1963 AMC 12

Time limit: 75 minutes

Typeset by: LIVE by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc12/1963

1. Which one of the following points is *not* on the graph of $y = \dfrac{x}{x+1}$?

A $(0,0)$

B $\left(-\dfrac{1}{2}, -1\right)$

C $\left(\dfrac{1}{2}, \dfrac{1}{3}\right)$

D $(-1,1)$

E $(-2,2)$

2. Let $n = x - y^{x-y}$. Find n when $x = 2$ and $y = -2$.

A -14

B 0

C 1

D 18

E 256

3. If the reciprocal of $x+1$ is $x-1$, then x equals:

- A 0
- B 1
- C -1
- D ± 1
- E none of these

4. For what value(s) of k does the pair of equations $y=x^2$ and $y=3x+k$ have two identical solutions?

- A $\frac{4}{9}$
- B $-\frac{4}{9}$
- C $\frac{9}{4}$
- D $-\frac{9}{4}$
- E $\frac{9}{4} \text{ or } -\frac{9}{4}$

5. If x and $\log_{10} x$ are real numbers and $\log_{10} x < 0$, then:

- A $x < 0$
- B $-1 < x < 1$
- C $0 < x \leq 1$
- D $-1 < x < 0$
- E $0 < x < 1$

6. Triangle BAD is right-angled at B. On AD there is a point C for which $AC = CD$ and $AB = BC$. The magnitude of angle DAB, in degrees, is:

- A $67\frac{1}{2}$
- B 60
- C 45
- D 30
- E $22\frac{1}{2}$

7. Given the four equations:

$$\begin{aligned} &(1) \quad 3y - 2x = 12, \\ &(2) \quad -2x - 3y = 10, \\ &(3) \quad 3y + 2x = 12, \\ &(4) \quad 2y + 3x = 10. \end{aligned}$$

The pair representing perpendicular lines is:

A (1) and (4)

B (1) and (3)

C (1) and (2)

D (2) and (4)

E (2) and (3)

8. The smallest positive integer x for which $1260x = N^3$, where N is an integer, is:

A 1050

B 1260

C 1260^2

D 7350

E $44{,}100$

9. In the expansion of $\left(a-\dfrac{1}{\sqrt{a}}\right)^7$ the coefficient of $a^{-\frac{1}{2}}$ is:

A -7

B 7

C -21

D 21

E 35

10. Point P is taken interior to a square with side-length a and such that it is equally distant from two consecutive vertices and from the side opposite these vertices. If d represents the common distance, then d equals:

A $\dfrac{3a}{5}$

B $\dfrac{5a}{8}$

C $\dfrac{3a}{8}$

D $\dfrac{a\sqrt{2}}{2}$

E $\dfrac{a}{2}$

11. The arithmetic mean of a set of 50 numbers is $38.$ If two numbers of the set, namely 45 and $55,$ are discarded, the arithmetic mean of the remaining set of numbers is:

A 38.5

B 37.5

C 37

D 36.5

E 36

12. Three vertices of parallelogram $PQRS$ are $P(-3,-2), Q(1,-5), R(9,1)$ with P and R diagonally opposite. The sum of the coordinates of vertex S is:

A 13

B 12

C 11

D 10

E 9

13. If $2^a + 2^b = 3^c + 3^d$, the number of integers a, b, c, d which can possibly be negative is, at most:

A 4

B 3

C 2

D 1

E 0

14. Given the equations $x^2 + kx + 6 = 0$ and $x^2 - kx + 6 = 0$. If, when the roots of the equations are suitably listed, each root of the second equation is 5 more than the corresponding root of the first equation, then k equals:

A 5

B -5

C 7

D -7

E none of these

15. A circle is inscribed in an equilateral triangle, and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is:

A $\sqrt{3} : 1$

B $\sqrt{3} : \sqrt{2}$

C $3\sqrt{3} : 2$

D $3 : \sqrt{2}$

E $3 : 2\sqrt{2}$

16. Three numbers a, b, c, none zero, form an arithmetic progression. Increasing a by 1 or increasing c by 2 results in a geometric progression. Then b equals:

A 16

B 14

C 12

D 10

E 8

17. The expression

$$\frac{\dfrac{a}{a+y}+\dfrac{y}{a-y}}{\dfrac{y}{a+y}-\dfrac{a}{a-y}},$$

a real, $a \neq 0,$ has the value -1 for:

A all but two real values of y

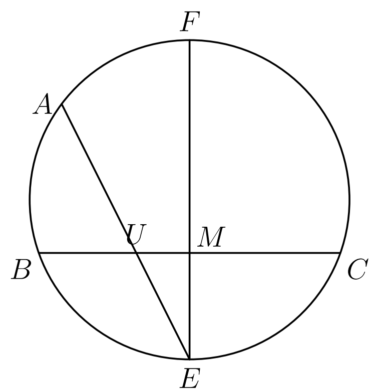
B only two real values of y

C all real values of y

D only one real value of y

E no real values of y

18. Chord EF is the perpendicular bisector of chord BC, intersecting it in M. Between B and M point U is taken, and EU extended meets the circle in A. Then, for any selection of U, as described, triangle EUM is similar to triangle:



A EFA

B EFC

C ABM

D ABU

E FMC

19. In counting n colored balls, some red and some black, it was found that 49 of the first 50 counted were red. Thereafter, 7 out of every 8 counted were red. If, in all, 90% or more of the balls counted were red, the maximum value of n is:

A 225

B 210

C 200

D 180

E 175

20. Two men at points R and S, 76 miles apart, set out at the same time to walk towards each other. The man at R walks uniformly at $4\frac{1}{2}$ miles per hour; the man at S walks at $3\frac{1}{4}$ miles per hour for the first hour, at $3\frac{3}{4}$ miles per hour for the second hour, and so on, in arithmetic progression. If the men meet x miles nearer R than S in an integral number of hours, then x is:

A 10

B 8

C 6

D 4

E 2

21. The expression $x^2 - y^2 - z^2 + 2yz + x + y - z$ has:

A no linear factor with integer coefficients and integer exponents

B the factor $-x + y + z$

C the factor $x - y - z + 1$

D the factor $x + y - z + 1$

E the factor $x - y + z + 1$

22. Acute-angled triangle ABC is inscribed in a circle with center at O; $\overset{\frown}{AB} = 120^\circ$ and $\overset{\frown}{BC} = 72^\circ$. A point E is taken in minor arc AC such that OE is perpendicular to AC. Then the ratio of the magnitudes of angles OBE and BAC is:

A $\frac{5}{18}$

B $\frac{2}{9}$

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{4}{9}$

23. A gives B as many cents as B has and C as many cents as C has. Similarly, B then gives A and C as many cents as each then has. C, similarly, then gives A and B as many cents as each then has. If each finally has 16 cents, with how many cents does A start?

A 24

B 26

C 28

D 30

E 32

24. Consider equations of the form $x^2 + bx + c = 0$. How many such equations have real roots and have coefficients b and c selected from the set of integers $\{1, 2, 3, 4, 5, 6\}$?

A 20

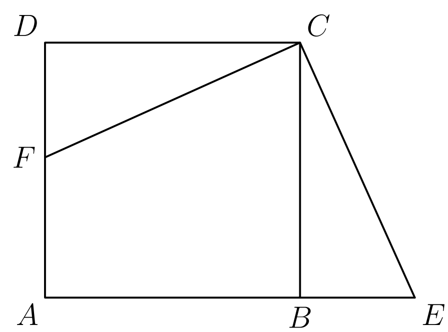
B 19

C 18

D 17

E 16

25. Point F is taken in side AD of square $ABCD$. At C a perpendicular is drawn to CF, meeting AB extended at E. The area of $ABCD$ is 256 square inches and the area of triangle CEF is 200 square inches. Then the number of inches in BE is:



A 12

B 14

C 15

D 16

E 20

26. Form I. Consider the statements

$$\begin{array}{ll} (1) & p \wedge \neg q \wedge r, \\ (2) & \neg p \wedge \neg q \wedge r, \\ (3) & p \wedge \neg q \wedge \neg r, \\ (4) & \neg p \wedge q \wedge r, \end{array}$$

where p, q, r are propositions. How many of these imply the truth of $(p \rightarrow q) \rightarrow r$?

Form II. Equivalently, test the four corresponding truth assignments against the statement "r is implied by the statement that p implies q."

