1982 AMC 12

Time limit: 75 minutes

Typeset by: LIVE by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc12/1982

1. When the polynomial $x^3 - 2$ is divided by the polynomial $x^2 - 2$, the remainder is

A 2

B -2

C $-2x - 2$

D $2x + 2$

E $2x - 2$

2. If a number eight times as large as x is increased by two, then one fourth of the result equals

A $2x + \frac{1}{2}$

B $x + \frac{1}{2}$

C $2x + 2$

D $2x + 4$

E $2x + 16$

3. Evaluate $(x^x)^{(x^x)}$ at $x = 2$.

A 16

B 64

C 256

D 1024

E $65{,}536$

4. The perimeter of a semicircular region, measured in centimeters, is numerically equal to its area, measured in square centimeters. The radius of the semicircle, measured in centimeters, is

A π

B $\frac{2}{\pi}$

C 1

D $\frac{1}{2}$

E $\frac{4}{\pi} + 2$

5. Two positive numbers x and y are in the ratio $a : b$, where $0 < a < b$. If $x + y = c$, then the smaller of x and y is

A $\frac{ac}{b}$

B $\frac{bc-ac}{b}$

C $\frac{ac}{a+b}$

D $\frac{bc}{a+b}$

E $\frac{ac}{b-a}$

6. The sum of all but one of the interior angles of a convex polygon equals 2570°. The remaining angle is

A 90°

B 105°

C 120°

D 130°

E 144°

7. If the operation $x * y$ is defined by $x * y = (x+1)(y+1) - 1,$ then which one of the following is false?

- A $x * y = y * x$ for all real x and $y.$
- B $x * (y+z)$ equals $(x * y) + (x * z)$ for all real $x, y,$ and $z.$
- C $(x-1) * (x+1)$ equals $(x * x) - 1$ for all real $x.$
- D $x * 0 = x$ for all real $x.$
- E $x * (y * z) = (x * y) * z$ for all real $x, y,$ and $z.$

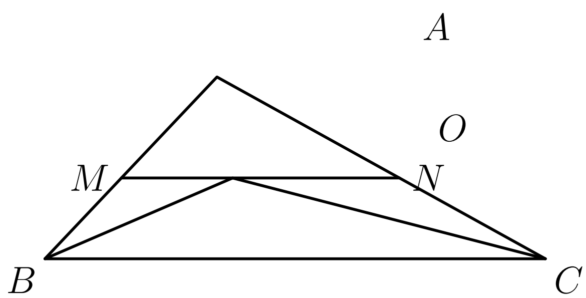
8. By definition $r! = r(r-1)\cdots 1$ and $\binom{j}{k} = \frac{j!}{k!(j-k)!},$ where r, j, k are positive integers and $k < j.$ If $\binom{n}{1}, \binom{n}{2}, \binom{n}{3}$ form an arithmetic progression with $n > 3,$ then n equals

- A 5
- B 7
- C 9
- D 11
- E 12

9. A vertical line divides the triangle with vertices $(0,0),(1,1)$ and $(9,1)$ in the xy-plane into two regions of equal area. The equation of the line is $x =$

- A 2.5
- B 3.0
- C 3.5
- D 4.0
- E 4.5

10. In the adjoining diagram, BO bisects $\angle CBA$, CO bisects $\angle ACB$, and MN is parallel to BC. If $AB = 12$, $BC = 24$, and $AC = 18$, then the perimeter of $\triangle AMN$ is



- A 30
- B 33
- C 36
- D 39
- E 42

11. How many integers with four different digits are there between $1{,}000$ and $9{,}999$ such that the absolute value of the difference between the first digit and the last digit is 2?

