?

A $\dfrac{3\sqrt{2}}{2}$

B $\dfrac{3\sqrt{3}}{2}$

[✓ C] $2\sqrt{2}$

D $2\sqrt{3}$

E $3\sqrt{2}$

Solution:

Place $A=(0,0,0),\ B=(3,0,0),\ C=(3,3,0),\ D=(0,3,0),\ E=(0,0,6)$. Then

$$P=(2,0,2),\quad Q=(0,2,2),\quad R=(1,1,4).$$

So $\vec{PQ}=(-2,2,0)$ and $\vec{PR}=(-1,1,2)$, giving $\vec{PQ}\times\vec{PR}=(4,4,0)$.

The area is $\dfrac{1}{2}\left|(4,4,0)\right|=\dfrac{1}{2}\cdot 4\sqrt{2}=2\sqrt{2}$.

Thus, **C** is the correct answer.

19. Raashan, Sylvia, and Ted play the following game. Each starts with $\$1$. A bell rings every 15 seconds, at which time each of the players who currently have money simultaneously chooses one of the other two players independently and at random and gives $\$1$ to that player. What is the probability that after the bell has rung 2019 times, each player will have $\$1$? (For example, Raashan and Ted may each decide to give $\$1$ to Sylvia, and Sylvia may decide to give her dollar to Ted, at which point Raashan will have $\$0$, Sylvia will have $\$2$, and Ted will have $\$1$, and that is the end of the first round of play. In the second round Raashan has no money to give, but Sylvia and Ted might choose each other to give their $\$1$ to, and the holdings will be the same at the end of the second round.)

A $\frac{1}{7}$

[✓ B] $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{1}{2}$

E $\frac{2}{3}$

Solution:

From $(1,1,1)$, each of the three players gives to one of two others, so there are 8 equally likely outcomes; only the 2 cyclic gift patterns return to $(1,1,1)$, a probability of $\frac{1}{4}$.

From a $(2,1,0)$ state the broke player gives nothing, and checking the 4 equally likely choices of the other two shows exactly one yields $(1,1,1)$, again probability $\frac{1}{4}$.

So after any ring the probability of $(1,1,1)$ is $\frac{1}{4}$, including after 2019 rings.

Thus, **B** is the correct answer.

20. Points $A(6, 13)$ and $B(12, 11)$ lie on circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x-axis. What is the area of ω?

- A $\frac{83\pi}{8}$
- B $\frac{21\pi}{2}$
- [✓ C] $\frac{85\pi}{8}$
- D $\frac{43\pi}{4}$
- E $\frac{87\pi}{8}$

Solution:

Let $P = (x, 0)$ be the intersection. Equal tangent lengths give $PA = PB$, so $(x - 6)^2 + 13^2 = (x - 12)^2 + 11^2$, yielding $x = 5$ and $P = (5, 0)$.

The center $O = (h, k)$ satisfies $OA \perp PA$ and $OB \perp PB$. With $PA = (1, 13)$ and $PB = (7, 11)$, these give $h + 13k = 175$ and $7h + 11k = 205$, so $O = \left(\frac{37}{4}, \frac{51}{4}\right)$.

Then $r^2 = OA^2 = \left(\frac{13}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{170}{16} = \frac{85}{8}$, so the area is $\frac{85\pi}{8}$.

Thus, **C** is the correct answer.

21. How many quadratic polynomials with real coefficients are there such that the set of roots equals the set of coefficients? (For clarification: If the polynomial is $ax^2 + bx + c,\ a \neq 0$, and the roots are r and s, then the requirement is that $\{a, b, c\} = \{r, s\}$.)

A 3

[✓ B] 4

C 5

D 6

E infinitely many

Solution:

The set $\{a, b, c\}$ must equal the two-element set $\{r, s\}$, so at least two coefficients coincide, and the roots are the two distinct coefficient values. By Vieta's formulas $r + s = -\frac{b}{a}$ and $rs = \frac{c}{a}$.

Working through the cases of which coefficients are equal yields the polynomials $x^2 + x - 2, -x^2 - x, x^2 - \frac{1}{2}x - \frac{1}{2}$, and $ux^2 + \frac{1}{u}x + u$ where u is the unique real root of $u^3 + u + 1 = 0$.

That is 4 polynomials in all.

Thus, **B** is the correct answer.

