1971 AMC 12

Time limit: 75 minutes

Typeset by: LIVE by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc12/1971

1. The number of digits in the number $N = 2^{12} \times 5^{8}$ is:

A 9

B 10

C 11

D 12

E 20

2. If b men take c days to lay f bricks, then the number of days it will take c men working at the same rate to lay b bricks is:

A fb^2

B $\frac{b}{f^2}$

C $\frac{f^2}{b}$

D $\frac{b^2}{f}$

E $\frac{f}{b^2}$

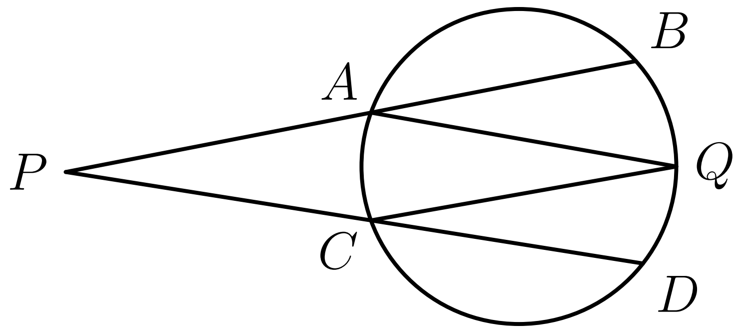
3. If the point $(x, -4)$ lies on the straight line joining the points $(0, 8)$ and $(-4, 0)$ in the xy-plane, then x is equal to:

- A -2
- B 2
- C -8
- D 6
- E -6

4. After simple interest for two months at 5% per annum was credited, a Boy Scout Troop had a total of $\$255.31$ in the Council Treasury. The interest credited was a number of dollars plus the following number of cents:

- A 11
- B 12
- C 13
- D 21
- E 31

5. Points $A, B, Q, D,$ and C lie on the circle shown, and the measures of arcs BQ and QD are 42° and 38°, respectively. The sum of the measures of angles P and Q is:



- A 80°
- B 62°
- C 40°
- D 46°
- E None of these

6. Let $*$ be a symbol denoting the binary operation on the set S of all nonzero real numbers as follows: for any $a, b \in S, a * b = 2ab.$ Which statement is not true?

- A $*$ is commutative over S
- B $*$ is associative over S
- C $\frac{1}{2}$ is an identity element for $*$ in S
- D Every element of S has an inverse for $*$
- E $\frac{1}{2a}$ is an inverse for $*$ of the element a of S

7. $2^{-(2k+1)} - 2^{-(2k-1)} + 2^{-2k}$ is equal to:

A 2^{-2k}

B $2^{-(2k-1)}$

C $-2^{-(2k+1)}$

D 0

E 2

8. The solution set of $6x^2 + 5x < 4$ is the set of all values of x such that:

A $-2 < x < 1$

B $-\frac{4}{3} < x < \frac{1}{2}$

C $-\frac{1}{2} < x < \frac{4}{3}$

D $x < \frac{1}{2}$ or $x > -\frac{4}{3}$

E $x < -\frac{4}{3}$ or $x > \frac{1}{2}$

9. An uncrossed belt is fitted without slack around two circular pulleys with radii of 14 inches and 4 inches. If the distance between the points of contact of the belt with the pulleys is 24 inches, then the distance between the centers of the pulleys in inches is:

A 24

B $2\sqrt{119}$

C 25

D 26

E $4\sqrt{35}$

10. Each of a group of 50 girls is blonde or brunette and is blue-eyed or brown-eyed. If 14 are blue-eyed blondes, 31 are brunettes, and 18 are brown-eyed, then the number of brown-eyed brunettes is:

A 5

B 7

C 9

D 11

E 13

11. The numeral 47 in base a represents the same number as 74 in base b. Assuming both bases are positive integers, the least possible value of $a + b$, written as a Roman numeral, is:

- A XIII
- B XV
- C XXI
- D XXIV
- E XVI

12. For each integer $N > 1$, define positive integers to be congruent if they leave the same nonnegative remainder when divided by N. If $69, 90,$ and 125 are congruent in one such system, then in that same system, 81 is congruent to:

- A 3
- B 4
- C 5
- D 7
- E 8

13. If $(1.0025)^{10}$ is evaluated correct to 5 decimal places, then the digit in the fifth decimal place is:

A 0

B 1

C 2

D 5

E 8

14. The number $2^{48}-1$ is exactly divisible by two numbers between 60 and $70.$ These numbers are:

A $61, 63$

B $61, 65$

C $63, 65$

D $63, 67$

E $67, 69$

15. An aquarium on a level table has rectangular faces and is 10 inches wide and 8 inches high. When it was tilted, the water in it just covered an 8-inch by 10-inch end but only three-fourths of the rectangular bottom. The depth of the water when the bottom was again made level was:

A $2\frac{1}{2}$ inches

B 3 inches

C $3\frac{1}{4}$ inches

D $3\frac{1}{2}$ inches

E 4 inches

16. After finding the average of 35 scores, a student carelessly included the average with the 35 scores and found the average of these 36 numbers. The ratio of the second average to the true average was:

A $1 : 1$

B $35 : 36$

C $36 : 35$

D $2 : 1$

E None of these

17. A circular disk is divided by $2n$ equally spaced radii $(n > 0)$ and one secant line. The maximum number of nonoverlapping areas into which the disk can be divided is:

A $2n+1$

B $2n+2$

C $3n-1$

D $3n$

E $3n+1$

18. The current in a river flows steadily at 3 miles per hour. A motorboat traveling at a constant rate in still water goes downstream 4 miles and then returns to its starting point. The trip takes one hour, excluding turning time. The ratio of the downstream rate to the upstream rate is:

A $4:3$

B $3:2$

C $5:3$

D $2:1$

E $5:2$

19. If the line $y = mx + 1$ intersects the ellipse $x^2 + 4y^2 = 1$ exactly once, then the value of m^2 is:

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{4}{5}$

E $\frac{5}{6}$

20. The sum of the squares of the roots of the equation $x^2 + 2hx = 3$ is $10.$ The absolute value of h is equal to:

A -1

B $\frac{1}{2}$

C $\frac{2}{3}$

D 2

E None of these

21. If

$$\log_2(\log_3(\log_4 x)) = 0,$$
$$\log_3(\log_4(\log_2 y)) = 0,$$
$$\log_4(\log_2(\log_3 z)) = 0,$$

then $x + y + z$ is equal to:

A 50

B 58

C 89

D 111

E 1296

22. If w is one of the imaginary roots of $x^3 = 1$, then $(1 - w + w^2)(1 + w - w^2)$ is equal to:

A 4

B w

C 2

D w^2

E 1

23. Teams A and B are playing a series of games. If either team has an equal chance to win any game, and Team A must win two games while Team B must win three games to win the series, then the odds favoring Team A to win the series are:

- A 11 to 5
- B 5 to 2
- C 8 to 3
- D 3 to 2
- E 13 to 6

24. Pascal's triangle is an array of positive integers, shown below, in which the first row is 1, the second row is two 1's, each row begins and ends with 1, and each other entry is the sum of the two entries above it.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \\ & & & & \text{etc.} & & & & \end{array}$$

The quotient of the number of entries in the first n rows which are not 1's and the number of 1's is:

A $\dfrac{n^2-n}{2n-1}$

B $\dfrac{n^2-n}{4n-2}$

C $\dfrac{n^2-2n}{2n-1}$

D $\dfrac{n^2-3n+2}{4n-2}$

E None of these

25. A teenage boy wrote his own age after his father's. From this new four-place number, he subtracted the absolute value of the difference of their ages to get $4{,}289$. The sum of their ages was:

A 48

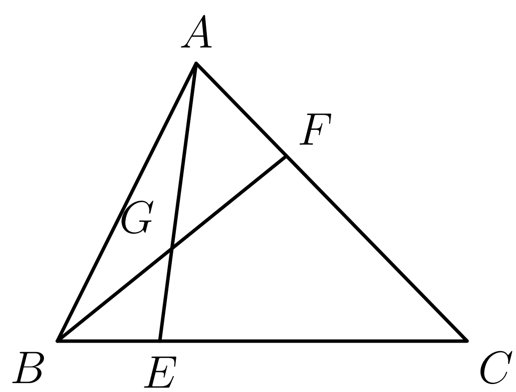
B 52

C 56

D 59

E 64

26. In triangle ABC, point F divides side AC in the ratio $1:2$. Let E be the point where side BC meets AG, where G is the midpoint of BF. Then E divides BC in the ratio:



A $1:4$

B $1:3$

C $2:5$

D $4:11$

E $3:8$

27. A box contains chips, each of which is red, white, or blue. The number of blue chips is at least half the number of white chips and at most one-third the number of red chips. The number which are white or blue is at least 55. The minimum number of red chips is:

A 24

B 33

C 45

D 54

E 57

28. Nine lines parallel to the base of a triangle divide the other sides each into 10 equal segments and the area into 10 distinct parts. If the area of the largest of these parts is 38, then the area of the original triangle is:

A 180

B 190

C 200

D 210

E 240

29. Given the progression

$$10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}},$$
$$10^{\frac{4}{11}}, \ldots, 10^{\frac{n}{11}},$$

the least positive integer n such that the product of the first n terms exceeds $100{,}000$ is:

A 7

B 8

C 9

D 10

E 11

30. Given the linear fractional transformation

$$f_1(x) = \frac{2x-1}{x+1},$$

define $f_{n+1}(x) = f_1(f_n(x))$ for $n = 1, 2, 3, \ldots$. Assuming $f_{35}(x) = f_5(x)$, it follows that $f_{28}(x)$ is equal to:

A x

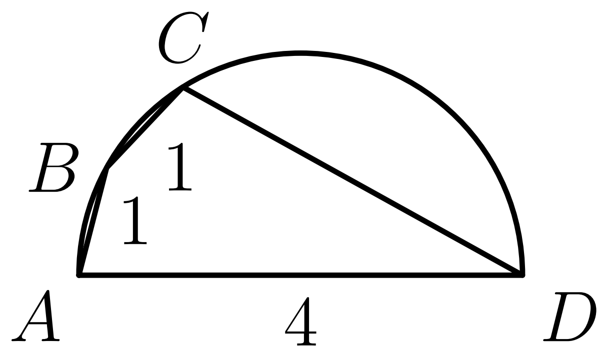
B $\frac{1}{x}$

C $\frac{x-1}{x}$

D $\frac{1}{1-x}$

E None of these

31. Quadrilateral $ABCD$ is inscribed in a circle with side AD, a diameter of length 4. If sides AB and BC each have length 1, then side CD has length:



A $\frac{7}{2}$

B $\frac{5\sqrt{2}}{2}$

C $\sqrt{11}$

D $\sqrt{13}$

E $2\sqrt{3}$

32. If

$$s = (1 + 2^{-\frac{1}{32}})(1 + 2^{-\frac{1}{16}}) \cdot (1 + 2^{-\frac{1}{8}})(1 + 2^{-\frac{1}{4}}) \cdot (1 + 2^{-\frac{1}{2}}),$$

then s is equal to:

A $\frac{1}{2}(1 - 2^{-\frac{1}{32}})^{-1}$

B $(1 - 2^{-\frac{1}{32}})^{-1}$

C $1 - 2^{-\frac{1}{32}}$

D $\frac{1}{2}(1 - 2^{-\frac{1}{32}})$

E $\frac{1}{2}$

33. If P is the product of n quantities in geometric progression, S their sum, and S' the sum of their reciprocals, then P in terms of S, S', and n is:

A $(SS')^{\frac{n}{2}}$

B $(\frac{S}{S'})^{\frac{n}{2}}$

C $(SS')^{n-2}$

D $(\frac{S}{S'})^{n}$

E $(\frac{S'}{S})^{\frac{n-1}{2}}$

34. An ordinary clock in a factory is running slow so that the minute hand passes the hour hand at the usual dial positions $(12 \text{ o'clock},$ etc.) but only every 69 minutes. At time and one-half for overtime, the extra pay to which a $\$4.00$-per-hour worker should be entitled after working a normal 8-hour day by that slow-running clock is:

A $\$2.30$

B $\$2.60$

C $\$2.80$

D $\$3.00$

E $\$3.30$

35. Each circle in an infinite sequence with decreasing radii is tangent externally to the one following it and to both sides of a given right angle. The ratio of the area of the first circle to the sum of the areas of all the other circles in the sequence is:

A $(4+3\sqrt{2}):4$

B $9\sqrt{2}:2$

C $(16+12\sqrt{2}):1$

D $(2+2\sqrt{2}):1$

E $(3+2\sqrt{2}):1$

Solutions: https://live.poshenloh.com/past-contests/amc12/1971/solutions