1966 AMC 12 Solutions

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1. Given that the ratio of $3x-4$ to $y+15$ is constant, and $y=3$ when $x=2$, then, when $y=12$, x equals:

A $\frac{1}{8}$

B $\frac{8}{7}$

C $\frac{7}{3}$

D $\frac{7}{2}$

E 8

Solution:

The given pair makes the ratio $\frac{3\cdot 2-4}{3+15}=\frac{1}{9}$. Therefore, when $y=12$,

$$\frac{3x-4}{27}=\frac{1}{9},$$

so $3x-4=3$ and $x=\frac{7}{3}$.

Therefore, the correct answer is **C**.

2. When the base of a triangle is increased 10% and the altitude to this base is decreased 10%, the change in area is:

A 1% increase

B $\frac{1}{2}\%$ increase

C 0%

D $\frac{1}{2}\%$ decrease

E 1% decrease

Solution:

The new area is $1.1 \cdot 0.9 = 0.99$ times the old area. It is therefore 1% smaller.

Thus, the correct answer is **E**.

3. If the arithmetic mean of two numbers is 6 and their geometric mean is $10,$ then an equation with the given two numbers as roots is:

A $x^2 + 12x + 100 = 0$

B $x^2 + 6x + 100 = 0$

C $x^2 - 12x - 10 = 0$

D $x^2 - 12x + 100 = 0$

E $x^2 - 6x + 100 = 0$

Solution:

If the numbers are $r, s,$ then $\frac{r+s}{2} = 6$ gives $r + s = 12,$ while $\sqrt{rs} = 10$ gives $rs = 100.$ Their monic root equation is $x^2 - (r + s)x + rs = 0,$ or $x^2 - 12x + 100 = 0.$

Therefore, the correct answer is **D**.

4. Circle I is circumscribed about a given square and circle II is inscribed in the given square. If r is the ratio of the area of circle I to that of circle II, then r equals:

A $\sqrt{2}$

B 2

C $\sqrt{3}$

D $2\sqrt{2}$

E $2\sqrt{3}$

Solution:

For square side s, the outer circle has radius $\frac{s}{\sqrt{2}}$, while the inner circle has radius $\frac{s}{2}$. Hence

$$r = \frac{\pi\left(\frac{s}{\sqrt{2}}\right)^2}{\pi\left(\frac{s}{2}\right)^2} = 2.$$

Therefore, the correct answer is **B**.

5. The number of values of x satisfying the equation

$$\frac{2x^2 - 10x}{x^2 - 5x} = x - 3$$

is:

- A zero
- B one
- C two
- D three
- E an integer greater than 3

Solution:

The denominator requires $x \neq 0, 5$. On that domain,

$$\frac{2x(x-5)}{x(x-5)} = 2.$$

The remaining equation $2 = x - 3$ gives $x = 5$, which is excluded. Thus there are no solutions.

Therefore, the correct answer is **A**.

6. AB is a diameter of a circle centered at O. C is a point on the circle such that angle BOC is 60°. If the diameter of the circle is 5 inches, the length of chord AC, expressed in inches, is:

A 3

B $\frac{5\sqrt{2}}{2}$

C $\frac{5\sqrt{3}}{2}$

D $3\sqrt{3}$

E none of these

Solution:

The radius is $\frac{5}{2}$, and $\angle AOC = 180^\circ - 60^\circ = 120^\circ$. A chord subtending 120° has length

$$2\left(\frac{5}{2}\right)\sin 60^\circ = \frac{5\sqrt{3}}{2}.$$

Therefore, the correct answer is **C**.

7. Let

$$\frac{35x-29}{x^2-3x+2}=\frac{N_1}{x-1}+\frac{N_2}{x-2}$$

be an identity in x. The numerical value of $N_1 N_2$ is:

A -246

B -210

C -29

D 210

E 246

Solution:

Combining the right side gives numerator

$$\begin{aligned} &N_1(x-2)+N_2(x-1)\\ &\quad=(N_1+N_2)x\\ &\qquad-(2N_1+N_2). \end{aligned}$$

Thus $N_1+N_2=35$ and $2N_1+N_2=29$. Hence $N_1=-6,\ N_2=41$, and $N_1N_2=-246$.

Therefore, the correct answer is **A**.

