1964 AMC 12 Solutions

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1. What is the value of $[\log_{10}(5\log_{10}100)]^2$?

A $\log_{10} 50$

B 25

C 10

D 2

E 1

Solution:

Since $\log_{10} 100 = 2$, the expression inside the square brackets is $\log_{10}(5 \cdot 2) = \log_{10} 10 = 1$. Its square is 1.

Therefore, the correct answer is **E**.

2. The graph of $x^2 - 4y^2 = 0$ is:

- A a parabola
- B an ellipse
- C a pair of straight lines
- D a point
- E none of these

Solution:

Factoring gives

$$x^2 - 4y^2 = (x - 2y)(x + 2y).$$

Thus every point lies on $x = 2y$ or $x = -2y$, a pair of straight lines.

Therefore, the correct answer is **C**.

3. When a positive integer x is divided by a positive integer y, the quotient is u and the remainder is v, where u and v are integers. What is the remainder when $x + 2uy$ is divided by y?

A 0

B $2u$

C $3u$

D v

E $2v$

Solution:

The division algorithm gives $x = uy + v$. Hence

$$x + 2uy = 3uy + v.$$

The first term is divisible by y, so the remainder is v.

Therefore, the correct answer is **D**.

4. The expression

$$\frac{P+Q}{P-Q}-\frac{P-Q}{P+Q},$$

where $P=x+y$ and $Q=x-y$, is equivalent to:

A $\dfrac{x^2-y^2}{xy}$

B $\dfrac{x^2-y^2}{2xy}$

C 1

D $\dfrac{x^2+y^2}{xy}$

E $\dfrac{x^2+y^2}{2xy}$

Solution:

Since $P+Q=2x$ and $P-Q=2y$, the expression becomes

$$\frac{x}{y}-\frac{y}{x}=\frac{x^2-y^2}{xy}.$$

Therefore, the correct answer is **A**.

5. If y varies directly as x, and if $y = 8$ when $x = 4$, the value of y when $x = -8$ is:

A -16

B -4

C -2

D $4k, k = \pm 1, \pm 2, \ldots$

E $16k, k = \pm 1, \pm 2, \ldots$

Solution:

Direct variation gives $y = kx$. From $8 = 4k, k = 2$. Therefore, when $x = -8, y = 2(-8) = -16$.

Thus, the correct answer is **A**.

6. If $x, 2x+2, 3x+3, \ldots$ are in geometric progression, the fourth term is:

A -27

B $-13\frac{1}{2}$

C 12

D $13\frac{1}{2}$

E 27

Solution:

The geometric-mean relation gives

$$(2x+2)^2 = x(3x+3).$$

Under the examination's ratio convention for a geometric progression, the consecutive ratios must be defined, so $x \neq -1$. Dividing by $x+1$ yields $4(x+1) = 3x$ and $x = -4$. The first three terms are $-4, -6, -9,$ with ratio $\frac{3}{2}$. The fourth is $-9(\frac{3}{2}) = -\frac{27}{2}$.

Therefore, the correct answer is **B**.

7. Let n be the number of real values of p for which the roots of

$$x^2 - px + p = 0$$

are equal. Then n equals:

A 0

B 1

C 2

D a finite number greater than 2

E an infinitely large number

Solution:

Equal roots require

$$(-p)^2 - 4p = p(p-4) = 0.$$

Thus $p = 0$ or $p = 4$, giving two real values.

Therefore, the correct answer is **C**.

8. The smaller root of the equation

$$\left(x-\frac{3}{4}\right)\left(x-\frac{3}{4}\right) + \left(x-\frac{3}{4}\right)\left(x-\frac{1}{2}\right)=0$$

is:

A $-\frac{3}{4}$

B $\frac{1}{2}$

C $\frac{5}{8}$

D $\frac{3}{4}$

E 1

Solution:

Combining the two terms and factoring gives

$$\left(x-\frac{3}{4}\right)\left(2x-\frac{5}{4}\right)=0.$$

Hence the roots are $\frac{3}{4}$ and $\frac{5}{8}$, of which $\frac{5}{8}$ is smaller.

