

# 1965 AMC 12

Time limit: 75 minutes

Typeset by: LIVE by Po-Shen Loh

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1. The number of real values of  $x$  satisfying the equation  $2^{2x^2-7x+5} = 1$  is:

A 0

B 1

C 2

D 4

E more than 4

2. A regular hexagon is inscribed in a circle. The ratio of the length of a side of the hexagon to the length of the shorter of the arcs intercepted by the side is:

A 1 : 1

B 1 : 6

C 1 :  $\pi$

D 3 :  $\pi$

E 6 :  $\pi$

3. The expression  $81^{-(2^{-2})}$  has the same value as:

A  $\frac{1}{81}$

B  $\frac{1}{3}$

C 3

D 81

E  $81^4$

4. Line  $l_2$  intersects line  $l_1$  and line  $l_3$  is parallel to  $l_1$ . The three lines are distinct and lie in a plane. The number of points equidistant from all three lines is:

A 0

B 1

C 2

D 4

E 8

5. When the repeating decimal  $0.\overline{363636} \dots$  is written in simplest fractional form, the sum of the numerator and denominator is:

- A 15
- B 45
- C 114
- D 135
- E 150

6. If  $10^{\log_{10} 9} = 8x + 5$ , then  $x$  equals:

- A 0
- B  $\frac{1}{2}$
- C  $\frac{5}{8}$
- D  $\frac{9}{8}$
- E  $\frac{2 \log_{10} 3 - 5}{8}$

7. The sum of the reciprocals of the roots of the equation  $ax^2 + bx + c = 0$  is:

A  $\frac{1}{a} + \frac{1}{b}$

B  $-\frac{c}{b}$

C  $\frac{c}{b}$

D  $-\frac{a}{b}$

E  $-\frac{b}{c}$

8. One side of a given triangle is 18 inches. Inside the triangle a line segment is drawn parallel to this side forming a trapezoid whose area is one-third of that of the triangle. The length of this segment, in inches, is:

A  $6\sqrt{6}$

B  $9\sqrt{2}$

C 12

D  $6\sqrt{3}$

E 9

9. The vertex of the parabola  $y = x^2 - 8x + c$  will be a point on the  $x$ -axis if the value of  $c$  is:

A  $-16$

B  $-4$

C  $4$

D  $8$

E  $16$

10. The statement  $x^2 - x - 6 < 0$  is equivalent to the statement:

A  $-2 < x < 3$

B  $x > -2$

C  $x < 3$

D  $x > 3$  and  $x < -2$

E  $x > 3$  or  $x < -2$

11. Consider the statements:

I:  $(\sqrt{-4})(\sqrt{-16}) = \sqrt{(-4)(-16)},$

II:  $\sqrt{(-4)(-16)} = \sqrt{64},$

and

III:  $\sqrt{64} = 8.$

Of these the following are *incorrect*:

A none

B I only

C II only

D III only

E I and III only

12. A rhombus is inscribed in triangle  $ABC$  in such a way that one of its vertices is  $A$  and two of its sides lie along  $AB$  and  $AC$ . If  $AC = 6$  inches,  $AB = 12$  inches, and  $BC = 8$  inches, the side of the rhombus, in inches, is:

A 2

B 3

C  $3\frac{1}{2}$

D 4

E 5

13. Let  $n$  be the number of number-pairs  $(x, y)$  which satisfy  $5y - 3x = 15$  and  $x^2 + y^2 \leq 16$ . Then  $n$  is:

- ☐ A 0
- ☐ B 1
- ☐ C 2
- ☐ D more than two, but finite
- ☐ E greater than any finite number

14. The sum of the numerical coefficients in the complete expansion of  $(x^2 - 2xy + y^2)^7$  is:

- ☐ A 0
- ☐ B 7
- ☐ C 14
- ☐ D 128
- ☐ E  $128^2$

15. The symbol  $25_b$  represents a two-digit number in the base  $b$ . If the number  $52_b$  is double the number  $25_b$ , then  $b$  is:

- A 7
- B 8
- C 9
- D 11
- E 12

16. Let line  $AC$  be perpendicular to line  $CE$ . Connect  $A$  to the midpoint  $D$  of  $CE$ , and connect  $E$  to the midpoint  $B$  of  $AC$ . If  $AD$  and  $EB$  intersect in point  $F$ , and  $BC = CD = 15$  inches, then the area of triangle  $DFE$ , in square inches, is:

- A 50
- B  $50\sqrt{2}$
- C 75
- D  $\frac{15}{2}\sqrt{105}$
- E 100

17. Given the true statement: The picnic on Sunday will not be held only if the weather is not fair. We can then conclude that:

- A If the picnic is held, Sunday's weather is undoubtedly fair.
- B If the picnic is not held, Sunday's weather is possibly unfair.
- C If it is not fair Sunday, the picnic will not be held.
- D If it is fair Sunday, the picnic may be held.
- E If it is fair Sunday, the picnic will be held.

18. If  $1 - y$  is used as an approximation to the value of  $\frac{1}{1 + y}$ ,  $|y| < 1$ , the ratio of the error made to the correct value is:

- A  $y$
- B  $y^2$
- C  $\frac{1}{1 + y}$
- D  $\frac{y}{1 + y}$
- E  $\frac{y^2}{1 + y}$

19. If  $x^4 + 4x^3 + 6px^2 + 4qx + r$  is exactly divisible by  $x^3 + 3x^2 + 9x + 3$ , the value of  $(p + q)r$  is:

A  $-18$

B  $12$

C  $15$

D  $27$

E  $45$

20. For every  $n$  the sum  $S_n$  of  $n$  terms of an arithmetic progression is  $2n + 3n^2$ . The  $r$ th term is:

A  $3r^2$

B  $3r^2 + 2r$

C  $6r - 1$

D  $5r + 5$

E  $6r + 2$

21. It is possible to choose  $x > \frac{2}{3}$  in such a way that the value of

$$\log_{10}(x^2 + 3) - 2 \log_{10} x$$

is:

- ☐ A negative
- ☐ B zero
- ☐ C one
- ☐ D smaller than any positive number that might be specified
- ☐ E greater than any positive number that might be specified

22. If  $a_2 \neq 0$  and  $r, s$  are the roots of  $a_0 + a_1x + a_2x^2 = 0$ , then the equality

$$a_0 + a_1x + a_2x^2 = a_0 \left(1 - \frac{x}{r}\right) \cdot \left(1 - \frac{x}{s}\right)$$

holds:

- ☐ A for all values of  $x, a_0 \neq 0$
- ☐ B for all values of  $x$
- ☐ C only when  $x = 0$
- ☐ D only when  $x = r$  or  $x = s$
- ☐ E only when  $x = r$  or  $x = s, a_0 \neq 0$

23. If we write  $|x^2 - 4| < N$  for all  $x$  such that  $|x - 2| < 0.01$ , the smallest value we can use for  $N$  is:

A 0.0301

B 0.0349

C 0.0399

D 0.0401

E 0.0499

24. Given the sequence  $10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}}, \dots, 10^{\frac{n}{11}}$ , the smallest value of  $n$  such that the product of the first  $n$  members of this sequence exceeds 100,000 is:

A 7

B 8

C 9

D 10

E 11

25. Let  $ABCD$  be a quadrilateral with  $AB$  extended to  $E$  so that  $AB = BE$ . Lines  $AC$  and  $CE$  are drawn to form angle  $ACE$ . For this angle to be a right angle it is necessary that quadrilateral  $ABCD$  have:

- ☐ A all angles equal
- ☐ B all sides equal
- ☐ C two pairs of equal sides
- ☐ D one pair of equal sides
- ☐ E one pair of equal angles

26. For the numbers  $a, b, c, d, e$  define  $m$  to be the arithmetic mean of all five numbers;  $k$  to be the arithmetic mean of  $a$  and  $b$ ;  $l$  to be the arithmetic mean of  $c, d$ , and  $e$ ; and  $p$  to be the arithmetic mean of  $k$  and  $l$ . Then, no matter how  $a, b, c, d, e$  are chosen, we shall always have:

- ☐ A  $m = p$
- ☐ B  $m \geq p$
- ☐ C  $m > p$
- ☐ D  $m < p$
- ☐ E none of these

27. When  $y^2 + my + 2$  is divided by  $y - 1$  the quotient is  $f(y)$  and the remainder is  $R_1$ . When  $y^2 + my + 2$  is divided by  $y + 1$  the quotient is  $g(y)$  and the remainder is  $R_2$ . If  $R_1 = R_2$ , then  $m$  is:

- ☐ A 0
- ☐ B 1
- ☐ C 2
- ☐ D  $-1$
- ☐ E an undetermined constant

28. An escalator (moving staircase) of  $n$  uniform steps visible at all times descends at constant speed. Two boys,  $A$  and  $Z$ , walk down the escalator steadily as it moves,  $A$  negotiating twice as many escalator steps per minute as  $Z$ .  $A$  reaches the bottom after taking 27 steps while  $Z$  reaches the bottom after taking 18 steps. Then  $n$  is:

- ☐ A 63
- ☐ B 54
- ☐ C 45
- ☐ D 36
- ☐ E 30

**29.** Of 28 students taking at least one subject the number taking Mathematics and English only equals the number taking Mathematics only. No student takes English only or History only, and six students take Mathematics and History, but no English. The number taking English and History only is five times the number taking all three subjects. If the number taking all three subjects is even and non-zero, the number taking English and Mathematics only is:

A 5

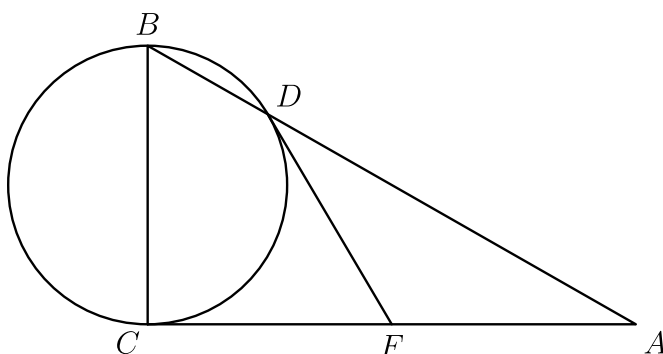
B 6

C 7

D 8

E 9

30. Let  $BC$  of right triangle  $ABC$  be the diameter of a circle intersecting hypotenuse  $AB$  in  $D$ . At  $D$  a tangent is drawn cutting leg  $CA$  in  $F$ . This information is *not* sufficient to prove that:



- ☐ A  $DF$  bisects  $CA$
  - ☐ B  $DF$  bisects  $\angle CDA$
  - ☐ C  $DF = FA$
  - ☐ D  $\angle A = \angle BCD$
  - ☐ E  $\angle CFD = 2\angle A$
31. The number of real values of  $x$  satisfying the equality  $(\log_a x)(\log_b x) = \log_a b$ , where  $a$  and  $b$  are positive constants different from 1, is:

- ☐ A 0
- ☐ B 1
- ☐ C 2
- ☐ D an integer greater than 2
- ☐ E not finite

32. An article costing  $C$  dollars is sold for \$100 at a loss of  $x$  percent of the selling price. It is then resold at a profit of  $x$  percent of the new selling price  $S'$ . If the difference between  $S'$  and  $C$  is  $1\frac{1}{9}$  dollars, then  $x$  is:

- A undetermined
- B  $\frac{80}{9}$
- C 10
- D  $\frac{95}{9}$
- E  $\frac{100}{9}$

33. If the number  $15!$ , that is,  $15 \cdot 14 \cdot 13 \cdots 1$ , ends with  $k$  zeros when given to the base 12 and ends with  $h$  zeros when given to the base 10, then  $k + h$  equals:

- A 5
- B 6
- C 7
- D 8
- E 9

34. For  $x \geq 0$ , the smallest value of  $\frac{4x^2 + 8x + 13}{6(1 + x)}$  is:

- A 1
- B 2
- C  $\frac{25}{12}$
- D  $\frac{13}{6}$
- E  $\frac{34}{5}$

35. The length of a rectangle is 5 inches and its width is less than 4 inches. The rectangle is folded so that two diagonally opposite vertices coincide. If the length of the crease is  $\sqrt{6}$ , then the width is:

- A  $\sqrt{2}$
- B  $\sqrt{3}$
- C 2
- D  $\sqrt{5}$
- E  $\sqrt{\frac{11}{2}}$

36. Given distinct straight lines  $OA$  and  $OB$ . From a point in  $OA$  a perpendicular is drawn to  $OB$ ; from the foot of this perpendicular a line is drawn perpendicular to  $OA$ . From the foot of this second perpendicular a line is drawn perpendicular to  $OB$ ; and so on indefinitely. The lengths of the first and second perpendiculars are  $a$  and  $b$ , respectively. Then the sum of the lengths of the perpendiculars approaches a limit as the number of perpendiculars grows beyond all bounds. This limit is:

A  $\frac{b}{a - b}$

B  $\frac{a}{a - b}$

C  $\frac{ab}{a - b}$

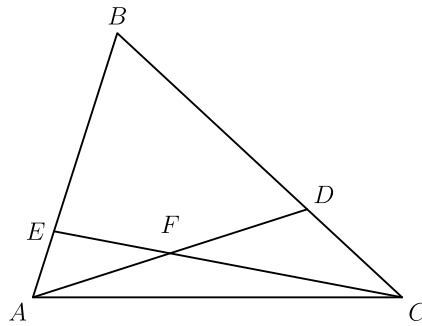
D  $\frac{b^2}{a - b}$

E  $\frac{a^2}{a - b}$

37. Point  $E$  is selected on side  $AB$  of triangle  $ABC$  in such a way that  $AE : EB = 1 : 3$ , and point  $D$  is selected on side  $BC$  so that  $CD : DB = 1 : 2$ . The point of intersection of  $AD$  and  $CE$  is  $F$ . Then

$$\frac{EF}{FC} + \frac{AF}{FD}$$

is:



- ☐ A  $\frac{4}{5}$
- ☐ B  $\frac{5}{4}$
- ☐ C  $\frac{3}{2}$
- ☐ D 2
- ☐ E  $\frac{5}{2}$

38. A takes  $m$  times as long to do a piece of work as  $B$  and  $C$  together;  $B$  takes  $n$  times as long as  $C$  and  $A$  together; and  $C$  takes  $x$  times as long as  $A$  and  $B$  together. Then  $x$ , in terms of  $m$  and  $n$ , is:

A  $\frac{2mn}{m+n}$

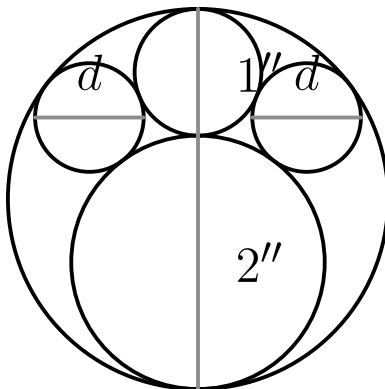
B  $\frac{1}{2(m+n)}$

C  $\frac{1}{m+n-mn}$

D  $\frac{1-mn}{m+n+2mn}$

E  $\frac{m+n+2}{mn-1}$

39. A foreman noticed an inspector checking a 3-inch hole with a 2-inch plug and a 1-inch plug and suggested that two more gauges be inserted to be sure that the fit was snug. If the new gauges are alike, then the diameter  $d$  of each, to the nearest hundredth of an inch, is:



- A 0.87
- B 0.86
- C 0.83
- D 0.75
- E 0.71
40. Let  $n$  be the number of integer values of  $x$  such that  $P = x^4 + 6x^3 + 11x^2 + 3x + 31$  is the square of an integer. Then  $n$  is:

- A 4
- B 3
- C 2
- D 1
- E 0

Solutions: <https://live.poshenloh.com/past-contests/amc12/1965/solutions>

