# 1962 AMC 12 Solutions

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**1.** The expression $\dfrac{1^{4y-1}}{5^{-1}+3^{-1}}$ is equal to:

A $\dfrac{4y-1}{8}$

B $8$

C $\dfrac{15}{2}$

D $\dfrac{15}{8}$

E $\dfrac{1}{8}$

## Solution:

The numerator is $1$, while

$$5^{-1}+3^{-1}=\frac{1}{5}+\frac{1}{3}=\frac{8}{15}.$$

Therefore the expression equals $\frac{1}{\frac{8}{15}}=\frac{15}{8}$.

Thus, the correct answer is **D**.

**2.** The expression $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}}$ is equal to:

A $\frac{\sqrt{3}}{6}$

B $-\frac{\sqrt{3}}{6}$

C $\frac{\sqrt{-3}}{6}$

D $\frac{5\sqrt{3}}{6}$

E $1$

## Solution:

We have

$$\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} = \frac{4-3}{2\sqrt{3}} = \frac{\sqrt{3}}{6}.$$

Therefore, the correct answer is **A**.

**3.** The first three terms of an arithmetic progression are $x - 1, x + 1, 2x + 3,$ in the order shown. The value of $x$ is:

A $-2$

B $0$

C $2$

D $4$

E undetermined

## Solution:

Equality of consecutive differences gives

$$2 = (2x + 3) - (x + 1) = x + 2,$$

so $x = 0.$

Thus, the correct answer is **B**.

**4.** If $8^x = 32$, then $x$ equals:

A $4$

B $\frac{5}{3}$

C $\frac{3}{2}$

D $\frac{3}{5}$

E $\frac{1}{4}$

## Solution:

Since $8 = 2^3$ and $32 = 2^5$, the equation becomes $2^{3x} = 2^5$. Hence $3x = 5$ and $x = \frac{5}{3}$.

Therefore, the correct answer is **B**.

**5.** If the radius of a circle is increased by $1$ unit, the ratio of the new circumference to the new diameter is:

A $\pi + 2$

B $\dfrac{2\pi + 1}{2}$

C $\pi$

D $\dfrac{2\pi - 1}{2}$

E $\pi - 2$

## Solution:

For every circle, regardless of its radius,

$$\frac{\text{circumference}}{\text{diameter}} = \frac{2\pi R}{2R} = \pi.$$

Thus, the correct answer is **C**.

**6.** A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ square inches. Expressed in inches the diagonal of the square is:

A $\frac{9}{2}$

B $2\sqrt{5}$

C $4\sqrt{2}$

D $\frac{9\sqrt{2}}{2}$

E none of these

## Solution:

If $s$ is the triangle's side, then

$$\frac{\sqrt{3}}{4}s^2 = 9\sqrt{3},$$

so $s = 6$. Its perimeter is $18$, making the square's side $\frac{18}{4} = \frac{9}{2}$. The square's diagonal is therefore $\frac{9\sqrt{2}}{2}$.

Thus, the correct answer is **D**.

**7.** Let the bisectors of the exterior angles at $B$ and $C$ of triangle $ABC$ meet at $D$. Then, if all measurements are in degrees, angle $BDC$ equals:

A $\frac{1}{2}(90-A)$

B $90-A$

C $\frac{1}{2}(180-A)$

D $180-A$

E $180-2A$

## Solution:

The angles of triangle $BDC$ at $B$ and $C$ are $90^\circ - \frac{B}{2}$ and $90^\circ - \frac{C}{2}$. Thus

$$\begin{aligned}\angle BDC &= 180^\circ - \left(90^\circ - \frac{B}{2}\right) \\ &\quad - \left(90^\circ - \frac{C}{2}\right) \\ &= \frac{B+C}{2} \\ &= \frac{180^\circ - A}{2}.\end{aligned}$$

Therefore, the correct answer is **C**.

**8.** Given the set of $n$ numbers, $n > 1$, of which one is $1 - \frac{1}{n}$ and all the others are $1$. The arithmetic mean of the $n$ numbers is:

A $1$

B $n - \frac{1}{n}$

C $n - \frac{1}{n^2}$

D $1 - \frac{1}{n^2}$

E $1 - \frac{1}{n} - \frac{1}{n^2}$

## Solution:

The sum is

$$(n-1) + \left(1 - \frac{1}{n}\right) = n - \frac{1}{n}.$$

Dividing by $n$ gives $1 - \frac{1}{n^2}$.

