1991 AMC 12

Time limit: 75 minutes

Typeset by: LIVE by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc12/1991

1. If for any three distinct numbers $a, b,$ and c we define

$$\boxed{a, b, c} = \frac{c+a}{c-b},$$

then $\boxed{1, -2, -3} =$

A -2

B $-\frac{2}{5}$

C $-\frac{1}{4}$

D $\frac{2}{5}$

E 2

2. $|3-\pi| =$

- A $\frac{1}{7}$
- B 0.14
- C $3-\pi$
- D $3+\pi$
- E $\pi-3$

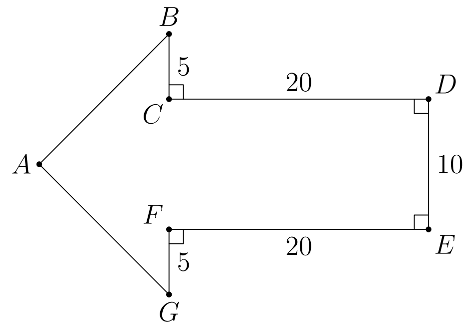
3. $\left(4^{-1}-3^{-1}\right)^{-1} =$

- A -12
- B -1
- C $\frac{1}{12}$
- D 1
- E 12

4. Which of the following triangles cannot exist?

- A An acute isosceles triangle
- B An isosceles right triangle
- C An obtuse right triangle
- D A scalene right triangle
- E A scalene obtuse triangle

5. In the arrow-shaped polygon shown, the angles at vertices $A, C, D, E,$ and F are right angles, $BC = FG = 5, CD = FE = 20, DE = 10,$ and $AB = AG$. The area of the polygon is closest to



A 288

B 291

C 294

D 297

E 300

6. If $x \geq 0$, then $\sqrt{x\sqrt{x\sqrt{x}}} =$

A $x\sqrt{x}$

B $x\sqrt[4]{x}$

C $\sqrt[8]{x}$

D $\sqrt[8]{x^3}$

E $\sqrt[8]{x^7}$

7. If $x = \frac{a}{b}, a \neq b,$ and $b \neq 0,$ then $\dfrac{a+b}{a-b} =$

A $\frac{x}{x+1}$

B $\frac{x+1}{x-1}$

C 1

D $x - \frac{1}{x}$

E $x + \frac{1}{x}$

8. Liquid X does not mix with water. Unless obstructed, it spreads out on the surface of water to form a circular film 0.1 cm thick. A rectangular box measuring 6 cm by 3 cm by 12 cm is filled with liquid X. Its contents are poured onto a large body of water. What will be the radius, in centimeters, of the resulting circular film?

A $\frac{\sqrt{216}}{\pi}$

B $\sqrt{\frac{216}{\pi}}$

C $\sqrt{\frac{2160}{\pi}}$

D $\frac{216}{\pi}$

E $\frac{2160}{\pi}$

9. From time $t = 0$ to time $t = 1$ a population increased by $i\%$, and from time $t = 1$ to time $t = 2$ the population increased by $j\%$. Therefore, from time $t = 0$ to time $t = 2$ the population increased by

A $(i + j)\%$

B $ij\%$

C $(i + ij)\%$

D $\left(i + j + \frac{ij}{100}\right)\%$

E $\left(i + j + \frac{i+j}{100}\right)\%$

10. Point P is 9 units from the center of a circle of radius 15. How many different chords of the circle contain P and have integer lengths?

A 11

B 12

C 13

D 14

E 29

11. Jack and Jill run 10 kilometers. They start at the same point, run 5 kilometers up a hill, and return to the starting point by the same route. Jack has a 10-minute head start and runs at the rate of 15 km/hr uphill and 20 km/hr downhill. Jill runs 16 km/hr uphill and 22 km/hr downhill. How far from the top of the hill are they when they pass going in opposite directions?

