1980 AMC 12

Time limit: 75 minutes

Typeset by: LIVE by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc12/1980

1. The largest whole number such that seven times the number is less than 100 is

A 12

B 13

C 14

D 15

E 16

2. The degree of $(x^2+1)^4(x^3+1)^3$ as a polynomial in x is

A 5

B 7

C 12

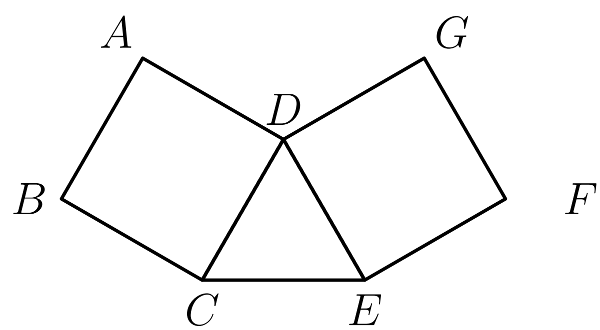
D 17

E 72

3. If the ratio of $2x - y$ to $x + y$ is $\frac{2}{3}$, what is the ratio of x to y?

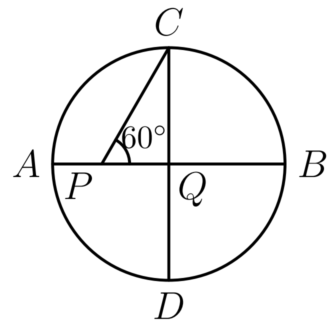
- A $\frac{1}{5}$
- B $\frac{4}{5}$
- C 1
- D $\frac{6}{5}$
- E $\frac{5}{4}$

4. In the adjoining figure, CDE is an equilateral triangle and $ABCD$ and $DEFG$ are squares. The measure of $\angle GDA$ is



- A 90°
- B 105°
- C 120°
- D 135°
- E 150°

5. If AB and CD are perpendicular diameters of circle Q, and $\angle QPC = 60^\circ$, then the length of PQ divided by the length of AQ is



A $\frac{\sqrt{3}}{2}$

B $\frac{\sqrt{3}}{3}$

C $\frac{\sqrt{2}}{2}$

D $\frac{1}{2}$

E $\frac{2}{3}$

6. A positive number x satisfies the inequality $\sqrt{x} < 2x$ if and only if

A $x > \frac{1}{4}$

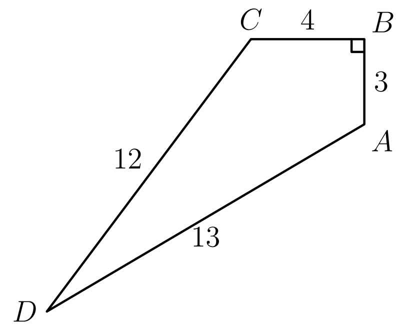
B $x > 2$

C $x > 4$

D $x < \frac{1}{4}$

E $x < 4$

7. Sides $AB, BC, CD,$ and DA of convex quadrilateral $ABCD$ have lengths $3, 4, 12,$ and $13,$ respectively; and $\angle CBA$ is a right angle. The area of the quadrilateral is



A 32

B 36

C 39

D 42

E 48

8. How many pairs (a, b) of nonzero real numbers satisfy the equation

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{a+b}?$$

A none

B 1

C 2

D one pair for each $b \neq 0$

E two pairs for each $b \neq 0$

9. A man walks x miles due west, turns 150° to his left and walks 3 miles in the new direction. If he finishes at a point $\sqrt{3}$ miles from his starting point, then x is

A $\sqrt{3}$

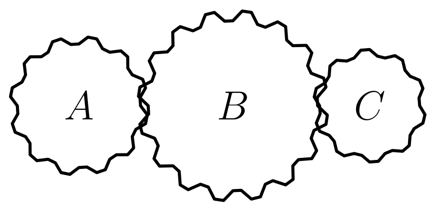
B $2\sqrt{3}$

C $\frac{3}{2}$

D 3

E not uniquely determined by the given information

10. The number of teeth in three meshed circular gears A, B, C are $x, y, z,$ respectively. (The teeth on all gears are the same size and regularly spaced as in the figure.) The angular speeds, in revolutions per minute, of A, B, C are in the proportion