22. Define a sequence recursively by $x_0 = 5$ and

$$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$$

for all nonnegative integers n. Let m be the least positive integer such that

$$x_m \le 4 + \frac{1}{2^{20}}.$$

In which of the following intervals does m lie?

A $[9, 26]$

B $[27, 80]$

[✓ C] $[81, 242]$

D $[243, 728]$

E $[729, \infty)$

Solution:

Let $a_n = x_n - 4$. A short computation gives

$$a_{n+1} = x_{n+1} - 4 = \frac{(x_n + 5)(x_n - 4)}{x_n + 6} = a_n \cdot \frac{x_n + 5}{x_n + 6}.$$

Starting from $a_0 = 1$, the terms stay positive and decrease. Because x_n decreases from 5 toward 4, each ratio $\frac{x_n + 5}{x_n + 6}$ lies strictly between $\frac{9}{10}$ and $\frac{10}{11}$. Hence a_m is squeezed between $\left(\frac{9}{10}\right)^m$ and $\left(\frac{10}{11}\right)^m$. Solving $a_m \le 2^{-20}$ puts m between about 132 and 146, which lies in $[81, 242]$.

Thus, **C** is the correct answer.

23. How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

A 55

B 60

[✓ C] 65

D 70

E 75

Solution:

No two 0s are adjacent, so the 0s are separated by blocks of 1s, each of size 1 or 2 (never 3). If there are k zeros, there are $k-1$ such blocks summing to $19-k$ ones.

The number of size-2 blocks is $(19-k)-(k-1)=20-2k,$ which must satisfy $0 \leq 20-2k \leq k-1,$ i.e. $7 \leq k \leq 10.$

Summing $\binom{k-1}{20-2k}$ over $k=7,8,9,10$ gives $\binom{6}{6}+\binom{7}{4}+\binom{8}{2}+\binom{9}{0}=1+35+28+1=65.$

Thus, **C** is the correct answer.

24. Let $\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$. Let S denote all points in the complex plane of the form $a + b\omega + c\omega^2$, where $0 \le a \le 1,\ 0 \le b \le 1$, and $0 \le c \le 1$. What is the area of S?

A $\frac{1}{2}\sqrt{3}$

B $\frac{3}{4}\sqrt{3}$

[✓ C] $\frac{3}{2}\sqrt{3}$

D $\frac{1}{2}\pi\sqrt{3}$

E π

Solution:

As a, b, c range over $[0, 1]$, the set S is the Minkowski sum of the three unit segments along $v_1 = 1 = (1, 0),\ v_2 = \omega = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right),\ v_3 = \omega^2 = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

This is a zonogon whose area is the sum of the cross-product magnitudes over pairs.

Each pair gives $|v_i \times v_j| = \frac{\sqrt{3}}{2}$.

Therefore the area is $3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$.

Thus, **C** is the correct answer.

25. Let $ABCD$ be a convex quadrilateral with $BC = 2$ and $CD = 6$. Suppose that the centroids of $\triangle ABC, \triangle BCD,$ and $\triangle ACD$ form the vertices of an equilateral triangle. What is the maximum possible value of the area of $ABCD$?

A 27

B $16\sqrt{3}$

[✓ C] $12 + 10\sqrt{3}$

D $9 + 12\sqrt{3}$

E 30

Solution:

The centroids are $\dfrac{A+B+C}{3}, \dfrac{B+C+D}{3}, \dfrac{A+C+D}{3}$. Their pairwise differences are $\dfrac{A-D}{3}, \dfrac{B-A}{3}, \dfrac{B-D}{3}$, so an equilateral centroid triangle forces $AB = BD = DA$; that is, $\triangle ABD$ is equilateral with side $s = BD$.

Splitting along BD,

$$[ABCD] = [ABD] + [BCD] = \frac{\sqrt{3}}{4}s^2 + \frac{1}{2} \cdot 2 \cdot 6 \sin C,$$

where $C = \angle BCD$. By the Law of Cosines $s^2 = 40 - 24\cos C$, so

$$[ABCD] = 10\sqrt{3} - 6\sqrt{3}\cos C + 6\sin C.$$

The expression $6\sin C - 6\sqrt{3}\cos C$ has maximum $\sqrt{6^2 + (6\sqrt{3})^2} = 12$, so the greatest area is $10\sqrt{3} + 12 = 12 + 10\sqrt{3}$.

Thus, **C** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc12/2019B