Therefore, the correct answer is **C**.

9. A jobber buys an article at $\$24$ less $12\frac{1}{2}\%$. He then wishes to sell the article at a gain of $33\frac{1}{3}\%$ of his cost after allowing a 20% discount on his marked price. At what price, in dollars, should the article be marked?

A 25.20

B 30.00

C 33.60

D 40.00

E none of these

Solution:

The cost is $24(1-\frac{1}{8})=21$ dollars. A gain of one third makes the desired selling price $21(\frac{4}{3})=28$ dollars. If M is the marked price, then $0.8M=28$, so $M=35$. This is not listed.

Thus, the correct answer is **E**.

10. Given a square side of length s. On a diagonal as base a triangle with three unequal sides is constructed so that its area equals that of the square. The length of the altitude drawn to the base is:

A $s\sqrt{2}$

B $\frac{s}{\sqrt{2}}$

C $2s$

D $2\sqrt{s}$

E $\frac{2}{\sqrt{s}}$

Solution:

The triangle's base is the square's diagonal, $s\sqrt{2}$. If its altitude is h, equality of areas gives

$$\frac{1}{2}(s\sqrt{2})h = s^2,$$

so $h = s\sqrt{2}$.

Therefore, the correct answer is **A**.

11. Given $2^x = 8^{y+1}$ and $9^y = 3^{x-9}$, find the value of $x + y$.

A 18

B 21

C 24

D 27

E 30

Solution:

Writing both equations with common bases gives

$$x = 3y + 3, \qquad 2y = x - 9.$$

Substitution yields $y = 6$ and $x = 21$, so $x + y = 27$.

Therefore, the correct answer is **D**.

12. Which of the following is the negation of the statement: For all x of a certain set, $x^2 > 0$?

A For all $x, x^2 < 0$

B For all $x, x^2 \leq 0$

C For no $x, x^2 > 0$

D For some $x, x^2 > 0$

E For some $x, x^2 \leq 0$

Solution:

The negation of a universal statement is an existential statement, and the negation of $x^2 > 0$ is $x^2 \leq 0$. Thus the negation is: for some $x,\ x^2 \leq 0$.

Therefore, the correct answer is **E**.

13. A circle is inscribed in a triangle with side lengths $8, 13$, and 17. Let the segments of the side of length 8, made by a point of tangency, be r and s, with $r < s$. What is the ratio $r : s$?

A $1:3$

B $2:5$

C $1:2$

D $2:3$

E $3:4$

Solution:

The semiperimeter is $\frac{8+13+17}{2} = 19$. At the endpoints of the side of length 8, the tangent lengths are $19 - 13 = 6$ and $19 - 17 = 2$. Thus $r : s = 2 : 6 = 1 : 3$.

Therefore, the correct answer is **A**.

14. A farmer bought 749 sheep. He sold 700 of them for the price paid for the 749 sheep. The remaining 49 sheep were sold at the same price per head as the other 700. Based on the cost, the percent gain on the entire transaction is:

- A 6.5
- B 6.75
- C 7
- D 7.5
- E 8

Solution:

Let the total cost be C. Since the first 700 sheep sell for C, each sheep sells for $\frac{C}{700}$. Total revenue is therefore $\frac{749C}{700} = 1.07C$, a 7% gain.

Thus, the correct answer is **C**.

15. A line through the point $(-a, 0)$ cuts from the second quadrant a triangular region with area T. The equation of the line is:

A $2Tx + a^2y + 2aT = 0$

B $2Tx - a^2y + 2aT = 0$

C $2Tx + a^2y - 2aT = 0$

D $2Tx - a^2y - 2aT = 0$

E none of these

Solution:

Let the y-intercept be b. The second-quadrant triangle has area $\frac{ab}{2} = T$, so $b = \frac{2T}{a}$. The intercept form is

$$\frac{x}{-a} + \frac{y}{\frac{2T}{a}} = 1.$$

Clearing denominators gives $2Tx - a^2y + 2aT = 0$.

Therefore, the correct answer is **B**.