8. The length of the common chord of two intersecting circles is 16 feet. If the radii are 10 feet and 17 feet, a possible value for the distance between the centers of the circles, expressed in feet, is:

- A 27
- B 21
- C $\sqrt{389}$
- D 15
- E undetermined

Solution:

The half-chord has length $8.$ The two perpendicular distances from the centers to its line are

$$\sqrt{10^2 - 8^2} = 6,$$
$$\sqrt{17^2 - 8^2} = 15.$$

If the centers lie on opposite sides of the chord, their distance is $6 + 15 = 21,$ which is a possible value.

Therefore, the correct answer is **B**.

9. If $x = (\log_8 2)^{(\log_2 8)}$, then $\log_3 x$ equals:

A -3

B $-\frac{1}{3}$

C $\frac{1}{3}$

D 3

E 9

Solution:

We have $\log_8 2 = \frac{1}{3}$ and $\log_2 8 = 3$. Therefore $x = (\frac{1}{3})^3 = 3^{-3}$, so $\log_3 x = -3.$

Thus, the correct answer is **A**.

10. If the sum of two numbers is 1 and their product is 1, then the sum of their cubes is:

A 2

B $-2-\frac{3\sqrt{3}i}{4}$

C 0

D $-\frac{3\sqrt{3}i}{4}$

E -2

Solution:

If the numbers are u, v, then

$$\begin{aligned} u^3+v^3 &= (u+v)^3-3uv(u+v) \\ &= 1^3-3=-2. \end{aligned}$$

Therefore, the correct answer is **E**.

11. The sides of triangle BAC are in the ratio $2:3:4$. BD is the angle-bisector drawn to the shortest side AC, dividing it into segments AD and CD. If the length of AC is 10, then the length of the longer segment of AC is:

A $3\frac{1}{2}$

B 5

C $5\frac{5}{7}$

D 6

E $7\frac{1}{2}$

Solution:

The sides adjacent to angle B are in ratio $3:4$. By the angle-bisector theorem, $AD : DC = 3:4$. The longer of the two parts is therefore

$$\frac{4}{3+4} \cdot 10 = \frac{40}{7} = 5\frac{5}{7}.$$

Therefore, the correct answer is **C**.

12. The number of real values of x that satisfy the equation

$$(2^{6x+3})(4^{3x+6}) = 8^{4x+5}$$

is:

A zero

B one

C two

D three

E greater than 3

Solution:

In base 2, the left exponent is

$$\begin{aligned}(6x+3)+2(3x+6)\\ = 12x+15,\end{aligned}$$

and the right exponent is $3(4x+5) = 12x+15$. The equation is an identity for every real x, so it has more than three real solutions.

Therefore, the correct answer is **E**.

13. The number of points with positive rational coordinates selected from the set of points in the xy-plane such that $x + y \leq 5$, is:

A 9

B 10

C 14

D 15

E infinite

Solution:

For example, set $y = 1$. Every positive rational $x \leq 4$ then gives an allowed point $(x, 1)$, and there are infinitely many such rationals.

Therefore, the correct answer is **E**.

14. The length of rectangle $ABCD$ is 5 inches and its width is 3 inches. Diagonal AC is divided into three equal segments by points E and F. The area of triangle BEF, expressed in square inches, is:

A $\frac{3}{2}$

B $\frac{5}{3}$

C $\frac{5}{2}$

D $\frac{1}{3}\sqrt{34}$

E $\frac{1}{3}\sqrt{68}$

Solution:

Take $A=(0,0)$, $B=(5,0)$, and $C=(5,3)$. Then $E=(\frac{5}{3},1)$ and $F=(\frac{10}{3},2)$. The determinant formula gives

$$\begin{aligned}[BEF] &= \frac{1}{2}\left|\left(-\frac{10}{3}\right)2-\left(-\frac{5}{3}\right)1\right| \\ &= \frac{5}{2}.\end{aligned}$$

Therefore, the correct answer is **C**.

15. If $x - y > x$ and $x + y < y$, then:

A $y < x$

B $x < y$

C $x < y < 0$

D $x < 0, y < 0$

E $x < 0, y > 0$

Solution:

From $x - y > x$ we get $-y > 0$, so $y < 0$. From $x + y < y$ we get $x < 0$.

Therefore, the correct answer is **D**.

