1996 AMC 12

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc12/1996

1. The addition below is incorrect. What is the largest digit that can be changed to make the addition correct?

$$\begin{array}{r} 641 \\ 852 \\ +\,973 \\ \hline 2456 \end{array}$$

A 4

B 5

C 6

D 7

E 8

2. Each day Walter gets $\$3$ for doing his chores or $\$5$ for doing them exceptionally well. After 10 days of doing his chores daily, Walter has received a total of $\$36.$ On how many days did Walter do them exceptionally well?

A 3

B 4

C 5

D 6

E 7

3. $\frac{(3!)!}{3!} =$

A 1

B 2

C 6

D 40

E 120

4. Six numbers from a list of nine integers are $7, 8, 3, 5, 9,$ and 5. The largest possible value of the median of all nine numbers in this list is

A 5

B 6

C 7

D 8

E 9

5. Given that $0 < a < b < c < d$, which of the following is the largest?

A $\frac{a+b}{c+d}$

B $\frac{a+d}{b+c}$

C $\frac{b+c}{a+d}$

D $\frac{b+d}{a+c}$

E $\frac{c+d}{a+b}$

6. If $f(x) = x^{x+1}(x+2)^{x+3}$, then $f(0) + f(-1) + f(-2) + f(-3) =$

A $-\frac{8}{9}$

B 0

C $\frac{8}{9}$

D 1

E $\frac{10}{9}$

7. A father takes his twins and a younger child out to dinner on the twins' birthday. The restaurant charges $\$4.95$ for the father and $\$0.45$ for each year of a child's age, where age is defined as the age at the most recent birthday. If the bill is $\$9.45$, which of the following could be the age of the youngest child?

A 1

B 2

C 3

D 4

E 5

8. If $3 = k \cdot 2^r$ and $15 = k \cdot 4^r$, then $r =$

A $-\log_2 5$

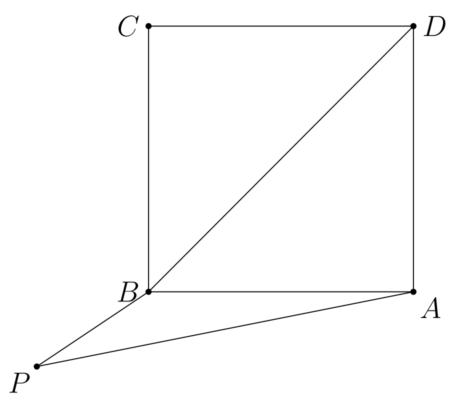
B $\log_5 2$

C $\log_{10} 5$

D $\log_2 5$

E $\frac{5}{2}$

9. Triangle PAB and square $ABCD$ are in perpendicular planes. Given that $PA = 3$, $PB = 4$, and $AB = 5$, what is PD?



A 5

B $\sqrt{34}$

C $\sqrt{41}$

D $2\sqrt{13}$

E 8

10. How many line segments have both their endpoints located at the vertices of a given cube?

A 12

B 15

C 24

D 28

E 56

11. Given a circle of radius 2, there are many line segments of length 2 that are tangent to the circle at their midpoints. Find the area of the region consisting of all such line segments.

A $\frac{\pi}{4}$

B $4 - \pi$

C $\frac{\pi}{2}$

D π

E 2π

12. A function f from the integers to the integers is defined as follows:

$$f(n) = \begin{cases} n+3 & \text{if } n \text{ is odd,} \\ \frac{n}{2} & \text{if } n \text{ is even.} \end{cases}$$

Suppose k is odd and $f(f(f(k))) = 27$. What is the sum of the digits of k?

A 3

B 6

C 9

D 12

E 15

13. Sunny runs at a steady rate, and Moonbeam runs m times as fast, where m is a number greater than 1. If Moonbeam gives Sunny a head start of h meters, how many meters must Moonbeam run to overtake Sunny?

A hm

B $\frac{h}{h+m}$

C $\frac{h}{m-1}$

D $\frac{hm}{m-1}$

E $\frac{h+m}{m-1}$

14. Let $E(n)$ denote the sum of the even digits of n. For example, $E(5681) = 6 + 8 = 14$. Find $E(1) + E(2) + E(3) + \cdots + E(100)$.