16. Let $f(x) = x^2 + 3x + 2$ and let S be the set of integers $\{0, 1, 2, \ldots, 25\}$. The number of members s of S such that $f(s)$ has remainder zero when divided by 6 is:

A 25

B 22

C 21

D 18

E 17

Solution:

We have $f(s) = (s+1)(s+2)$, a product of consecutive integers, so it is always even. It is divisible by 3 unless $s \equiv 0 \pmod{3}$. Among $0, 1, \ldots, 25$, nine values are multiples of 3. Thus $26 - 9 = 17$ values work.

Therefore, the correct answer is **E**.

17. Given the distinct points $P(x_1, y_1), Q(x_2, y_2)$ and $R(x_1 + x_2, y_1 + y_2)$. Line segments are drawn connecting these points to each other and to the origin O. Of the three possibilities: (1) parallelogram, (2) straight line, (3) trapezoid, figure $OPRQ$, depending upon the location of the points P, Q, and R, can be:

- A (1) only
- B (2) only
- C (3) only
- D (1) or (2) only
- E all three

Solution:

The relation $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{OQ}$ makes opposite sides of $OPRQ$ parallel and equal whenever P and Q are not collinear, so the figure is a parallelogram. If the two vectors are dependent, all four points lie on a straight line. It cannot be a genuine trapezoid.

Thus, the correct answer is **D**.

18. Let n be the number of pairs of values of b and c such that $3x + by + c = 0$ and $cx - 2y + 12 = 0$ have the same graph. Then n is:

A 0

B 1

C 2

D finite but more than 2

E greater than any finite number

Solution:

For the same graph, $(c, -2, 12) = k(3, b, c)$. Thus $c = 3k$ and $12 = kc = 3k^2$, so $k = \pm 2$. Each value determines one pair through $b = -\frac{2}{k}$ and $c = 3k$. Hence there are two pairs.

Therefore, the correct answer is **C**.

19. If $2x - 3y - z = 0$ and $x + 3y - 14z = 0, z \neq 0,$ the numerical value of $\dfrac{x^2 + 3xy}{y^2 + z^2}$ is:

A 7

B 2

C 0

D $-\dfrac{20}{17}$

E -2

Solution:

Solving the equations gives $y = 3z$ and $x = 5z$. Therefore

$$\frac{x^2 + 3xy}{y^2 + z^2} = \frac{25z^2 + 45z^2}{9z^2 + z^2} = 7.$$

Thus, the correct answer is **A**.

20. The sum of the numerical coefficients of all the terms in the expansion of $(x-2y)^{18}$ is:

A 0

B 1

C 19

D -1

E -19

Solution:

Set $x=y=1$. The sum of all numerical coefficients is then

$$(1-2)^{18}=(-1)^{18}=1.$$

Therefore, the correct answer is **B**.

21. If $\log_{b^2} x + \log_{x^2} b = 1$, where $b > 0, b \neq 1$, and $x \neq 1$, then x equals:

A $\dfrac{1}{b^2}$

B $\dfrac{1}{b}$

C b^2

D b

E $\sqrt{b}$

Solution:

Let $t = \log_b x$. Then $\log_{b^2} x = \frac{t}{2}$ and $\log_{x^2} b = \frac{1}{2t}$. Thus

$$t + \frac{1}{t} = 2,$$

so $(t-1)^2 = 0$ and $t = 1$. Hence $x = b$.

Therefore, the correct answer is **D**.

22. Given parallelogram $ABCD$ with E the midpoint of diagonal BD. Point E is connected to a point F in DA so that $DF = \frac{1}{3}DA$. What is the ratio of the area of triangle DFE to the area of quadrilateral $ABEF$?

A $1:2$

B $1:3$

C $1:5$

D $1:6$

E $1:7$

Solution:

Let the parallelogram's area be $S = |u \times v|$, with $A = 0,\ B = u,\ D = v$. Then $E = \frac{u+v}{2}$ and $F = \frac{2v}{3}$. Determinants give $[DFE] = \frac{S}{12}$. For quadrilateral $ABEF$, the two determinant contributions give

$$[ABEF] = \frac{S}{4} + \frac{S}{6} = \frac{5S}{12}.$$

The desired ratio is $1:5$.

