

1970 AMC 12

Time limit: 75 minutes

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1. The fourth power of $\sqrt{1 + \sqrt{1 + \sqrt{1}}}$ is:

A $\sqrt{2} + \sqrt{3}$

B $\frac{1}{2}(7 + 3\sqrt{5})$

C $1 + 2\sqrt{3}$

D 3

E $3 + 2\sqrt{2}$

2. A square and a circle have equal perimeters. The ratio of the area of the circle to the area of the square is:

A $\frac{4}{\pi}$

B $\frac{\pi}{\sqrt{2}}$

C $\frac{4}{1}$

D $\frac{\sqrt{2}}{\pi}$

E $\frac{\pi}{4}$

3. If $x = 1 + 2^p$ and $y = 1 + 2^{-p}$, then y in terms of x is:

A $\frac{x+1}{x-1}$

B $\frac{x+2}{x-1}$

C $\frac{x}{x-1}$

D $2 - x$

E $\frac{x-1}{x}$

4. Let S be the set of all numbers which are the sum of the squares of three consecutive integers. Then we can say that:

A No member of S is divisible by 2

B No member of S is divisible by 3 but some member is divisible by 11

C No member of S is divisible by 3 or by 5

D No member of S is divisible by 3 or by 7

E None of these

5. If $f(x) = \frac{x^4 + x^2}{x + 1}$, then $f(i)$, where $i = \sqrt{-1}$, is equal to:

A $1 + i$

B 1

C -1

D 0

E $-1 - i$

6. The smallest value of $x^2 + 8x$ for real values of x is:

A -16.25

B -16

C -15

D -8

E None of these

7. Inside square $ABCD$ with side s , quarter-circle arcs with radii s and centers at A and B are drawn. These arcs intersect at a point X inside the square. How far is X from side CD ?

A $\frac{1}{2}s(\sqrt{3} + 4)$

B $\frac{1}{2}s\sqrt{3}$

C $\frac{1}{2}s(1 + \sqrt{3})$

D $\frac{1}{2}s(\sqrt{3} - 1)$

E $\frac{1}{2}s(2 - \sqrt{3})$

8. If $a = \log_8 225$ and $b = \log_2 15$, then:

A $a = \frac{b}{2}$

B $a = \frac{2b}{3}$

C $a = b$

D $b = \frac{a}{2}$

E $a = \frac{3b}{2}$

9. Points P and Q are on line segment AB , and both points are on the same side of the midpoint of AB . Point P divides AB in the ratio $2 : 3$, and Q divides AB in the ratio $3 : 4$. If $PQ = 2$, then the length of segment AB is:

- A 12
- B 28
- C 70
- D 75
- E 105

10. Let $F = 0.4818181\dots$ be an infinite repeating decimal with the digits 8 and 1 repeating. When F is written as a fraction in lowest terms, the denominator exceeds the numerator by:

- A 13
- B 14
- C 29
- D 57
- E 126

11. If two factors of $2x^3 - hx + k$ are $x + 2$ and $x - 1$, the value of $|2h - 3k|$ is:

A 4

B 3

C 2

D 1

E 0

12. A circle with radius r is tangent to sides AB , AD , and CD of rectangle $ABCD$ and passes through the midpoint of diagonal AC . The area of the rectangle, in terms of r , is:

A $4r^2$

B $6r^2$

C $8r^2$

D $12r^2$

E $20r^2$

13. Given the binary operation $*$ defined by $a * b = a^b$ for all positive numbers a and b . Then for all positive a, b, c, n , we have:

A $a * b = b * a$

B $a * (b * c) = (a * b) * c$

C $a * (b^n) = (a * n) * b$

D $(a * b)^n = a * (bn)$

E None of these

14. Consider $x^2 + px + q = 0$, where p and q are positive numbers. If the roots of this equation differ by 1, then p equals:

A $\sqrt{4q + 1}$

B $q - 1$

C $-\sqrt{4q + 1}$

D $q + 1$

E $\sqrt{4q - 1}$

15. Lines in the xy -plane are drawn through the point $(3, 4)$ and the trisection points of the line segment joining the points $(-4, 5)$ and $(5, -1)$. One of these lines has the equation:

A $3x - 2y - 1 = 0$

B $4x - 5y + 8 = 0$

C $5x + 2y - 23 = 0$

D $x + 7y - 31 = 0$

E $x - 4y + 13 = 0$

16. If $F(n)$ is a function such that $F(1) = F(2) = F(3) = 1$, and such that

$$F(n+1) = \frac{F(n)F(n-1) + 1}{F(n-2)}$$

for $n \geq 3$, then $F(6)$ is equal to:

A 2

B 3

C 7

D 11

E 26

17. If $r > 0$, then for all p and q such that $pq \neq 0$ and $pr > qr$, we have:

A $-p > -q$

B $-p > q$

C $1 > -\frac{q}{p}$

D $1 < \frac{q}{p}$

E None of these

18. $\sqrt{3 + 2\sqrt{2}} - \sqrt{3 - 2\sqrt{2}}$ is equal to:

A 2

B $2\sqrt{3}$

C $4\sqrt{2}$

D $\sqrt{6}$

E $2\sqrt{2}$

19. The sum of an infinite geometric series with common ratio r such that $|r| < 1$ is 15, and the sum of the squares of the terms of this series is 45. The first term of the series is:

- ☐ A 12
- ☐ B 10
- ☐ C 5
- ☐ D 3
- ☐ E 2

20. Lines HK and BC lie in a plane. M is the midpoint of line segment BC , and BH and CK are perpendicular to HK . Then we:

- ☐ A always have $MH = MK$
- ☐ B always have $MH > BK$
- ☐ C sometimes have $MH = MK$ but not always
- ☐ D always have $MH > MB$
- ☐ E always have $BH < BC$

21. On an auto trip, the distance read from the instrument panel was 450 miles. With snow tires on for the return trip over the same route, the reading was 440 miles. Find, to the nearest hundredth of an inch, the increase in radius of the wheels if the original radius was 15 inches.