Thus, the correct answer is **D**.

**9.** When $x^9 - x$ is factored as completely as possible into polynomials and monomials with real integral coefficients, the number of factors is:

- A more than $5$
- B $5$
- C $4$
- D $3$
- E $2$

## Solution:

Factoring over the integers gives

$$x^9 - x = x(x-1)(x+1) \\ \cdot (x^2+1)(x^4+1).$$

The last two nonconstant factors are irreducible over the integers, so there are $5$ factors.

Thus, the correct answer is **B**.

**10.** A man drives $150$ miles to the seashore in $3$ hours and $20$ minutes. He returns from the shore to the starting point in $4$ hours and $10$ minutes. Let $r$ be the average rate for the entire trip. Then the average rate for the trip going exceeds $r$, in miles per hour, by:

A $5$

B $4\frac{1}{2}$

C $4$

D $2$

E $1$

## Solution:

The outbound rate is $\frac{150}{\frac{10}{3}} = 45$ miles per hour. The entire trip covers $300$ miles in

$$3\frac{1}{3} + 4\frac{1}{6} = 7\frac{1}{2}$$

hours, so $r = \frac{300}{\frac{15}{2}} = 40$. The difference is $45 - 40 = 5.$

Thus, the correct answer is **A**.

**11.** The difference between the larger root and the smaller root of

$$x^2 - px + \frac{p^2 - 1}{4} = 0$$

is:

A $0$

B $1$

C $2$

D $p$

E $p + 1$

## Solution:

The discriminant is

$$p^2 - 4\left(\frac{p^2 - 1}{4}\right) = 1.$$

The roots are $\frac{p+1}{2}$ and $\frac{p-1}{2}$, whose difference is $1$.

Therefore, the correct answer is **B**.

**12.** When $\left(1-\frac{1}{a}\right)^6$ is expanded the sum of the last three coefficients is:

- A $22$
- B $11$
- C $10$
- D $-10$
- E $-11$

## Solution:

The last three terms have coefficients

$$\binom{6}{4}, \quad -\binom{6}{5}, \quad \binom{6}{6},$$

so their sum is $15-6+1=10$.

Thus, the correct answer is **C**.

**13.** $R$ varies directly as $S$ and inversely as $T$. When $R = \dfrac{4}{3}$ and $T = \dfrac{9}{14}$, $S = \dfrac{3}{7}$. Find $S$ when $R = \sqrt{48}$ and $T = \sqrt{75}$.

- A $28$
- B $30$
- C $40$
- D $42$
- E $60$

## Solution:

Because $R = \frac{kS}{T}$, we have $S = \frac{RT}{k}$. Hence

$$\begin{aligned}\frac{S_2}{S_1} &= \frac{R_2 T_2}{R_1 T_1} \\ &= \frac{\sqrt{48}\sqrt{75}}{(\frac{4}{3})(\frac{9}{14})} \\ &= \frac{60}{\frac{6}{7}} = 70.\end{aligned}$$

Therefore $S_2 = (\frac{3}{7}) \cdot 70 = 30.$

Thus, the correct answer is **B**.

**14.** Let $s$ be the limiting sum of the geometric series $4-\frac{8}{3}+\frac{16}{9}-\cdots$, as the number of terms increases without bound. Then $s$ equals:

A a number between $0$ and $1$

B $2.4$

C $2.5$

D $3.6$

E $12$

## Solution:

The first term is $4$ and the common ratio is $-\frac{2}{3}$. Thus

$$s=\frac{4}{1-(-\frac{2}{3})}=\frac{12}{5}=2.4.$$

Therefore, the correct answer is **B**.

**15.** Given triangle $ABC$ with base $AB$ fixed in length and position. As the vertex $C$ moves on a straight line, the intersection point of the three medians moves on:

- A a circle
- B a parabola
- C an ellipse
- D a straight line
- E a curve here not listed

## Solution:

Let $A, B, C$ also denote their position vectors. The centroid is

$$G = \frac{A + B + C}{3}.$$

Since $A$ and $B$ are fixed, this is a translation and scaling by $\frac{1}{3}$ of the position of $C$. A straight-line locus therefore maps to a straight-line locus.