16. If

$$\frac{4^x}{2^{x+y}} = 8,$$
$$\frac{9^{x+y}}{3^{5y}} = 243,$$

x and y are real numbers, then xy equals:

A $1\frac{3}{4}$

B 4

C 6

D 12

E -4

Solution:

The first equation becomes $2^{x-y} = 2^3$, so $x - y = 3$. The second becomes $3^{2x-3y} = 3^5$, so $2x - 3y = 5$. Solving gives $y = 1,\ x = 4$, and hence $xy = 4$.

Therefore, the correct answer is **B**.

17. The number of distinct points common to the curves $x^2 + 4y^2 = 1$ and $4x^2 + y^2 = 4$ is:

A 0

B 1

C 2

D 3

E 4

Solution:

From the first equation, $x^2 = 1 - 4y^2$. Substitution into the second gives

$$4(1 - 4y^2) + y^2 = 4,$$

so $y = 0$. Then $x^2 = 1$, giving the two points $(1, 0)$ and $(-1, 0)$.

Therefore, the correct answer is **C**.

18. In a given arithmetic sequence the first term is 2, the last term is 29, and the sum of all the terms is 155. The common difference is:

A 3

B 2

C $\frac{27}{19}$

D $\frac{13}{9}$

E $\frac{23}{38}$

Solution:

The sum formula gives $155 = \frac{31n}{2}$, so $n = 10$. Therefore $29 = 2 + 9d$, and $d = 3$.

Thus, the correct answer is **A**.

19. Let s_1 be the sum of the first n terms of the arithmetic sequence $8, 12, \ldots$ and let s_2 be the sum of the first n terms of the arithmetic sequence $17, 19, \ldots$. Then $s_1 = s_2$ for:

A no value of n

B one value of n

C two values of n

D four values of n

E more than four values of n

Solution:

The two sums are

$$s_1 = 2n^2 + 6n,$$
$$s_2 = n^2 + 16n.$$

Equality gives $n(n-10) = 0$. Since a number of terms is positive, only $n = 10$ works, so there is one value.

Therefore, the correct answer is **B**.

20. If the proposition "$a = 0$" is true, the negation of the proposition "For real values of a and b, if $a = 0$, then $ab = 0$" is:

A. If $a \neq 0$, then $ab \neq 0$

B. If $a \neq 0$, then $ab = 0$

C. If $a = 0$, then $ab \neq 0$

D. If $ab \neq 0$, then $a \neq 0$

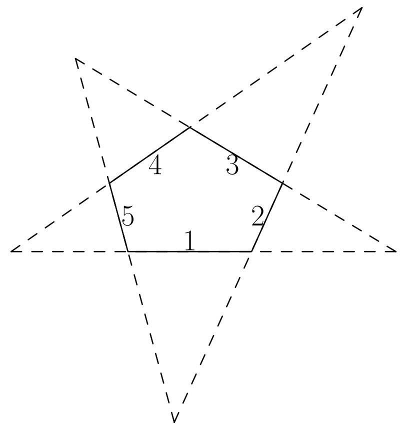
E. If $ab = 0$, then $a \neq 0$

Solution:

The stated premise $a = 0$ is assumed true. Negating the implication therefore negates its conclusion: $ab = 0$ becomes $ab \neq 0$. Thus the required statement is "If $a = 0$, then $ab \neq 0$."

Therefore, the correct answer is **C**.

21. An "n-pointed star" is formed as follows: the sides of a convex polygon are numbered consecutively $1, 2, \ldots, k, \ldots, n, n \geq 5$; for all n values of k, sides k and $k+2$ are non-parallel, sides $n+1$ and $n+2$ being respectively identical with sides 1 and 2; prolong the n pairs of sides numbered k and $k+2$ until they meet. (A figure is shown for the case $n = 5$.)



Let S be the degree-sum of the interior angles at the n points of the star; then S equals:

A 180

B 360

C $180(n+2)$

D $180(n-2)$

E $180(n-4)$

Solution:

Let the original polygon's interior angles be $a_1, \ldots, a_n$. The star angle made from sides k and $k+2$ equals

$$a_k + a_{k+1} - 180^\circ.$$

Summing cyclically counts each a_k twice. Hence

$$\begin{aligned} S &= 2\sum_{k=1}^{n} a_k - 180n \\ &= 2 \cdot 180(n-2) - 180n \\ &= 180(n-4). \end{aligned}$$

Therefore, the correct answer is **E**.