A 0.33

B 0.34

C 0.35

D 0.38

E 0.66

22. If the sum of the first $3n$ positive integers is 150 more than the sum of the first n positive integers, then the sum of the first $4n$ positive integers is:

A 300

B 350

C 400

D 450

E 600

23. The number $10!$ (10 is written in base 10), when written in the base 12 system, ends in exactly k zeros. The value of k is:

- ☐ A 1
- ☐ B 2
- ☐ C 3
- ☐ D 4
- ☐ E 5

24. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 2, then the area of the hexagon is:

- ☐ A 2
- ☐ B 3
- ☐ C 4
- ☐ D 6
- ☐ E 12

25. For every real number x , let $\lfloor x \rfloor$ be the greatest integer which is less than or equal to x . If the postal rate for first class mail is six cents for every ounce or portion thereof, then the cost in cents of first-class postage on a letter weighing W ounces is always:

- A $6W$
- B $6\lfloor W \rfloor$
- C $6(\lfloor W \rfloor - 1)$
- D $6(\lfloor W \rfloor + 1)$
- E $-6\lfloor -W \rfloor$

26. The number of distinct points in the xy -plane common to the graphs of

$$(x + y - 5)(2x - 3y + 5) = 0$$

and

$$(x - y + 1)(3x + 2y - 12) = 0$$

is:

- A 0
- B 1
- C 2
- D 3
- E 4
- F infinite

27. In a triangle, the area is numerically equal to the perimeter. What is the radius of the inscribed circle?

- A 2
- B 3
- C 4
- D 5
- E 6

28. In triangle ABC , the median from vertex A is perpendicular to the median from vertex B . If the lengths of sides AC and BC are 6 and 7, respectively, then the length of side AB is:

- A $\sqrt{17}$
- B 4
- C $4\frac{1}{2}$
- D $2\sqrt{5}$
- E $4\frac{1}{4}$

29. It is now between 10:00 and 11:00 o'clock, and six minutes from now, the minute hand of a watch will be exactly opposite the place where the hour hand was three minutes ago. What is the exact time now?

A $10:05\frac{5}{11}$

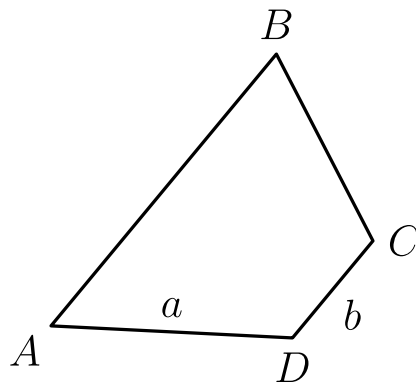
B $10:07\frac{1}{2}$

C 10:10

D 10:15

E $10:17\frac{1}{2}$

30. In the accompanying figure, segments AB and CD are parallel, the measure of angle D is twice that of angle B , and the measures of segments AD and CD are a and b , respectively. Then the measure of AB is equal to:



A $\frac{1}{2}a + 2b$

B $\frac{3}{2}b + \frac{3}{4}a$

C $2a - b$

D $4b - \frac{1}{2}a$

E $a + b$

31. If a number is selected at random from the set of all five-digit numbers in which the sum of the digits is equal to 43, what is the probability that this number will be divisible by 11?

A $\frac{2}{5}$

B $\frac{1}{5}$

C $\frac{1}{6}$

D $\frac{1}{11}$

E $\frac{1}{15}$

32. A and B travel around a circular track at uniform speeds in opposite directions, starting from diametrically opposite points. If they start at the same time, meet first after B has travelled 100 yards, and meet a second time 60 yards before A completes one lap, then the circumference of the track in yards is:

A 400

B 440

C 480

D 560

E 880

33. Find the sum of the digits of all the numerals in the sequence $1, 2, 3, 4, \dots, 10000$.

A 180,001

B 154,756

C 45,001

D 154,755

E 270,001

34. The greatest integer that will divide 13,511, 13,903, and 14,589 and leave the same remainder is:

A 28

B 49

C 98

D an odd multiple of 7 greater than 49

E an even multiple of 7 greater than 98

35. A retiring employee receives an annual pension proportional to the square root of the number of years of his service. Had he served a years more, his pension would have been p dollars greater, whereas, had he served b years more ($b \neq a$), his pension would have been q dollars greater than the original annual pension. Find his annual pension in terms of a , b , p , and q .

A $\frac{p^2 - q^2}{2(a - b)}$

B $\frac{(p - q)^2}{2\sqrt{ab}}$

C $\frac{ap^2 - bq^2}{2(ap - bq)}$

D $\frac{aq^2 - bp^2}{2(bp - aq)}$

E $\sqrt{(a - b)(p - q)}$

Solutions: <https://live.poshenloh.com/past-contests/amc12/1970/solutions>