A 200

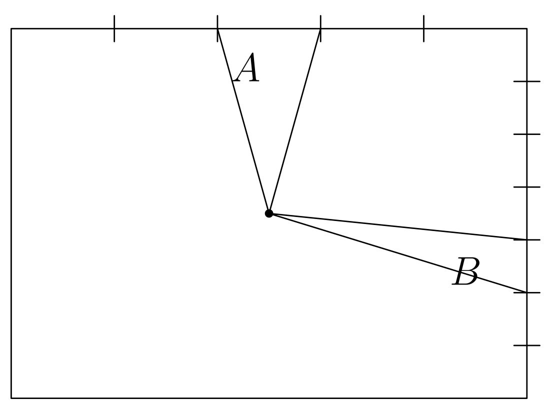
B 360

C 400

D 900

E 2250

15. Two opposite sides of a rectangle are each divided into n congruent segments, and the endpoints of one segment are joined to the center to form triangle A. The other sides are each divided into m congruent segments, and the endpoints of one of these segments are joined to the center to form triangle B. [See figure for $n = 5, m = 7$.] What is the ratio of the area of triangle A to the area of triangle B?



A 1

B $\frac{m}{n}$

C $\frac{n}{m}$

D $\frac{2m}{n}$

E $\frac{2n}{m}$

16. A fair standard six-sided dice is tossed three times. Given that the sum of the first two tosses equals the third, what is the probability that at least one 2 is tossed?

A $\frac{1}{6}$

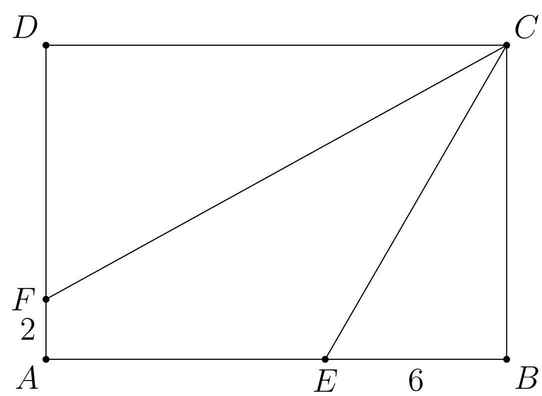
B $\frac{91}{216}$

C $\frac{1}{2}$

D $\frac{8}{15}$

E $\frac{7}{12}$

17. In rectangle $ABCD$, angle C is trisected by $\overline{CF}$ and $\overline{CE}$, where E is on $\overline{AB}$, F is on $\overline{AD}$, $BE = 6$, and $AF = 2$. Which of the following is closest to the area of the rectangle $ABCD$?



A 110

B 120

C 130

D 140

E 150

18. A circle of radius 2 has center at $(2, 0)$. A circle of radius 1 has center at $(5, 0)$. A line is tangent to the two circles at points in the first quadrant. Which of the following is closest to the y-intercept of the line?

A $\frac{\sqrt{2}}{4}$

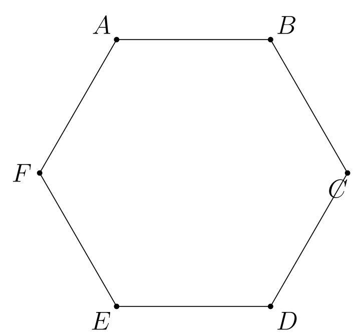
B $\frac{\sqrt{8}}{3}$

C $1 + \sqrt{3}$

D $2\sqrt{2}$

E 3

19. The midpoints of the sides of a regular hexagon $ABCDEF$ are joined to form a smaller hexagon. What fraction of the area of $ABCDEF$ is enclosed by the smaller hexagon?



A $\frac{1}{2}$

B $\frac{\sqrt{3}}{3}$

C $\frac{2}{3}$

D $\frac{3}{4}$

E $\frac{\sqrt{3}}{2}$

20. In the xy-plane, what is the length of the shortest path from $(0,0)$ to $(12,16)$ that does not go inside the circle $(x-6)^2+(y-8)^2=25$?