22. Consider the statements:

$(\mathrm{I})\ \sqrt{a^2+b^2} = 0$ and $(\mathrm{II})\ \sqrt{a^2+b^2} = ab,$

$(\mathrm{III})\ \sqrt{a^2+b^2} = a+b$ and $(\mathrm{IV})\ \sqrt{a^2+b^2} = a-b,$

where we allow a and b to be real or complex numbers. Those statements for which there exist solutions other than $a=0$ and $b=0$, are:

A $(\mathrm{I}), (\mathrm{II}), (\mathrm{III}), (\mathrm{IV})$

B $(\mathrm{II}), (\mathrm{III}), (\mathrm{IV})$ only

C $(\mathrm{I}), (\mathrm{III}), (\mathrm{IV})$ only

D $(\mathrm{III}), (\mathrm{IV})$ only

E (I) only

Solution:

Every statement has a nonzero solution. For (I), use $(a,b) = (1,i)$. For (II), use $a = b = \sqrt{2}$, giving $\sqrt{4} = 2 = ab$. For both (III) and (IV), use $(a,b) = (1,0)$. Thus all four statements qualify.

Therefore, the correct answer is **A**.

23. If x is real and $4y^2 + 4xy + x + 6 = 0$, then the complete set of values of x for which y is real, is:

A $x \leq -2$ or $x \geq 3$

B $x \leq 2$ or $x \geq 3$

C $x \leq -3$ or $x \geq 2$

D $-3 \leq x \leq 2$

E $-2 \leq x \leq 3$

Solution:

As a quadratic in y, the equation has discriminant

$$(4x)^2 - 16(x+6)$$
$$= 16(x-3)(x+2).$$

This is nonnegative exactly when $x \leq -2$ or $x \geq 3$.

Therefore, the correct answer is **A**.

24. If $\log_M N = \log_N M, M \neq N, MN > 0$, and $M \neq 1, N \neq 1$, then MN equals:

- A $\frac{1}{2}$
- B 1
- C 2
- D 10
- E a number greater than 2 and less than 10

Solution:

Change of base gives

$$\frac{\ln N}{\ln M} = \frac{\ln M}{\ln N},$$

so $(\ln M)^2 = (\ln N)^2$. Equality of the logs themselves would give $M = N$, which is excluded. Hence $\ln M = -\ln N$, and $\ln(MN) = 0$. Thus $MN = 1$.

Therefore, the correct answer is **B**.

25. If $F(n+1)=\frac{2F(n)+1}{2}$ for $n=1,2,\ldots,$ and $F(1)=2,$ then $F(101)$ equals:

A 49

B 50

C 51

D 52

E 53

Solution:

Each step adds $\frac{1}{2}$. There are 100 steps from $F(1)$ to $F(101)$, so

$$F(101)=2+\frac{100}{2}=52.$$

Therefore, the correct answer is **D**.

26. Let m be a positive integer and let the lines $13x + 11y = 700$ and $y = mx - 1$ intersect in a point whose coordinates are integers. Then m can be:

- A 4 only
- B 5 only
- C 6 only
- D 7 only
- E one of the integers $4, 5, 6, 7$ and one other positive integer

Solution:

Substitution gives

$$(13 + 11m)x = 711.$$

Thus $13 + 11m$ must be a positive divisor of $711 = 3^2 \cdot 79$. Among divisors greater than 13, only 79 is congruent to $13 \pmod{11}$. Hence $13 + 11m = 79$, so $m = 6$.

Therefore, the correct answer is **C**.

27. At his usual rate a man rows 15 miles downstream in five hours less time than it takes him to return. If he doubles his usual rate, the time downstream is only one hour less than the time upstream. In miles per hour, the rate of the stream's current is:

A 2

B $\frac{5}{2}$

C 3

D $\frac{7}{2}$

E 4

Solution:

The two time differences give

$$\begin{aligned}\frac{15}{v-c}-\frac{15}{v+c}&=5,\\ \frac{15}{2v-c}-\frac{15}{2v+c}&=1.\end{aligned}$$

These simplify to $v^2-c^2=6c$ and $4v^2-c^2=30c$. Substituting $v^2=c^2+6c$ into the second gives $3c^2-6c=0$. Since $c>0,\ c=2.$

Therefore, the correct answer is **A**.