A 0

B 1

C 2

D 3

E 4

27. Six straight lines are drawn in a plane with no two parallel and no three concurrent. The number of regions into which they divide the plane is:

A 16

B 20

C 22

D 24

E 26

28. Given the equation $3x^2 - 4x + k = 0$ with real roots. The value of k for which the product of the roots of the equation is a maximum is:

A $\frac{16}{9}$

B $\frac{16}{3}$

C $\frac{4}{9}$

D $\frac{4}{3}$

E $-\frac{4}{3}$

29. A particle projected vertically upward reaches, at the end of t seconds, an elevation of s feet where $s = 160t - 16t^2$. The highest elevation is:

A 800

B 640

C 400

D 320

E 160

30. Let

$$F = \log \frac{1+x}{1-x}.$$

Find a new function G by replacing each x in F by

$$\frac{3x + x^3}{1 + 3x^2},$$

and simplify. The simplified expression G is equal to:

A $-F$

B F

C $3F$

D F^3

E $F^3 - F$

31. The number of solutions in positive integers of $2x + 3y = 763$ is:

A 255

B 254

C 128

D 127

E 0

32. The dimensions of a rectangle R are a and $b, a < b$. It is required to obtain a rectangle with dimensions x and $y, x < a, y < a$, so that its perimeter is one-third that of R, and its area is one-third that of R. The number of such (different) rectangles is:

A 0

B 1

C 2

D 4

E infinitely many

33. Given the line $y = \frac{3}{4}x + 6$ and a line L parallel to the given line and 4 units from it. A possible equation for L is:

A $y = \frac{3}{4}x + 1$

B $y = \frac{3}{4}x$

C $y = \frac{3}{4}x - \frac{2}{3}$

D $y = \frac{3}{4}x - 1$

E $y = \frac{3}{4}x + 2$

34. In triangle ABC, side $a = \sqrt{3}$, side $b = \sqrt{3}$, and side $c > 3$. Let x be the largest number such that the magnitude, in degrees, of the angle opposite side c exceeds x. Then x equals:

A 150

B 120

C 105

D 90

E 60

35. The lengths of the sides of a triangle are integers, and its area is also an integer. One side is 21 and the perimeter is 48. The shortest side is:

A 8

B 10

C 12

D 14

E 16

36. A person starting with 64 cents and making 6 bets, wins three times and loses three times, the wins and losses occurring in random order. The chance for a win is equal to the chance for a loss. If each wager is for half the money remaining at the time of the bet, then the final result is:

A a loss of $27¢$

B a gain of $27¢$

C a loss of $37¢$

D neither a gain nor a loss

E a gain or a loss depending upon the order in which the wins and losses occur

37. Given points $P_1, P_2, \ldots, P_7$ on a straight line, in the order stated (not necessarily evenly spaced). Let P be an arbitrarily selected point on the line and let s be the sum of the undirected lengths

$$PP_1,\ PP_2,\ \ldots,\ PP_7.$$

Then s is smallest if and only if the point P is:

A midway between P_1 and P_7

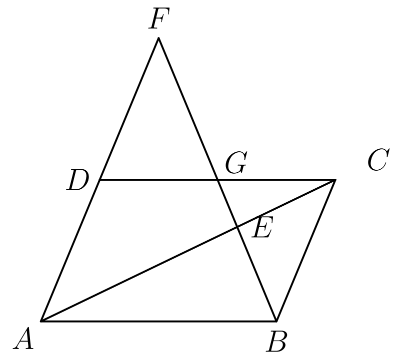
B midway between P_2 and P_6

C midway between P_3 and P_5

D at P_4

E at P_1

38. Point F is taken on the extension of side AD of parallelogram $ABCD$. BF intersects diagonal AC at E and side DC at G. If $EF = 32$ and $GF = 24$, then BE equals:



A 4

B 8

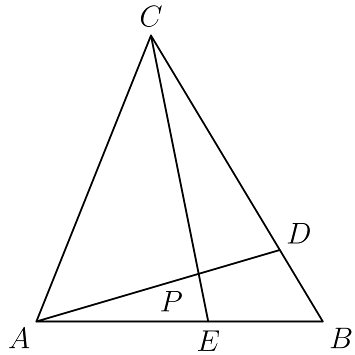
C 10

D 12

E 16

39. In triangle ABC lines CE and AD are drawn so that $\frac{CD}{DB} = \frac{3}{1}$ and $\frac{AE}{EB} = \frac{3}{2}$.

Let $r = \frac{CP}{PE}$, where P is the intersection point of CE and AD. Then r equals:



A 3

B $\frac{3}{2}$

C 4

D 5

E $\frac{5}{2}$

40. If x is a number satisfying the equation $\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3$, then x^2 is between:

A 55 and 65

B 65 and 75

C 75 and 85

D 85 and 95

E 95 and 105

Solutions: https://live.poshenloh.com/past-contests/amc12/1963/solutions