A $x : y : z$

B $z : y : x$

C $y : z : x$

D $yz : xz : xy$

E $xz : yx : zy$

11. If the sum of the first 10 terms and the sum of the first 100 terms of a given arithmetic progression are 100 and 10, respectively, then the sum of the first 110 terms is

A 90

B -90

C 110

D -110

E -100

12. The equations of L_1 and L_2 are $y = mx$ and $y = nx$, respectively. Suppose L_1 makes twice as large an angle with the horizontal (measured counterclockwise from the positive x-axis) as does L_2, and that L_1 has 4 times the slope of L_2. If L_1 is not horizontal, then mn is

A $\frac{\sqrt{2}}{2}$

B $-\frac{\sqrt{2}}{2}$

C 2

D -2

E not uniquely determined by the given information

13. A bug (of negligible size) starts at the origin on the coordinate plane. First it moves 1 unit right to $(1, 0)$. Then it makes a 90° turn counterclockwise and travels $\frac{1}{2}$ a unit to $(1, \frac{1}{2})$. If it continues in this fashion, each time making a 90° turn counterclockwise and traveling half as far as in the previous move, to which of the following points will it come closest?

A $(\frac{2}{3}, \frac{2}{3})$

B $(\frac{4}{5}, \frac{2}{5})$

C $(\frac{2}{3}, \frac{4}{5})$

D $(\frac{2}{3}, \frac{1}{3})$

E $(\frac{2}{5}, \frac{4}{5})$

14. If the function f defined by

$$f(x) = \frac{cx}{2x+3},$$
$$x \neq -\frac{3}{2},$$
$$c \text{ a constant},$$

satisfies $f(f(x)) = x$ for all real numbers x except $-\frac{3}{2}$, then c is

A -3

B $-\frac{3}{2}$

C $\frac{3}{2}$

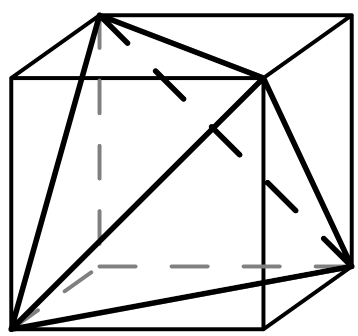
D 3

E not uniquely determined by the given information

15. A store prices an item in dollars and cents so that when 4% sales tax is added no rounding is necessary because the result is exactly n dollars, where n is a positive integer. The smallest value of n is

- A 1
- B 13
- C 25
- D 26
- E 100

16. Four of the eight vertices of a cube are vertices of a regular tetrahedron. Find the ratio of the surface area of the cube to the surface area of the tetrahedron.



- A $\sqrt{2}$
- B $\sqrt{3}$
- C $\sqrt{\frac{3}{2}}$
- D $\frac{2}{\sqrt{3}}$
- E 2

17. Given that $i^2 = -1$, for how many integers n is $(n+i)^4$ an integer?

A none

B 1

C 2

D 3

E 4

18. If $b > 1, \sin x > 0, \cos x > 0$ and $\log_b \sin x = a$, then $\log_b \cos x$ equals

A $2\log_b(1 - b^{\frac{a}{2}})$

B $\sqrt{1-a^2}$

C b^{a^2}

D $\frac{1}{2}\log_b(1 - b^{2a})$

E none of these

19. Let C_1, C_2, and C_3 be three parallel chords of a circle on the same side of the center. The distance between C_1 and C_2 is the same as the distance between C_2 and C_3. The lengths of the chords are $20, 16$, and 8. The radius of the circle is

A 12

B $4\sqrt{7}$

C $\frac{5\sqrt{65}}{3}$

D $\frac{5\sqrt{22}}{2}$

E not uniquely determined by the given information

20. A box contains 2 pennies, 4 nickels and 6 dimes. Six coins are drawn without replacement, with each coin having an equal probability of being chosen. What is the probability that the value of the coins drawn is at least 50 cents?

