1985 AMC 12

Time limit: 75 minutes

Typeset by: LIVE by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc12/1985

1. If $2x + 1 = 8$, then $4x + 1 =$

A 15

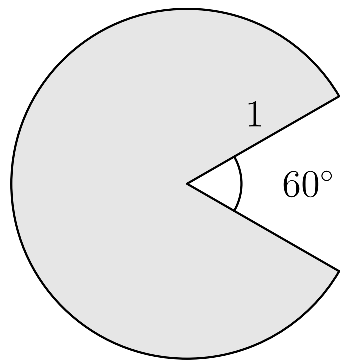
B 16

C 17

D 18

E 19

2. In an arcade game, the “monster” is the shaded sector of a circle of radius 1 cm, as shown in the figure. The missing piece (the mouth) has central angle 60°. What is the perimeter of the monster in cm?



A $\pi + 2$

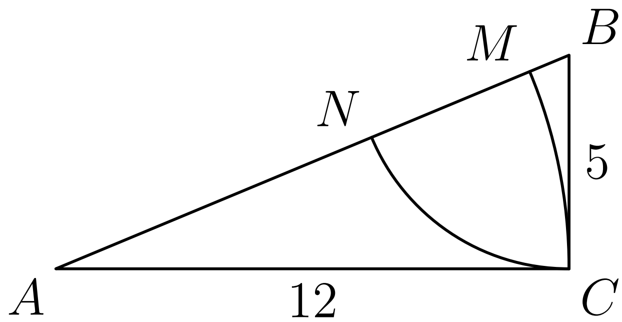
B 2π

C $\frac{5}{3}\pi$

D $\frac{5}{6}\pi + 2$

E $\frac{5}{3}\pi + 2$

3. In right $\triangle ABC$ with legs 5 and 12, arcs of circles are drawn, one with center A and radius 12, the other with center B and radius 5. They intersect the hypotenuse in M and N. Then MN has length



- A 2
- B $\frac{13}{5}$
- C 3
- D 4
- E $\frac{24}{5}$

4. A large bag of coins contains pennies, dimes and quarters. There are twice as many dimes as pennies and three times as many quarters as dimes. An amount of money which could be in the bag is

- A $\$306$
- B $\$333$
- C $\$342$
- D $\$348$
- E $\$360$

5. Which terms must be removed from the sum

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$$

if the sum of the remaining terms is to equal 1?

A $\frac{1}{4}$ and $\frac{1}{8}$

B $\frac{1}{4}$ and $\frac{1}{12}$

C $\frac{1}{8}$ and $\frac{1}{12}$

D $\frac{1}{6}$ and $\frac{1}{10}$

E $\frac{1}{8}$ and $\frac{1}{10}$

6. One student in a class of boys and girls is to be chosen to represent the class. Each student is equally likely to be chosen and the probability that a boy is chosen is $\frac{2}{3}$ of the probability that a girl is chosen. The ratio of the number of boys to the total number of boys and girls is

A $\frac{1}{3}$

B $\frac{2}{5}$

C $\frac{1}{2}$

D $\frac{3}{5}$

E $\frac{2}{3}$

7. In some computer languages (such as APL), when there are no parentheses in an algebraic expression, the operations are grouped from right to left. Thus, $a \times b - c$ in such languages means the same as $a(b - c)$ in ordinary algebraic notation. If $a \div b - c + d$ is evaluated in such a language, the result in ordinary algebraic notation would be

A $\frac{a}{b} - c + d$

B $\frac{a}{b} - c - d$

C $\frac{d+c-b}{a}$

D $\frac{a}{b-c+d}$

E $\frac{a}{b-c-d}$

8. Let a, a', b, b' be real numbers with a and a' nonzero. The solution to $ax + b = 0$ is less than the solution to $a'x + b' = 0$ if and only if

A $a'b < ab'$

B $ab' < a'b$

C $ab < a'b'$

D $\frac{b}{a} < \frac{b'}{a'}$

E $\frac{b'}{a'} < \frac{b}{a}$

9. The odd positive integers, $1, 3, 5, 7, \ldots$, are arranged in five columns continuing with the pattern shown. Counting from the left, the column in which 1985 appears is the

	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	33	35	37	39
47	45	43	41	
	49	51	53	55

A first

B second

C third

D fourth

E fifth

10. An arbitrary circle can intersect the graph of $y = \sin x$ in

A at most 2 points

B at most 4 points

C at most 6 points

D at most 8 points

E more than 16 points

11. How many distinguishable rearrangements of the letters in CONTEST have both the vowels first? (For instance, OETCNST is one such arrangement, but OTETSNC is not.)