28. Five points O, A, B, C, D are taken in order on a straight line with distances $OA = a$, $OB = b, OC = c$, and $OD = d$. P is a point on the line between B and C and such that $AP : PD = BP : PC$. Then OP equals:

A $\frac{b^2 - bc}{a - b + c - d}$

B $\frac{ac - bd}{a - b + c - d}$

C $-\frac{bd + ac}{a - b + c - d}$

D $\frac{bc + ad}{a + b + c + d}$

E $\frac{ac - bd}{a + b + c + d}$

Solution:

Let $p = OP$. The given ratio becomes

$$\frac{p - a}{d - p} = \frac{p - b}{c - p}.$$

Cross-multiplication cancels the p^2 terms and yields

$$p(a - b + c - d) = ac - bd.$$

Therefore $p = \frac{ac-bd}{a-b+c-d}$.

Thus, the correct answer is **B**.

29. The number of positive integers less than 1000 divisible by neither 5 nor 7 is:

- A 688
- B 686
- C 684
- D 658
- E 630

Solution:

By inclusion-exclusion, the count is

$$\begin{aligned} &999 - \left\lfloor \frac{999}{5} \right\rfloor - \left\lfloor \frac{999}{7} \right\rfloor \\ &\quad + \left\lfloor \frac{999}{35} \right\rfloor \\ &= 999 - 199 - 142 + 28 \\ &= 686. \end{aligned}$$

Therefore, the correct answer is **B**.

30. If three of the roots of $x^4 + ax^2 + bx + c = 0$ are $1, 2,$ and $3,$ then the value of $a + c$ is:

A 35

B 24

C -12

D -61

E -63

Solution:

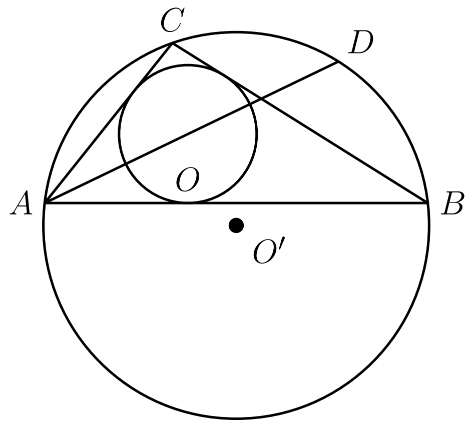
The fourth root is -6 because the root sum is zero. The sum of pairwise products is

$$\begin{aligned} a &= 2 + 3 - 6 + 6 \\ &\quad - 12 - 18 = -25, \end{aligned}$$

while the product is $c = (1)(2)(3)(-6) = -36.$ Thus $a + c = -61.$

Therefore, the correct answer is **D**.

31. Triangle ABC is inscribed in a circle with center O'. A circle with center O is inscribed in triangle ABC. AO is drawn, and extended to intersect the larger circle in D. Then we must have:



A $CD = BD = O'D$

B $AO = CO = OD$

C $CD = CO = BD$

D $CD = OD = BD$

E $O'B = O'C = OD$

Solution:

Because AO is an angle bisector, the inscribed angles $\angle BAD$ and $\angle CAD$ are equal. Hence arcs BD and CD, and therefore chords BD and CD, are equal.

Let $\angle BAD = \alpha$ and $\angle BCO = \beta$. Then $\angle OCD = \alpha + \beta$. In triangle AOC, the exterior angle $\angle COD$ also equals $\alpha + \beta$. Thus $CD = OD$, so $CD = OD = BD$.

Therefore, the correct answer is **D**.

32. Let M be the midpoint of side AB of triangle ABC. Let P be a point on AB between A and M, and let MD be drawn parallel to PC and intersecting BC at D. If the ratio of the area of triangle BPD to that of triangle ABC is denoted by r, then:

A $\frac{1}{2} < r < 1$ depending upon the position of P

B $r = \frac{1}{2}$ independent of the position of P

C $\frac{1}{2} \le r < 1$ depending upon the position of P

D $\frac{1}{3} < r < \frac{2}{3}$ depending upon the position of P

E $r = \frac{1}{3}$ independent of the position of P

Solution:

Since $MD \parallel PC$, triangles MDP and MDC have the same base MD and equal altitudes, so their areas are equal. Hence

$$\begin{aligned}[BPD] &= [BMD] + [MDP] \\ &= [BMD] + [MDC] \\ &= [BMC].\end{aligned}$$

Because CM is a median, $[BMC] = \frac{[ABC]}{2}$. Thus $r = \frac{1}{2}$ for every allowed P.