Thus, the correct answer is **D**.

**16.** Given rectangle $R_1$ with one side $2$ inches and area $12$ square inches. Rectangle $R_2$ with diagonal $15$ inches is similar to $R_1$. Expressed in square inches the area of $R_2$ is:

A $\frac{9}{2}$

B $36$

C $\frac{135}{2}$

D $9\sqrt{10}$

E $\frac{27\sqrt{10}}{4}$

## Solution:

The diagonal of $R_1$ is $\sqrt{2^2+6^2}=2\sqrt{10}$. Therefore

$$\frac{[R_2]}{[R_1]}=\left(\frac{15}{2\sqrt{10}}\right)^2=\frac{45}{8}.$$

Thus $[R_2]=12(\frac{45}{8})=\frac{135}{2}$.

Therefore, the correct answer is **C**.

**17.** If $a = \log_8 225$ and $b = \log_2 15$, then $a$, in terms of $b$, is:

A $\frac{b}{2}$

B $\frac{2b}{3}$

C $b$

D $\frac{3b}{2}$

E $2b$

## Solution:

Changing to base $2$ gives

$$\begin{aligned} a &= \frac{\log_2(15^2)}{\log_2(2^3)} \\ &= \frac{2\log_2 15}{3} = \frac{2b}{3}. \end{aligned}$$

Thus, the correct answer is **B**.

**18.** A regular dodecagon ($12$ sides) is inscribed in a circle with radius $r$ inches. The area of the dodecagon, in square inches, is:

- A $3r^2$
- B $2r^2$
- C $\frac{3r^2\sqrt{3}}{4}$
- D $r^2\sqrt{3}$
- E $3r^2\sqrt{3}$

## Solution:

Each of the $12$ central triangles has area

$$\frac{1}{2}r^2 \sin 30^\circ = \frac{r^2}{4}.$$

Their total area is $12(\frac{r^2}{4}) = 3r^2$.

Thus, the correct answer is **A**.

**19.** If the parabola $y = ax^2 + bx + c$ passes through the points $(-1, 12)$, $(0, 5)$, and $(2, -3)$, the value of $a + b + c$ is:

A $-4$

B $-2$

C $0$

D $1$

E $2$

## Solution:

The point $(0, 5)$ gives $c = 5$. The other two points give

$$a - b = 7, \qquad 4a + 2b = -8.$$

Solving yields $a = 1,\ b = -6$. Hence $a + b + c = 1 - 6 + 5 = 0$.

Thus, the correct answer is **C**.

**20.** The angles of a pentagon are in arithmetic progression. One of the angles, in degrees, must be:

- A $108$
- B $90$
- C $72$
- D $54$
- E $36$

## Solution:

The average of the five angles is $\frac{540^\circ}{5} = 108^\circ$. For five terms in arithmetic progression, the middle term equals the average, so one angle must be $108^\circ$.

Thus, the correct answer is **A**.

**21.** It is given that one root of $2x^2 + rx + s = 0$, with $r$ and $s$ real numbers, is $3 + 2i$ $(i = \sqrt{-1})$. The value of $s$ is:

A undetermined

B $5$

C $6$

D $-13$

E $26$

## Solution:

The other root is $3 - 2i$. Their product is

$$(3 + 2i)(3 - 2i) = 9 + 4 = 13.$$

Vieta's formula gives $\frac{s}{2} = 13$, so $s = 26$.

Therefore, the correct answer is **E**.

**22.** The number $121_b$, written in the integral base $b$, is the square of an integer, for:

A $b = 10$, only

B $b = 10$ and $b = 5$, only

C $2 \leq b \leq 10$

D $b > 2$

E no value of (b)

## Solution:

In ordinary notation,

$$121_b = b^2 + 2b + 1 = (b+1)^2.$$

This is a square for every allowable base. Since digit $2$ occurs, precisely the integral bases $b > 2$ are allowable.

Thus, the correct answer is **D**.

**23.** In triangle $ABC, CD$ is the altitude to $AB$ and $AE$ is the altitude to $BC$. If the lengths of $AB, CD$, and $AE$ are known, the length of $DB$ is:

A not determined by the information given

B determined only if $A$ is an acute angle

C determined only if $B$ is an acute angle

D determined only if $ABC$ is an acute triangle

E none of these is correct

## Solution:

Let $AB = c,\ CD = h$, and $AE = e$. Equating two area formulas gives

$$\frac{1}{2}ch = \frac{1}{2}(BC)e,$$

so $BC = \frac{ch}{e}$. Since triangle $BCD$ is right at $D$,

$$DB = \sqrt{BC^2 - CD^2}.$$

This determines $DB$ whether the original triangle is acute, right, or obtuse, so none of choices A-D is correct.