Therefore, the correct answer is **C**.

23. Two numbers are such that their difference, their sum, and their product are to one another as $1 : 7 : 24$. The product of the two numbers is:

- A 6
- B 12
- C 24
- D 48
- E 96

Solution:

Let the difference be k and the sum be $7k$. The numbers are $4k$ and $3k$, so their product is $12k^2$. But the ratio also says the product is $24k$. For distinct numbers $k \neq 0$, so $k = 2$ and the product is 48.

Therefore, the correct answer is **D**.

24. Let $y = (x-a)^2 + (x-b)^2, a, b$ constants. For what value of x is y a minimum?

A $\dfrac{a+b}{2}$

B $a+b$

C $\sqrt{ab}$

D $\sqrt{\dfrac{a^2+b^2}{2}}$

E $\dfrac{a+b}{2ab}$

Solution:

Expanding and completing the square gives

$$y = 2\left(x - \frac{a+b}{2}\right)^2 + \frac{(a-b)^2}{2}.$$

The squared term is minimized at $x = \frac{a+b}{2}$.

Therefore, the correct answer is **A**.

25. The set of values of m for which $x^2 + 3xy + x + my - m$ has two factors, with integer coefficients, which are linear in x and y, is precisely:

A $0, 12, -12$

B $0, 12$

C $12, -12$

D 12

E 0

Solution:

Write the factorization as $(x + ay + b)(x + cy + d)$. Comparing the y^2 and xy coefficients gives $ac = 0$ and $a + c = 3$, so take $a = 0,\ c = 3$. The other coefficients give

$$b + d = 1, \qquad bd = -m,$$
$$3b = m.$$

Hence $b(d + 3) = 0$. If $b = 0$, then $m = 0$; otherwise $d = -3,\ b = 4$, and $m = 12$. Both values produce valid integer factorizations.

Therefore, the correct answer is **B**.

26. In a ten-mile race First beats Second by 2 miles and First beats Third by 4 miles. If the runners maintain constant speeds throughout the race, by how many miles does Second beat Third?

A 2

B $2\frac{1}{4}$

C $2\frac{1}{2}$

D $2\frac{3}{4}$

E 3

Solution:

Relative to First's speed, Second's speed is $\frac{8}{10} = \frac{4}{5}$ and Third's is $\frac{6}{10} = \frac{3}{5}$. Thus Third runs $\frac{3}{4}$ as fast as Second. When Second finishes 10 miles, Third has run $10(\frac{3}{4}) = 7.5$ miles, so Second wins by 2.5 miles.

Therefore, the correct answer is **C**.

27. If x is a real number and $|x-4|+|x-3|<a$, where $a>0$, then:

A $0<a<0.01$

B $0.01<a<1$

C $0<a<1$

D $0<a\leq 1$

E $a>1$

Solution:

By the triangle inequality,

$$|x-4|+|x-3|\geq|4-3|=1.$$

Equality holds for every x between 3 and 4. Therefore the strict inequality has a real solution exactly when $a>1$.

Thus, the correct answer is **E**.

28. The sum of n terms of an arithmetic progression is 153, and the common difference is 2. If the first term is an integer, and $n > 1$, then the number of possible values for n is:

A 2

B 3

C 4

D 5

E 6

Solution:

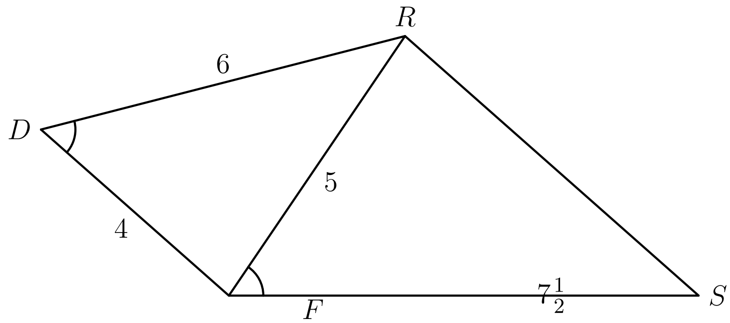
The arithmetic-series formula simplifies to

$$153 = n(a + n - 1).$$

Thus n must divide 153, and every such n gives an integer a. The divisors greater than 1 are $3, 9, 17, 51, 153,$ so there are five possibilities.