Therefore, the correct answer is **B**.

33. If $ab \neq 0$ and $|a| \neq |b|$, the number of distinct values of x satisfying the equation

$$\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}$$

is:

A zero

B one

C two

D three

E four

Solution:

Let $N = (a+b)x - a^2 - b^2$. Combining each side gives

$$N\left(\frac{1}{ab} - \frac{1}{(x-a)(x-b)}\right) = 0.$$

The first factor gives $x = \frac{a^2+b^2}{a+b}$. The second gives $(x-a)(x-b) = ab$, or $x(x-a-b) = 0$, giving $x = 0$ and $x = a+b$. The hypotheses ensure that all three are defined and distinct.

Therefore, the correct answer is **D**.

34. Let r be the speed in miles per hour at which a wheel, 11 feet in circumference, travels. If the time for a complete rotation of the wheel is shortened by $\frac{1}{4}$ of a second, the speed r is increased by 5 miles per hour. Then r is:

- A 9
- B 10
- C $10\frac{1}{2}$
- D 11
- E 12

Solution:

Since 11 feet is $\frac{11}{5280}$ mile, one rotation at r mph takes

$$\frac{3600 \cdot 11}{5280r} = \frac{7.5}{r}$$

seconds. Thus

$$\frac{7.5}{r} - \frac{7.5}{r+5} = \frac{1}{4}.$$

This reduces to $r^2 + 5r - 150 = 0$, or $(r-10)(r+15) = 0$. The positive speed is $r = 10$.

Therefore, the correct answer is **B**.

35. Let O be an interior point of triangle ABC, and let $s_1 = OA + OB + OC$. If $s_2 = AB + BC + CA$, then:

A for every triangle $s_1 > \frac{1}{2}s_2, s_1 \leq s_2$

B for every triangle $s_1 \geq \frac{1}{2}s_2, s_1 < s_2$

C for every triangle $s_1 > \frac{1}{2}s_2, s_1 < s_2$

D for every triangle $s_1 \geq \frac{1}{2}s_2, s_1 \leq s_2$

E neither (A) nor (B) nor (C) nor (D) applies to every triangle

Solution:

Adding the three strict triangle inequalities

$$AB < OA + OB,$$
$$BC < OB + OC,$$
$$CA < OC + OA$$

gives $s_2 < 2s_1$.

For the upper bound, extend AO to D on BC. Then $AO + OD < AC + CD$ and $OB < OD + DB$, so cancellation gives $OA + OB < AC + CB$. Cycling and adding yields $2s_1 < 2s_2$. Therefore $s_1 > \frac{s_2}{2}$ and $s_1 < s_2$.

Thus, the correct answer is **C**.

36. Let

$$(1 + x + x^2)^n = a_0 + a_1 x \\ + a_2 x^2 + \cdots \\ + a_{2n} x^{2n}$$

be an identity in x. If we let $s = a_0 + a_2 + a_4 + \cdots + a_{2n}$, then s equals:

A 2^n

B $2^n + 1$

C $\frac{3^n - 1}{2}$

D $\frac{3^n}{2}$

E $\frac{3^n + 1}{2}$

Solution:

Let $P(x) = (1 + x + x^2)^n$. Then $P(1) = 3^n$ is the sum of all coefficients, while $P(-1) = 1$ is the even-coefficient sum minus the odd-coefficient sum. Therefore

$$s = \frac{P(1) + P(-1)}{2} = \frac{3^n + 1}{2}.$$

Therefore, the correct answer is **E**.

37. Three men, Alpha, Beta, and Gamma, working together, do a job in 6 hours less time than Alpha alone, in 1 hour less time than Beta alone, and in one-half the time needed by Gamma when working alone. Let h be the number of hours needed by Alpha and Beta, working together, to do the job. Then h equals:

A $\frac{5}{2}$

B $\frac{3}{2}$

C $\frac{4}{3}$

D $\frac{5}{4}$

E $\frac{3}{4}$

Solution:

If all three together take t hours, then

$$\frac{1}{t+6}+\frac{1}{t+1}+\frac{1}{2t}=\frac{1}{t}.$$

This simplifies to $3t^2+7t-6=0$, so $t=\frac{2}{3}$. Alpha and Beta alone take $\frac{20}{3}$ and $\frac{5}{3}$ hours, so their combined rate is $\frac{3}{20}+\frac{3}{5}=\frac{3}{4}$. Hence $h=\frac{4}{3}$.