A 60

B 120

C 240

D 720

E 2520

12. Let p, q and r be distinct prime numbers, where 1 is not considered a prime. Which of the following is the smallest positive perfect cube having $n = pq^2r^4$ as a divisor?

A $p^8q^8r^8$

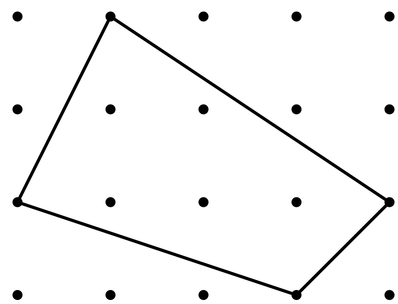
B $(pq^2r^2)^3$

C $(p^2q^2r^2)^3$

D $(pqr^2)^3$

E $4p^3q^3r^3$

13. Pegs are put in a board 1 unit apart both horizontally and vertically. A rubber band is stretched over 4 pegs as shown in the figure, forming a quadrilateral. Its area in square units is



A 4

B 4.5

C 5

D 5.5

E 6

14. Exactly three of the interior angles of a convex polygon are obtuse. What is the maximum number of sides of such a polygon?

A 4

B 5

C 6

D 7

E 8

15. If a and b are positive numbers such that $a^b = b^a$ and $b = 9a,$ then the value of a is

A 9

B $\frac{1}{9}$

C $\sqrt[9]{9}$

D $\sqrt[3]{9}$

E $\sqrt[4]{3}$

16. If $A = 20^\circ$ and $B = 25^\circ$, then the value of $(1 + \tan A)(1 + \tan B)$ is

A $\sqrt{3}$

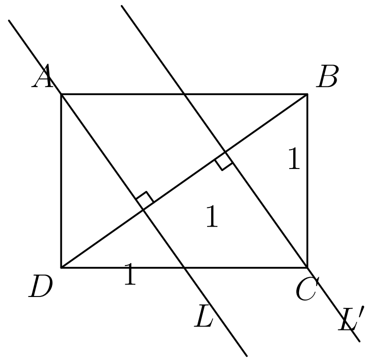
B 2

C $1 + \sqrt{2}$

D $2(\tan A + \tan B)$

E none of these

17. Diagonal DB of rectangle $ABCD$ is divided into three segments of length 1 by parallel lines L and L' that pass through A and C and are perpendicular to DB. The area of $ABCD$, rounded to one decimal place, is



- A 4.1
- B 4.2
- C 4.3
- D 4.4
- E 4.5

18. Six bags of marbles contain $18, 19, 21, 23, 25$ and 34 marbles, respectively. One bag contains chipped marbles only. The other 5 bags contain no chipped marbles. Jane takes three of the bags and George takes two of the others. Only the bag of chipped marbles remains. If Jane gets twice as many marbles as George, how many chipped marbles are there?

A 18

B 19

C 21

D 23

E 25

19. Consider the graphs of $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and x and y are real variables. In how many points do the two graphs intersect?

A exactly 4

B exactly 2

C at least 1, but the number varies for different positive values of A

D 0 for at least one positive value of A

E none of these

20. A wooden cube with edge length n units (where n is an integer > 2) is painted black all over. By slices parallel to its faces, the cube is cut into n^3 smaller cubes each of unit edge length. If the number of smaller cubes with just one face painted black is equal to the number of smaller cubes completely free of paint, what is n?