Therefore, the correct answer is **D**.

29. In this figure $\angle RFS = \angle FDR, FD = 4$ inches, $DR = 6$ inches, $FR = 5$ inches, and $FS = 7\frac{1}{2}$ inches. The length of $RS,$ in inches, is:



A undetermined

B 4

C $5\frac{1}{2}$

D 6

E $6\frac{1}{4}$

Solution:

At the equal included angles,

$$\frac{FR}{FD} = \frac{5}{4},$$
$$\frac{FS}{DR} = \frac{7.5}{6} = \frac{5}{4}.$$

Thus $\triangle RFS \sim \triangle FDR$ by SAS. Side RS corresponds to $FR = 5,$ so $RS = (\frac{5}{4}) \cdot 5 = \frac{25}{4} = 6\frac{1}{4}.$

Therefore, the correct answer is **E**.

30. If

$$(7+4\sqrt{3})x^2+(2+\sqrt{3})x \\ -2=0,$$

the larger root minus the smaller root is:

A $-2+3\sqrt{3}$

B $2-\sqrt{3}$

C $6+3\sqrt{3}$

D $6-3\sqrt{3}$

E $3\sqrt{3}+2$

Solution:

Let $A=7+4\sqrt{3}=(2+\sqrt{3})^2$ and $B=2+\sqrt{3}$. The discriminant is

$$B^2-4A(-2)=A+8A=9A.$$

Hence the root difference is

$$\frac{\sqrt{9A}}{A}=\frac{3}{2+\sqrt{3}}=6-3\sqrt{3}.$$

Therefore, the correct answer is **D**.

31. Let

$$f(n) = \frac{5+3\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-3\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

Then $f(n+1) - f(n-1)$, expressed in terms of $f(n)$, equals:

A $\frac{1}{2}f(n)$

B $f(n)$

C $2f(n)+1$

D $f^2(n)$

E $\frac{1}{2}\left(f^2(n)-1\right)$

Solution:

Each base $r = \frac{1\pm\sqrt{5}}{2}$ satisfies $r^2 = r+1$, so $r^2 - 1 = r$. Therefore

$$r^{n+1} - r^{n-1} = r^{n-1}(r^2-1) = r^n.$$

Applying this identity term by term to the defining linear combination gives $f(n+1) - f(n-1) = f(n)$.

Thus, the correct answer is **B**.

32. If $\dfrac{a+b}{b+c} = \dfrac{c+d}{d+a}$, then:

A a must equal c

B $a+b+c+d$ must equal zero

C either $a=c$ or $a+b+c+d=0$, or both

D $a+b+c+d \neq 0$ if $a=c$

E $a(b+c+d) = c(a+b+d)$

Solution:

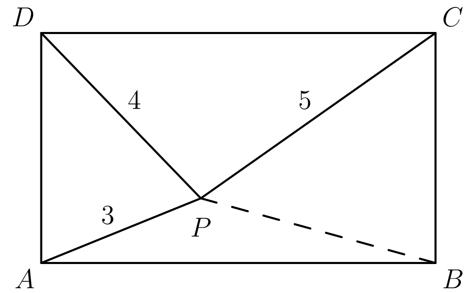
Cross-multiplication gives $(a+b)(a+d) = (b+c)(c+d)$. Subtracting the right side and factoring yields

$$(a-c)(a+b+c+d) = 0.$$

Hence either $a=c$ or the sum is zero, and both may occur.

Therefore, the correct answer is **C**.