A $10\sqrt{3}$

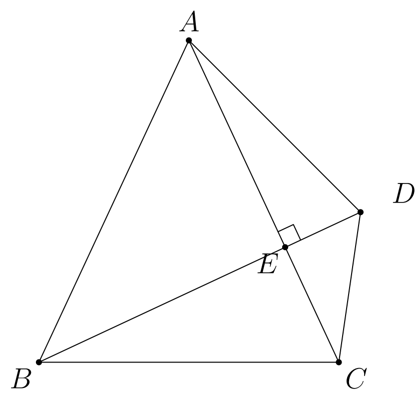
B $10\sqrt{5}$

C $10\sqrt{3}+\frac{5\pi}{3}$

D $\frac{40\sqrt{3}}{3}$

E $10+5\pi$

21. Triangles ABC and ABD are isosceles with $AB=AC=BD$, and $\overline{BD}$ intersects $\overline{AC}$ at E. If $\overline{BD}\perp\overline{AC}$, then $\angle C+\angle D$ is



A 115°

B 120°

C 130°

D 135°

E not uniquely determined

22. Four distinct points, $A, B, C,$ and $D,$ are to be selected from 1996 points evenly spaced around a circle. All quadruples are equally likely to be chosen. What is the probability that the chord $\overline{AB}$ intersects the chord $\overline{CD}$?

- A $\frac{1}{4}$
- B $\frac{1}{3}$
- C $\frac{1}{2}$
- D $\frac{2}{3}$
- E $\frac{3}{4}$

23. The sum of the lengths of the twelve edges of a rectangular box is $140,$ and the distance from one corner of the box to the farthest corner is $21.$ The total surface area of the box is

- A 776
- B 784
- C 798
- D 800
- E 812

24. The sequence

$$1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2,$$
$$2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, \ldots$$

consists of 1's separated by blocks of 2's with n 2's in the nth block. The sum of the first 1234 terms of this sequence is

A 1996

B 2419

C 2429

D 2439

E 2449

25. Given that $x^2 + y^2 = 14x + 6y + 6$, what is the largest possible value that $3x + 4y$ can have?

A 72

B 73

C 74

D 75

E 76

26. An urn contains marbles of four colors: red, white, blue, and green. When four marbles are drawn without replacement, the following events are equally likely:

(a) the selection of four red marbles;

(b) the selection of one white and three red marbles;

(c) the selection of one white, one blue, and two red marbles; and

(d) the selection of one marble of each color.

What is the smallest number of marbles satisfying the given condition?

A 19

B 21

C 46

D 69

E more than 69

27. Consider two solid spherical balls, one centered at $(0, 0, \frac{21}{2})$ with radius 6, and the other centered at $(0, 0, 1)$ with radius $\frac{9}{2}$. How many points (x, y, z) with only integer coordinates (lattice points) are there in the intersection of the balls?

A 7

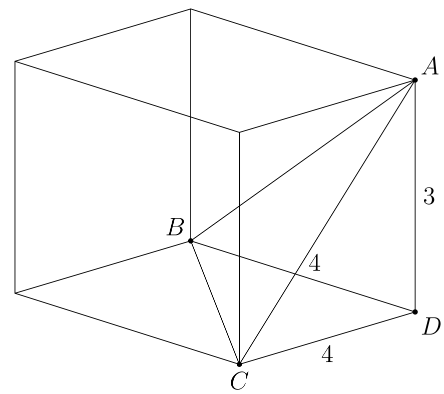
B 9

C 11

D 13

E 15

28. On a $4 \times 4 \times 3$ rectangular parallelepiped, vertices $A, B,$ and C are adjacent to vertex D. The perpendicular distance from D to the plane containing $A, B,$ and C is closest to



A 1.6

B 1.9

C 2.1

D 2.7

E 2.9

29. If n is a positive integer such that $2n$ has 28 positive divisors and $3n$ has 30 positive divisors, then how many positive divisors does $6n$ have?

A 32

B 34

C 35

D 36

E 38

30. A hexagon inscribed in a circle has three consecutive sides each of length 3 and three consecutive sides each of length 5. The chord of the circle that divides the hexagon into two trapezoids, one with three sides each of length 3 and the other with three sides each of length 5, has length equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

A 309

B 349

C 369

D 389

E 409

Solutions: https://live.poshenloh.com/past-contests/amc12/1996/solutions