Thus, the correct answer is **E**.

**24.** Three machines $P, Q,$ and $R,$ working together, can do a job in $x$ hours. When working alone, $P$ needs an additional $6$ hours to do the job; $Q,$ one additional hour; and $R, x$ additional hours. The value of $x$ is:

A $\frac{2}{3}$

B $\frac{11}{12}$

C $\frac{3}{2}$

D $2$

E $3$

## Solution:

The rates satisfy

$$\frac{1}{x+6} + \frac{1}{x+1} + \frac{1}{2x} = \frac{1}{x}.$$

Thus

$$\frac{1}{x+6} + \frac{1}{x+1} = \frac{1}{2x},$$

which simplifies to $3x^2 + 7x - 6 = 0$, or $(3x-2)(x+3) = 0$. A time must be positive, so $x = \frac{2}{3}$.

Therefore, the correct answer is **A**.

**25.** Given square $ABCD$ with side 8 feet. A circle is drawn through vertices $A$ and $D$ and tangent to side $BC$. The radius of the circle, in feet, is:

- A $4$
- B $4\sqrt{2}$
- C $5$
- D $5\sqrt{2}$
- E $6$

## Solution:

Place the center at $(h, 4)$. Since the circle passes through $A$,

$$r^2 = h^2 + 16.$$

Tangency to $x = 8$ gives $r = 8 - h$. Therefore $h^2 + 16 = (8 - h)^2$, so $h = 3$ and $r = 5$.

Thus, the correct answer is **C**.

**26.** For any real value of $x$ the maximum value of $8x - 3x^2$ is:

A $0$

B $\frac{8}{3}$

C $4$

D $5$

E $\frac{16}{3}$

## Solution:

Completing the square,

$$8x - 3x^2 = \frac{16}{3} - 3\left(x - \frac{4}{3}\right)^2.$$

The square term is nonnegative, so the maximum is $\frac{16}{3}$.

Thus, the correct answer is **E**.

**27.** Let $a \,@\, b$ represent the operation on two numbers, $a$ and $b$, which selects the larger of the two numbers, with $a \,@\, a = a$. Let $a \,!\, b$ represent the operation which selects the smaller of the two numbers, with $a \,!\, a = a$. Which of the following three rules is (are) correct?

$$\begin{aligned} &(1) \quad a \,@\, b = b \,@\, a, \\ &(2) \quad a \,@\, (b \,@\, c) = (a \,@\, b) \,@\, c, \\ &(3) \quad a \,!\, (b \,@\, c) \\ &\qquad\quad = (a \,!\, b) \,@\, (a \,!\, c). \end{aligned}$$

A $(1)$ only

B $(2)$ only

C $(1)$ and $(2)$ only

D $(1)$ and $(3)$ only

E all three

## Solution:

Maximum is commutative and associative, so $(1)$ and $(2)$ hold. Rule $(3)$ is the distributive identity

$$\begin{aligned} &\min(a, \max(b, c)) \\ &\quad = \max(\min(a, b), \min(a, c)). \end{aligned}$$

If $a \geq \max(b, c)$, both sides equal $\max(b, c)$; otherwise both sides equal $a$ or the larger of $b, c$ that remains below $a$. Thus $(3)$ also holds.

Therefore, the correct answer is **E**.

**28.** The set of $x$-values satisfying the equation

$$x^{\log_{10} x} = \frac{x^3}{100}$$

consists of:

A $\frac{1}{10}$, only

B $10$, only

C $100$, only

D $10$ or $100$, only

E more than two real numbers

## Solution:

Set $y = \log_{10} x$, so $x = 10^y$. Taking base-$10$ logarithms gives

$$y^2 = 3y - 2.$$

Hence $(y-1)(y-2) = 0$, so $x = 10$ or $x = 100$.

Thus, the correct answer is **D**.