A $\frac{5}{4}$ km

B $\frac{35}{27}$ km

C $\frac{27}{20}$ km

D $\frac{7}{3}$ km

E $\frac{28}{9}$ km

12. The measures (in degrees) of the interior angles of a convex hexagon form an arithmetic sequence of positive integers. Let m° be the measure of the largest interior angle of the hexagon. The largest possible value of m° is

A 165°

B 167°

C 170°

D 175°

E 179°

13. Horses X, Y, and Z are entered in a three-horse race in which ties are not possible. If the odds against X winning are 3-to-1 and the odds against Y winning are 2-to-3, what are the odds against Z winning? (By "odds against H winning are p-to-q" we mean that the probability of H winning the race is $\frac{q}{p+q}$.)

- A 3-to-20
- B 5-to-6
- C 8-to-5
- D 17-to-3
- E 20-to-3

14. If x is the cube of a positive integer and d is the number of positive integers that are divisors of x, then d could be

- A 200
- B 201
- C 202
- D 203
- E 204

15. A circular table has exactly 60 chairs around it. There are N people seated at this table in such a way that the next person to be seated must sit next to someone. The smallest possible value of N is

A 15

B 20

C 30

D 40

E 58

16. One hundred students at Century High School participated in the AHSME last year, and their mean score was $100.$ The number of non-seniors taking the AHSME was 50% more than the number of seniors, and the mean score of the seniors was 50% higher than that of the non-seniors. What was the mean score of the seniors?

A 100

B 112.5

C 120

D 125

E 150

17. A positive integer N is a palindrome if the integer obtained by reversing the sequence of digits of N is equal to N. The year 1991 is the only year in the current century with the following two properties:

(a) It is a palindrome.

(b) It factors as a product of a 2-digit prime palindrome and a 3-digit prime palindrome.

How many years in the millenium between 1000 and 2000 (including the year 1991) have properties (a) and (b)?

A 1

B 2

C 3

D 4

E 5

18. If S is the set of points z in the complex plane such that $(3+4i)z$ is a real number, then S is a

A right triangle

B circle

C hyperbola

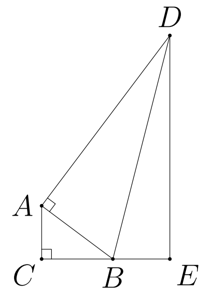
D line

E parabola

19. Triangle ABC has a right angle at C, $AC = 3$, and $BC = 4$. Triangle ABD has a right angle at A and $AD = 12$. Points C and D are on opposite sides of AB. The line through D parallel to AC meets CB extended at E. If

$$\frac{DE}{DB} = \frac{m}{n},$$

where m and n are relatively prime positive integers, then $m + n =$



A 25

B 128

C 153

D 243

E 256

20. The sum of all real x such that

$$\begin{aligned}(2^x-4)^3+(4^x-2)^3 \\ =(4^x+2^x-6)^3\end{aligned}$$

is

A $\frac{3}{2}$

B 2

C $\frac{5}{2}$

D 3

E $\frac{7}{2}$

21. If

$$f\left(\frac{x}{x-1}\right)=\frac{1}{x}$$
$$\text{for all } x \neq 0,1$$

and $0<\theta<\frac{\pi}{2}$, then $f(\sec^2\theta)=$

A $\sin^2\theta$

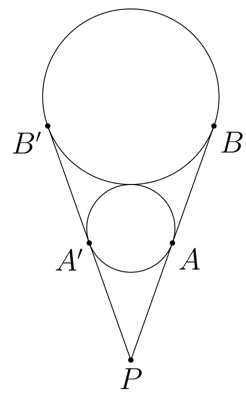
B $\cos^2\theta$

C $\tan^2\theta$

D $\cot^2\theta$

E $\csc^2\theta$

22. Two circles are externally tangent. Lines PAB and $PA'B'$ are common tangents with A and A' on the smaller circle and B and B' on the larger circle. If $PA = AB = 4$, then the area of the smaller circle is



A 1.44π

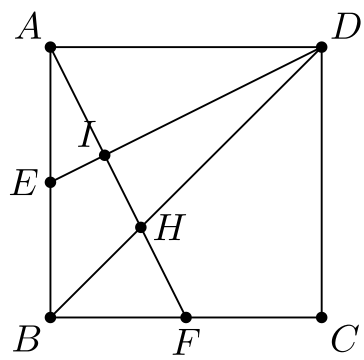
B 2π

C 2.56π

D $\sqrt{8}\pi$

E 4π

23. If $ABCD$ is a 2×2 square, E is the midpoint of AB, F is the midpoint of BC, AF and DE intersect at I, and BD and AF intersect at H, then the area of quadrilateral $BEIH$ is