A 5

B 6

C 7

D 8

E none of these

21. How many integers x satisfy the equation

$$(x^2 - x - 1)^{x+2} = 1?$$

A 2

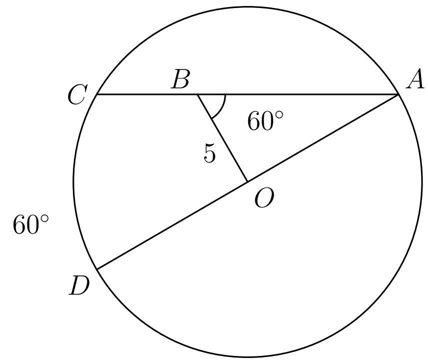
B 3

C 4

D 5

E none of these

22. In a circle with center O, AD is a diameter, ABC is a chord, $BO = 5$ and $\angle ABO = \overset{\frown}{CD} = 60^\circ$. Then the length of BC is



A 3

B $3 + \sqrt{3}$

C $5 - \frac{\sqrt{3}}{2}$

D 5

E none of the above

23. If

$$x = \frac{-1 + i\sqrt{3}}{2},$$
$$y = \frac{-1 - i\sqrt{3}}{2},$$

where $i^2 = -1$, then which of the following is not correct?

A $x^5 + y^5 = -1$

B $x^7 + y^7 = -1$

C $x^9 + y^9 = -1$

D $x^{11} + y^{11} = -1$

E $x^{13} + y^{13} = -1$

24. A non-zero digit is chosen in such a way that the probability of choosing digit d is $\log_{10}(d+1) - \log_{10} d$. The probability that the digit 2 is chosen is exactly $\frac{1}{2}$ the probability that the digit chosen is in the set

A $\{2, 3\}$

B $\{3, 4\}$

C $\{4, 5, 6, 7, 8\}$

D $\{5, 6, 7, 8, 9\}$

E $\{4, 5, 6, 7, 8, 9\}$

25. The volume of a certain rectangular solid is $8\ \mathrm{cm}^3$, its total surface area is $32\ \mathrm{cm}^2$, and its three dimensions are in geometric progression. The sum of the lengths in cm of all the edges of this solid is

- A 28
- B 32
- C 36
- D 40
- E 44

26. Find the least positive integer n for which $\frac{n-13}{5n+6}$ is a non-zero reducible fraction.

- A 45
- B 68
- C 155
- D 226
- E none of these

27. Consider a sequence $x_1, x_2, x_3, \ldots,$ defined by

$$x_1 = \sqrt[3]{3},$$
$$x_2 = (\sqrt[3]{3})^{\sqrt[3]{3}}.$$

and in general

$$x_n = (x_{n-1})^{\sqrt[3]{3}} \qquad \text{for } n > 1.$$

What is the smallest value of n for which x_n is an integer?

A 2

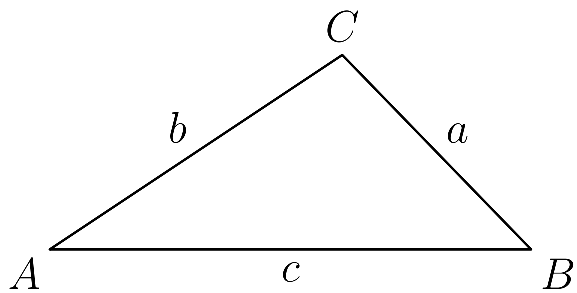
B 3

C 4

D 9

E 27

28. In $\triangle ABC$, we have $\angle C = 3\angle A$, $a = 27$ and $c = 48$. What is b?



A 33

B 35

C 37

D 39

E not uniquely determined

29. In their base 10 representations, the integer a consists of a sequence of 1985 eights and the integer b consists of a sequence of 1985 fives. What is the sum of the digits of the base 10 representation of the integer $9ab$?

A 15880

B 17856

C 17865

D 17874

E 19851

30. Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. Then the number of real solutions to $4x^2 - 40\lfloor x \rfloor + 51 = 0$ is

A 0

B 1

C 2

D 3

E 4

Solutions: https://live.poshenloh.com/past-contests/amc12/1985/solutions