**29.** Which of the following sets of $x$-values satisfy the inequality $2x^2 + x < 6$?

- A $-2 < x < \dfrac{3}{2}$
- B $x > \dfrac{3}{2}$ or $x < -2$
- C $x < \dfrac{3}{2}$
- D $\dfrac{3}{2} < x < 2$
- E $x < -2$

## Solution:

The inequality is $2x^2 + x - 6 < 0$. Factoring gives $(2x - 3)(x + 2) < 0$. The product is negative between its roots, so $-2 < x < \frac{3}{2}$.

Thus, the correct answer is **A**.

**30.** Consider the statements:

$(1)$ $p$ and $q$ are both true
$(2)$ $p$ is true and $q$ is false
$(3)$ $p$ is false and $q$ is true
$(4)$ $p$ is false and $q$ is false.

How many of these imply the negation of the statement "$p$ and $q$ are both true"?

A $0$

B $1$

C $2$

D $3$

E $4$

## Solution:

The negation of "$p$ and $q$ are both true" holds whenever at least one of $p, q$ is false. This occurs in cases $(2)$, $(3)$, and $(4)$, for a total of $3$.

Thus, the correct answer is **D**.

**31.** The ratio of the interior angles of two regular polygons with sides of unit length is $3 : 2$. How many such pairs are there?

A $1$

B $2$

C $3$

D $4$

E infinitely many

## Solution:

Let the smaller and larger polygons have $n$ and $N$ sides. Then

$$\frac{\frac{N-2}{N}}{\frac{n-2}{n}} = \frac{3}{2},$$

which gives $N = \frac{4n}{6-n}$. Positivity and $N > n$ require $n = 3, 4,$ or $5$. These give $N = 4, 8,$ and $20,$ respectively. Thus there are $3$ pairs.

Therefore, the correct answer is **C**.

**32.** If $x_{k+1} = x_k + \dfrac{1}{2}$ for $k = 1, 2, \ldots, n-1$ and $x_1 = 1$, find $x_1 + x_2 + \cdots + x_n$.

A $\dfrac{n+1}{2}$

B $\dfrac{n+3}{2}$

C $\dfrac{n^2-1}{2}$

D $\dfrac{n^2+n}{4}$

E $\dfrac{n^2+3n}{4}$

## Solution:

We have $x_n = 1 + \frac{n-1}{2} = \frac{n+1}{2}$. Therefore

$$\begin{aligned} x_1 + \cdots + x_n &= \frac{n}{2}\left(1 + \frac{n+1}{2}\right) \\ &= \frac{n^2+3n}{4}. \end{aligned}$$

Thus, the correct answer is **E**.

**33.** The set of $x$-values satisfying the inequality $2 \leq |x-1| \leq 5$ is:

A $-4 \leq x \leq -1$ or $3 \leq x \leq 6$

B $3 \leq x \leq 6$ or $-6 \leq x \leq -3$

C $x \leq -1$ or $x \geq 3$

D $-1 \leq x \leq 3$

E $-4 \leq x \leq 6$

## Solution:

For $x-1 \geq 0,$ the bounds give $3 \leq x \leq 6.$ For $x-1 \leq 0,$ they give $-4 \leq x \leq -1.$ The solution is the union of these two intervals.

Thus, the correct answer is **A**.

**34.** For what real values of $K$ does $x = K^2(x-1)(x-2)$ have real roots?

A none

B $-2 < K < 1$

C $-2\sqrt{2} < K < 2\sqrt{2}$

D $K > 1$ or $K < -2$

E all

## Solution:

Rearranging gives

$$K^2x^2 - (3K^2+1)x + 2K^2 = 0.$$

Its discriminant is

$$\begin{aligned}(3K^2+1)^2 - 8K^4 \\ = K^4 + 6K^2 + 1 > 0\end{aligned}$$

for every real $K$. This also covers $K = 0$, when the original equation gives $x = 0$.

Therefore, the correct answer is **E**.

**35.** A man on his way to dinner shortly after $6:00$ p.m. observes that the hands of his watch form an angle of $110^\circ$. Returning before $7:00$ p.m. he notices that again the hands of his watch form an angle of $110^\circ$. The number of minutes that he has been away is:

A $36\frac{2}{3}$

B $40$

C $42$

D $42.4$

E $45$

## Solution:

The two observations lie on opposite sides of the instant when the hands coincide. Their signed angular separation changes by $220^\circ$. Since the minute hand gains on the hour hand at $6 - 0.5 = 5.5^\circ$ per minute, the elapsed time is

$$\frac{220}{5.5} = 40$$

minutes.