- A $\frac{1}{3}$
- B $\frac{2}{5}$
- C $\frac{7}{15}$
- D $\frac{8}{15}$
- E $\frac{3}{5}$

24. The graph, G, of $y = \log_{10} x$ is rotated 90° counter-clockwise about the origin to obtain a new graph G'. Which of the following is an equation for G'?

- A $y = \log_{10}\left(\frac{x+90}{9}\right)$
- B $y = \log_x 10$
- C $y = \frac{1}{x+1}$
- D $y = 10^{-x}$
- E $y = 10^x$

25. If $T_n = 1 + 2 + 3 + \cdots + n$ and

$$P_n = \frac{T_2}{T_2 - 1} \cdot \frac{T_3}{T_3 - 1} \cdot \frac{T_4}{T_4 - 1} \cdots \frac{T_n}{T_n - 1}$$

for $n = 2, 3, 4, \ldots$, then P_{1991} is closest to which of the following numbers?

A 2.0

B 2.3

C 2.6

D 2.9

E 3.2

26. An n-digit positive integer is *cute* if its n digits are an arrangement of the set $\{1, 2, \ldots, n\}$ and its first k digits form an integer that is divisible by k, for $k = 1, 2, \ldots, n$. For example, 321 is a cute 3-digit integer because 1 divides 3, 2 divides 32, and 3 divides 321. How many cute 6-digit integers are there?

A 0

B 1

C 2

D 3

E 4

27. If

$$x+\sqrt{x^2-1}+\frac{1}{x-\sqrt{x^2-1}} = 20,$$

then

$$x^2+\sqrt{x^4-1}+\frac{1}{x^2+\sqrt{x^4-1}} =$$

- A 5.05
- B 20
- C 51.005
- D 61.25
- E 400

28. Initially an urn contains 100 black marbles and 100 white marbles. Repeatedly, three marbles are removed from the urn and replaced from a pile outside the urn as follows:

Marbles removed	**Replaced with**
3 black	1 black
2 black, 1 white	1 black, 1 white
1 black, 2 white	2 white
3 white	1 black, 1 white

Which of the following sets of marbles could be the contents of the urn after repeated applications of this procedure?

A 2 black marbles

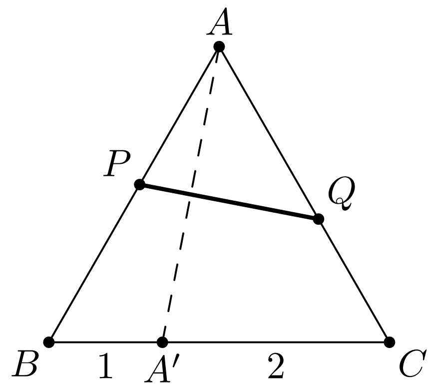
B 2 white marbles

C 1 black marble

D 1 black and 1 white marble

E 1 white marble

29. Equilateral triangle ABC has been creased and folded so that vertex A now rests at A' on BC as shown. If $BA' = 1$ and $A'C = 2$, then the length of crease PQ is



A $\frac{8}{5}$

B $\frac{7}{20}\sqrt{21}$

C $\frac{1+\sqrt{5}}{2}$

D $\frac{13}{8}$

E $\sqrt{3}$

30. For any set S, let $|S|$ denote the number of elements in S, and let $n(S)$ be the number of subsets of S, including the empty set and the set S itself. If A, B, and C are sets for which

$$\begin{gathered} n(A)+n(B)+n(C) \\ =n(A \cup B \cup C), \\ |A|=|B|=100, \end{gathered}$$

then what is the minimum possible value of $|A \cap B \cap C|$?

A 96

B 97

C 98

D 99

E 100

Solutions: https://live.poshenloh.com/past-contests/amc12/1991/solutions