Therefore, the correct answer is **C**.

38. In triangle ABC the medians AM and CN to sides BC and AB, respectively, intersect in point O. P is the midpoint of side AC, and MP intersects CN in Q. If the area of triangle OMQ is n, then the area of triangle ABC is:

A $16n$

B $18n$

C $21n$

D $24n$

E $27n$

Solution:

Area ratios are affine-invariant, so take $A = (0,0)$, $B = (2,0)$, and $C = (0,2)$. Then

$$M = (1,1),$$
$$O = (\frac{2}{3}, \frac{2}{3}),$$
$$P = (0,1).$$

Line MP is $y = 1$, and it meets median CN at $Q = (\frac{1}{2}, 1)$. Thus $[OMQ] = \frac{1}{12}$, while $[ABC] = 2$. Their ratio is 24, so $[ABC] = 24n$.

Therefore, the correct answer is **D**.

39. In base R_1 the expanded fraction F_1 becomes $0.373737\ldots$ and the expanded fraction F_2 becomes $0.737373\ldots$. In base R_2 fraction F_1, when expanded, becomes $0.252525\ldots$, while fraction F_2 becomes $0.525252\ldots$. The sum of R_1 and R_2, each written in base ten, is:

A 24

B 22

C 21

D 20

E 19

Solution:

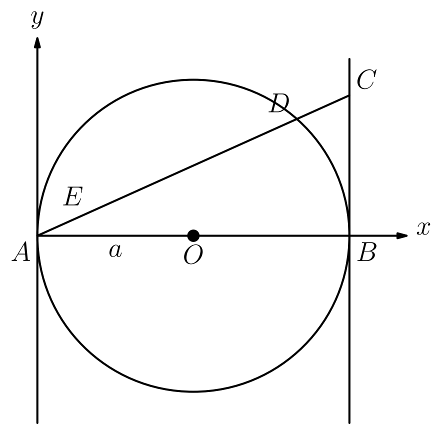
The two descriptions give

$$\begin{aligned} F_1 &= \frac{3R_1+7}{R_1^2-1} \\ &= \frac{2R_2+5}{R_2^2-1}, \\ F_2 &= \frac{7R_1+3}{R_1^2-1} \\ &= \frac{5R_2+2}{R_2^2-1}. \end{aligned}$$

Adding yields $\frac{10}{R_1-1} = \frac{7}{R_2-1}$, or $10R_2 - 7R_1 = 3$. Subtracting yields $\frac{4}{R_1+1} = \frac{3}{R_2+1}$, or $4R_2 - 3R_1 = -1$. Solving gives $R_1 = 11,\ R_2 = 8$, whose sum is 19.

Therefore, the correct answer is **E**.

40. In this figure AB is a diameter of a circle, centered at O, with radius a. A chord AD is drawn and extended to meet the tangent to the circle at B, in point C. Point E is taken on AC so that $AE = DC$. If the coordinates of E are (x, y), then:



A $y^2 = \dfrac{x^3}{2a - x}$

B $y^2 = \dfrac{x^3}{2a + x}$

C $y^4 = \dfrac{x^2}{2a - x}$

D $x^2 = \dfrac{y^2}{2a - x}$

E $x^2 = \dfrac{y^2}{2a + x}$

Solution:

Drop perpendiculars EM and DN to AB. Since $AE = DC$ and the tangents through A, B are parallel, their projections give $NB = x$. Hence $AN = 2a - x$. In right triangle ADB, the altitude theorem gives

$$DN^2 = x(2a - x).$$

Similar triangles AME and AND give $\frac{DN}{2a-x} = \frac{y}{x}$, so $DN = \frac{y(2a-x)}{x}$. Substitution and cancellation yield

$$y^2 = \frac{x^3}{2a - x}.$$

Therefore, the correct answer is **A**.

Problems: https://live.poshenloh.com/past-contests/amc12/1966