A 672

B 784

C 840

D 896

E $1{,}008$

12. Let $f(x) = ax^7 + bx^3 + cx - 5$, where $a, b,$ and c are constants. If $f(-7) = 7$, then $f(7)$ equals

A -17

B -7

C 14

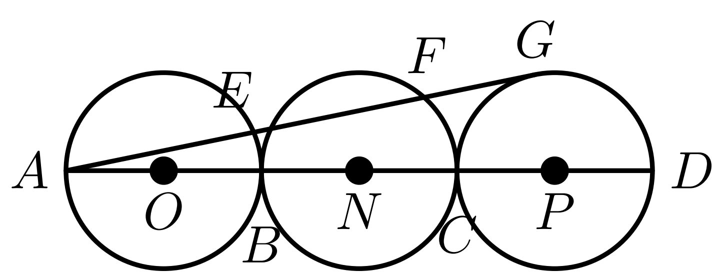
D 21

E not uniquely determined

13. If $a > 1, b > 1$ and $p = \frac{\log_b(\log_b a)}{\log_b a}$, then a^p equals

- A 1
- B b
- C $\log_a b$
- D $\log_b a$
- E $a^{\log_b a}$

14. In the adjoining figure, points B and C lie on line segment AD, and AB, BC, and CD are diameters of circles O, N, and P, respectively. Circles O, N, and P all have radius 15, and the line AG is tangent to circle P at G. If AG intersects circle N at points E and F, then chord EF has length



- A 20
- B $15\sqrt{2}$
- C 24
- D 25
- E none of these

15. Let $\lfloor z \rfloor$ denote the greatest integer not exceeding z. Let x and y satisfy the simultaneous equations

$$y = 2\lfloor x \rfloor + 3,$$
$$y = 3\lfloor x - 2 \rfloor + 5.$$

If x is not an integer, then $x + y$ is

A an integer

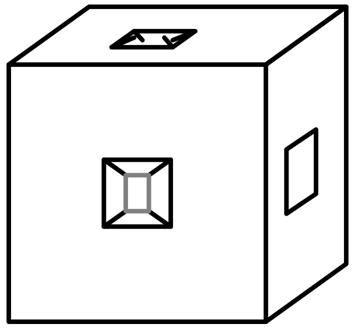
B between 4 and 5

C between -4 and 4

D between 15 and 16

E 16.5

16. In the adjoining figure, a wooden cube has edges of length 3 meters. Square holes of side one meter, centered in each face, are cut through to the opposite face. The edges of the holes are parallel to the edges of the cube. The entire surface area including the inside, in square meters, is



A 54

B 72

C 76

D 84

E 86

17. How many real numbers x satisfy the equation $3^{2x+2} - 3^{x+3} - 3^{x} + 3 = 0$?

A 0

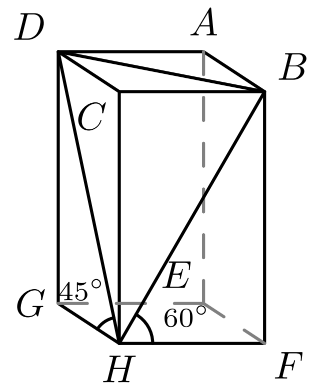
B 1

C 2

D 3

E 4

18. In the adjoining figure of a rectangular solid, $\angle DHG = 45^\circ$ and $\angle FHB = 60^\circ$. Find the cosine of $\angle BHD$.



A $\frac{\sqrt{3}}{6}$

B $\frac{\sqrt{2}}{6}$

C $\frac{\sqrt{6}}{3}$

D $\frac{\sqrt{6}}{4}$

E $\frac{\sqrt{6}-\sqrt{2}}{4}$

19. Let $f(x) = |x-2| + |x-4| - |2x-6|$, for $2 \leq x \leq 8$. The sum of the largest and smallest values of $f(x)$ is

A 1

B 2

C 4

D 6

E none of these

20. The number of pairs of positive integers (x, y) which satisfy the equation $x^2 + y^2 = x^3$ is

A 0

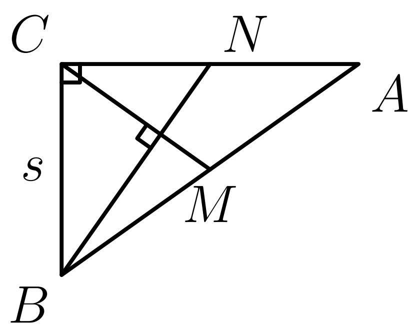
B 1

C 2

D not finite

E none of these

21. In the adjoining figure, the triangle ABC is a right triangle with $\angle BCA = 90^\circ$. Median CM is perpendicular to median BN, and side $BC = s$. The length of BN is