33. P is a point interior to rectangle $ABCD$ and such that $PA = 3$ inches, $PD = 4$ inches, and $PC = 5$ inches. Then PB, in inches, equals:



A $2\sqrt{3}$

B $3\sqrt{2}$

C $3\sqrt{3}$

D $4\sqrt{2}$

E 2

Solution:

For any point in a rectangle, the British flag theorem gives

$$PA^2 + PC^2 = PB^2 + PD^2.$$

Thus $9 + 25 = PB^2 + 16$, so $PB^2 = 18$ and $PB = 3\sqrt{2}$.

Therefore, the correct answer is **B**.

34. If n is a multiple of 4, the sum

$$s = 1 + 2i + 3i^2 + \cdots \\ + (n+1)i^n,$$

where $i = \sqrt{-1}$, equals:

A $1 + i$

B $\frac{1}{2}(n+2)$

C $\frac{1}{2}(n+2-ni)$

D $\frac{1}{2}\left[(n+1)(1-i)+2\right]$

E $\frac{1}{8}(n^2+8-4ni)$

Solution:

Write $n = 4m$. For $j = 0, \ldots, m-1$, the four terms with exponents $4j$ through $4j + 3$ sum to

$$\begin{aligned}&(4j+1) + (4j+2)i \\ &\quad - (4j+3) - (4j+4)i \\ &= -2 - 2i.\end{aligned}$$

The final term is $4m + 1$. Therefore

$$\begin{aligned}s &= m(-2-2i) + (4m+1) \\ &= \frac{n+2-ni}{2}.\end{aligned}$$

Thus, the correct answer is **C**.

35. The sides of a triangle are of lengths $13, 14$, and 15. The altitudes of the triangle meet at point H. If AD is the altitude to the side of length 14, what is the ratio $HD : HA$?

A $3 : 11$

B $5 : 11$

C $1 : 2$

D $2 : 3$

E $25 : 33$

Solution:

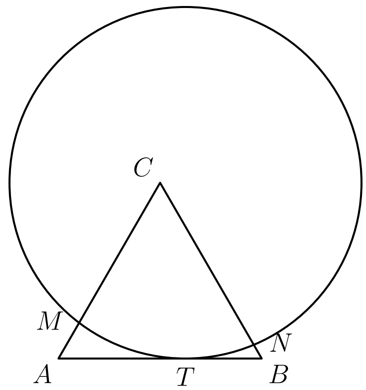
Heron's formula gives area 84, so $AD = \frac{2(84)}{14} = 12$. The adjacent 13-side has projection $\sqrt{13^2 - 12^2} = 5$, leaving 9 on the base. Put $D = (0,0),\ A = (0,12),\ B = (-5,0),\ C = (9,0)$. Line AC has slope $-\frac{4}{3}$, so the altitude through B has slope $\frac{3}{4}$ and meets AD at height $\frac{15}{4}$. Thus

$$HD = \frac{15}{4},$$
$$HA = 12 - \frac{15}{4} = \frac{33}{4},$$

giving $HD : HA = 5 : 11$.

Therefore, the correct answer is **B**.

36. In this figure the radius of the circle is equal to the altitude of the equilateral triangle ABC. The circle is made to roll along the side AB, remaining tangent to it at a variable point T and intersecting lines AC and BC in variable points M and N, respectively. Let n be the number of degrees in arc MTN. Then n, for all permissible positions of the circle:



A varies from $30°$ to $90°$

B varies from $30°$ to $60°$

C varies from $60°$ to $90°$

D remains constant at $30°$

E remains constant at $60°$

Solution:

Let O be the circle's center. Both O and C are one triangle altitude above AB, so $CO \parallel AB$. Extend NC through C to meet the circle again at D. Since CD is opposite to CN,

$$\begin{aligned}\angle MCD &= 180° - \angle MCN \\ &= 120°.\end{aligned}$$

The line CO, parallel to AB, bisects this angle. Reflection across CO fixes the circle and interchanges rays CM and CD, so it interchanges M and D. Hence $CM = CD$, and isosceles triangle MCD has base angles $30°$.

Because D, C, N are collinear, $\angle MDN = 30^\circ$. This inscribed angle subtends arc MTN, whose measure is therefore 60° for every permissible circle position.