A $\frac{37}{924}$

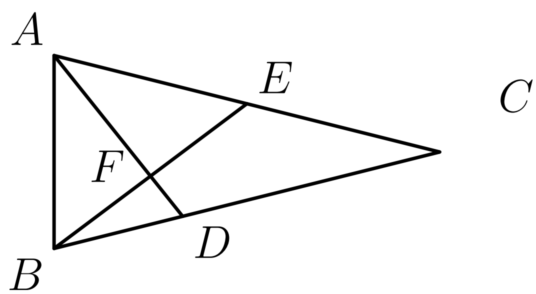
B $\frac{91}{924}$

C $\frac{127}{924}$

D $\frac{132}{924}$

E none of these

21. In triangle $ABC, \angle CBA = 72^\circ, E$ is the midpoint of side AC, and D is a point on side BC such that $2BD = DC$; AD and BE intersect at F. The ratio of the area of $\triangle BDF$ to the area of quadrilateral $FDCE$ is



A $\frac{1}{5}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{2}{5}$

E none of these

22. For each real number x, let $f(x)$ be the minimum of the numbers $4x + 1, x + 2,$ and $-2x + 4$. Then the maximum value of $f(x)$ is

A $\frac{1}{3}$

B $\frac{1}{2}$

C $\frac{2}{3}$

D $\frac{5}{2}$

E $\frac{8}{3}$

23. Line segments drawn from the vertex opposite the hypotenuse of a right triangle to the points trisecting the hypotenuse have lengths $\sin x$ and $\cos x$, where x is a real number such that $0 < x < \frac{\pi}{2}$. The length of the hypotenuse is

A $\frac{4}{3}$

B $\frac{3}{2}$

C $\frac{3\sqrt{5}}{5}$

D $\frac{2\sqrt{5}}{3}$

E not uniquely determined by the given information

24. For some real number r, the polynomial $8x^3 - 4x^2 - 42x + 45$ is divisible by $(x - r)^2$. Which of the following numbers is closest to r?

A 1.22

B 1.32

C 1.42

D 1.52

E 1.62

25. In the nondecreasing sequence of odd integers

$$(a_1, a_2, a_3, \ldots) = (1, 3, 3, 3, \\ 5, 5, 5, 5, 5, \ldots)$$

each positive odd integer k appears k times. It is a fact that there are integers b, c, and d such that, for all positive integers n,

$$a_n = b\left\lfloor \sqrt{n+c} \right\rfloor + d,$$

where $\lfloor x \rfloor$ denotes the largest integer not exceeding x. The sum $b + c + d$ equals

A 0

B 1

C 2

D 3

E 4

26. Four balls of radius 1 are mutually tangent, three resting on the floor and the fourth resting on the others. A tetrahedron, each of whose edges has length s, is circumscribed around the balls. Then s equals

A $4\sqrt{2}$

B $4\sqrt{3}$

C $2\sqrt{6}$

D $1 + 2\sqrt{6}$

E $2 + 2\sqrt{6}$

27. The sum

$$\sqrt[3]{5+2\sqrt{13}}+\sqrt[3]{5-2\sqrt{13}}$$

equals

A $\frac{3}{2}$

B $\frac{\sqrt[3]{65}}{4}$

C $\frac{1+\sqrt[6]{13}}{2}$

D $\sqrt[3]{2}$

E none of these

28. The polynomial $x^{2n}+1+(x+1)^{2n}$ is not divisible by x^2+x+1 if n equals

A 17

B 20

C 21

D 64

E 65

29. How many ordered triples (x, y, z) of integers satisfy the system of equations below?

$$\begin{aligned} x^2 - 3xy + 2y^2 - z^2 &= 31, \\ -x^2 + 6yz + 2z^2 &= 44, \\ x^2 + xy + 8z^2 &= 100. \end{aligned}$$

A 0

B 1

C 2

D a finite number greater than two

E infinitely many

30. A six digit number (base 10) is *squarish* if it satisfies the following conditions:

(i) none of its digits is zero;

(ii) it is a perfect square; and

(iii) the first two digits, the middle two digits and the last two digits of the number are all perfect squares when considered as two digit numbers.

How many squarish numbers are there?

A 0

B 2

C 3

D 8

E 9

Solutions: https://live.poshenloh.com/past-contests/amc12/1980/solutions