A $s\sqrt{2}$

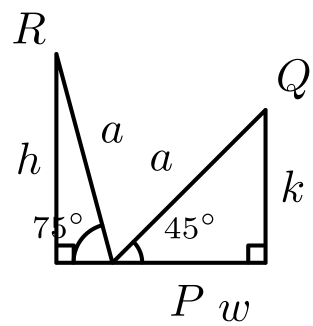
B $\frac{3}{2}s\sqrt{2}$

C $2s\sqrt{2}$

D $\frac{s\sqrt{5}}{2}$

E $\frac{s\sqrt{6}}{2}$

22. In a narrow alley of width w a ladder of length a is placed with its foot at a point P between the walls. Resting against one wall at Q, a distance k above the ground, the ladder makes a 45° angle with the ground. Resting against the other wall at R, a distance h above the ground, the ladder makes a 75° angle with the ground. The width w is equal to



A a

B RQ

C k

D $\frac{h+k}{2}$

E h

23. The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle. The cosine of the smallest angle is

A $\frac{3}{4}$

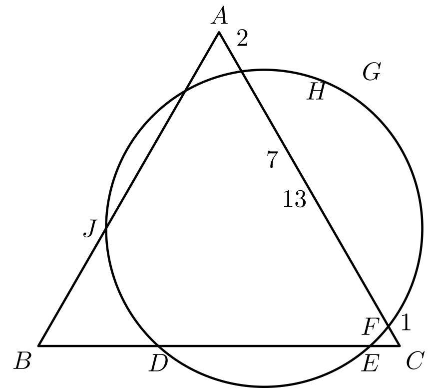
B $\frac{7}{10}$

C $\frac{2}{3}$

D $\frac{9}{14}$

E none of these

24. In the adjoining figure, the circle meets the sides of an equilateral triangle at six points. If $AG = 2, GF = 13, FC = 1$ and $HJ = 7$, then DE equals



A $2\sqrt{22}$

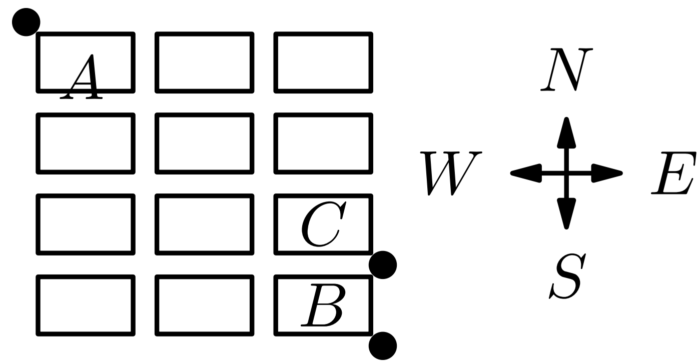
B $7\sqrt{3}$

C 9

D 10

E 13

25. The adjoining figure is a map of part of a city: the small rectangles are blocks and the spaces in between are streets. Each morning a student walks from intersection A to intersection B, always walking along streets shown, always going east or south. For variety, at each intersection where he has a choice, he chooses with probability $\frac{1}{2}$ (independent of all other choices) whether to go east or south. Find the probability that, on any given morning, he walks through intersection C.



A $\frac{11}{32}$

B $\frac{1}{2}$

C $\frac{4}{7}$

D $\frac{21}{32}$

E $\frac{3}{4}$

26. If the base 8 representation of a perfect square is $ab3c$, where $a \neq 0$, then c is

A 0

B 1

C 3

D 4

E not uniquely determined

27. Suppose $z = a + bi$ is a solution of the polynomial equation

$$c_4 z^4 + i c_3 z^3 + c_2 z^2 \\ + i c_1 z + c_0 = 0,$$

where $c_0, c_1, c_2, c_3, c_4, a,$ and b are real constants and $i^2 = -1$. Which one of the following must also be a solution?

A $-a - bi$

B $a - bi$

C $-a + bi$

D $b + ai$

E none of these

28. A set of consecutive positive integers beginning with 1 is written on a blackboard. One number is erased. The average (arithmetic mean) of the remaining numbers is $35\frac{7}{17}$. What number was erased?

A 6

B 7

C 8

D 9

E can not be determined

29. Let $x, y,$ and z be three positive real numbers whose sum is $1.$ If no one of these numbers is more than twice any other, then the minimum possible value of the product xyz is

A $\frac{1}{32}$

B $\frac{1}{36}$

C $\frac{4}{125}$

D $\frac{1}{127}$

E none of these

30. Find the units digit in the decimal expansion of

$$(15+\sqrt{220})^{19}+(15+\sqrt{220})^{82}.$$

A 0

B 2

C 5

D 9

E none of these

Solutions: https://live.poshenloh.com/past-contests/amc12/1982/solutions