Thus, the correct answer is **E**.

37. Given two positive numbers a, b such that $a < b$. Let A.M. be their arithmetic mean and let G.M. be their positive geometric mean. Then A.M. minus G.M. is always less than:

A $\dfrac{(b+a)^2}{ab}$

B $\dfrac{(b+a)^2}{8b}$

C $\dfrac{(b-a)^2}{ab}$

D $\dfrac{(b-a)^2}{8a}$

E $\dfrac{(b-a)^2}{8b}$

Solution:

We have

$$\frac{a+b}{2} - \sqrt{ab} = \frac{(\sqrt{b} - \sqrt{a})^2}{2}.$$

After canceling the positive factor $(\sqrt{b} - \sqrt{a})^2$, comparison with choice D reduces to

$$\frac{1}{2} < \frac{(\sqrt{b} + \sqrt{a})^2}{8a}.$$

This is true because $\sqrt{b} + \sqrt{a} > 2\sqrt{a}$. Thus A.M. minus G.M. is always less than the expression in choice D.

Therefore, the correct answer is **D**.

38. The sides PQ and PR of triangle PQR are respectively of lengths 4 inches and 7 inches. The median PM is $3\frac{1}{2}$ inches. Then QR, in inches, is:

- A 6
- B 7
- C 8
- D 9
- E 10

Solution:

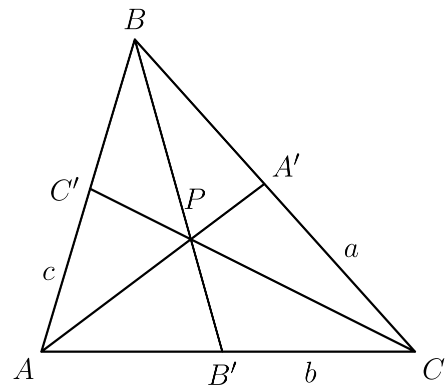
Apollonius's theorem gives

$$\begin{aligned} QR^2 &= 2(4^2+7^2) - 4\left(\frac{7}{2}\right)^2 \\ &= 81. \end{aligned}$$

Therefore $QR = 9.$

Thus, the correct answer is **D**.

39. The magnitudes of the sides of triangle ABC are a, b, c, as shown, with $c \leq b \leq a$. Through interior point P and the vertices A, B, C, lines are drawn meeting the opposite sides in A', B', C', respectively. Let $s = AA' + BB' + CC'$. Then, for all positions of point P, s is less than:



A $2a + b$

B $2a + c$

C $2b + c$

D $a + 2b$

E $a + b + c$

Solution:

For a fixed vertex, squared distance is a convex function along the opposite side, so its maximum occurs at an endpoint. Since each cevian endpoint is interior to its side,

$$AA' < \max(AB, AC) = b,$$
$$BB' < \max(BA, BC) = a,$$
$$CC' < \max(CA, CB) = a.$$

Adding gives $s < 2a + b$.

Therefore, the correct answer is **A**.

40. A watch loses $2\frac{1}{2}$ minutes per day. It is set right at 1 P.M. on March $15.$ Let n be the positive correction, in minutes, to be added to the time shown by the watch at a given time. When the watch shows 9 A.M. on March $21,$ n equals:

A $14\frac{14}{23}$

B $14\frac{1}{14}$

C $13\frac{101}{115}$

D $13\frac{83}{115}$

E $13\frac{13}{23}$

Solution:

The watch runs at $\frac{1437.5}{1440} = \frac{575}{576}$ of the correct rate. The displayed elapsed time is 5 days 20 hours, or 8400 minutes. Thus the real elapsed time is $8400(\frac{576}{575})$, and the correction is

$$\begin{aligned} n &= 8400\left(\frac{576}{575} - 1\right) \\ &= \frac{8400}{575} = \frac{336}{23} \\ &= 14\frac{14}{23}. \end{aligned}$$

Therefore, the correct answer is **A**.

Problems: https://live.poshenloh.com/past-contests/amc12/1964