Thus, the correct answer is **B**.

**36.** If both $x$ and $y$ are integers, how many pairs of solutions are there of the equation $(x-8)(x-10)=2^y$?

A $0$

B $1$

C $2$

D $3$

E more than $3$

## Solution:

Put $n=x-9$. Then

$$2^y=n^2-1=(n-1)(n+1).$$

Since the product is a power of $2$, both factors must have no odd prime divisor. The only consecutive even integers differing by $2$ that are both signed powers of $2$ are $(-4,-2)$ and $(2,4)$. Thus $n=\pm 3,\ y=3$, giving $x=6$ or $12$. There are $2$ ordered pairs $(x,y)$.

Therefore, the correct answer is **C**.

**37.** $ABCD$ is a square with side of unit length. Points $E$ and $F$ are taken respectively on sides $AB$ and $AD$ so that $AE = AF$ and the quadrilateral $CDFE$ has maximum area. In square units this maximum area is:

A $\frac{1}{2}$

B $\frac{9}{16}$

C $\frac{19}{32}$

D $\frac{5}{8}$

E $\frac{2}{3}$

## Solution:

Set $A = (0,0), B = (1,0), C = (1,1)$, and $D = (0,1)$. Then $E = (t,0)$ and $F = (0,t)$. The shoelace formula gives

$$\begin{aligned}[CDFE] &= \frac{1+t-t^2}{2} \\ &= \frac{5}{8} - \frac{1}{2}\left(t - \frac{1}{2}\right)^2.\end{aligned}$$

Its maximum is $\frac{5}{8}$.

Thus, the correct answer is **D**.

**38.** The population of Nosuch Junction at one time was a perfect square. Later, with an increase of $100$, the population was one more than a perfect square. Now, with an additional increase of $100$, the population is again a perfect square.

The original population is a multiple of:

- A $3$
- B $7$
- C $9$
- D $11$
- E $17$

## Solution:

Let the original population be $a^2$. Then

$$b^2 - a^2 = 99, \qquad c^2 - a^2 = 200.$$

From $(b-a)(b+a) = 99$, the positive possibilities for $a$ are $49, 15$, and $1$. Checking the second condition, only $a = 49$ works, since

$$49^2 + 200 = 2601 = 51^2.$$

The population is $49^2 = 7^4$, a multiple of $7$.

Thus, the correct answer is **B**.

**39.** The medians $AN$ and $BP$ of a triangle with unequal sides are, respectively, $3$ inches and $6$ inches long. Its area is $3\sqrt{15}$ square inches. The length of the third median, in inches, is:

- A $4$
- B $3\sqrt{3}$
- C $3\sqrt{6}$
- D $6\sqrt{3}$
- E $6\sqrt{6}$

## Solution:

The triangle whose sides are the three medians has area

$$\frac{3}{4}\left(3\sqrt{15}\right) = \frac{9\sqrt{15}}{4}.$$

If $\theta$ is the included angle between its sides $3$ and $6$, then

$$9\sin\theta = \frac{9\sqrt{15}}{4},$$

so $\cos\theta = \pm\frac{1}{4}$. By the law of cosines, the third median $m$ satisfies

$$m^2 = 3^2 + 6^2 - 2(3)(6)\cos\theta,$$

giving $m^2 = 36$ or $54$. The value $m = 6$ would make two medians, and hence two sides, equal. Because the triangle has unequal sides, $m = \sqrt{54} = 3\sqrt{6}$.

Therefore, the correct answer is **C**.

**40.** The limiting sum of the infinite series

$$\frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + \cdots,$$

whose $n$th term is $\frac{n}{10^n}$, is:

A $\frac{1}{9}$

B $\frac{10}{81}$

C $\frac{1}{8}$

D $\frac{17}{72}$

E larger than any finite quantity

## Solution:

For $|x| < 1$,

$$\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2}.$$

Taking $x = \frac{1}{10}$ gives

$$\frac{\frac{1}{10}}{(\frac{9}{10})^2} = \frac{10}{81}.$$

Thus, the correct answer is **B**.

Problems: https://live.poshenloh.com/past-contests/amc12/1962