1962 AMC 12

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc12/1962

1. The expression $\frac{1^{4y-1}}{5^{-1}+3^{-1}}$ is equal to:

A $\frac{4y-1}{8}$

B 8

C $\frac{15}{2}$

D $\frac{15}{8}$

E $\frac{1}{8}$

2. The expression $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}}$ is equal to:

A $\frac{\sqrt{3}}{6}$

B $-\frac{\sqrt{3}}{6}$

C $\frac{\sqrt{-3}}{6}$

D $\frac{5\sqrt{3}}{6}$

E 1

3. The first three terms of an arithmetic progression are $x-1, x+1, 2x+3$, in the order shown. The value of x is:

A -2

B 0

C 2

D 4

E undetermined

4. If $8^x = 32$, then x equals:

A 4

B $\frac{5}{3}$

C $\frac{3}{2}$

D $\frac{3}{5}$

E $\frac{1}{4}$

5. If the radius of a circle is increased by 1 unit, the ratio of the new circumference to the new diameter is:

A $\pi + 2$

B $\frac{2\pi + 1}{2}$

C π

D $\frac{2\pi - 1}{2}$

E $\pi - 2$

6. A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ square inches. Expressed in inches the diagonal of the square is:

A $\frac{9}{2}$

B $2\sqrt{5}$

C $4\sqrt{2}$

D $\frac{9\sqrt{2}}{2}$

E none of these

7. Let the bisectors of the exterior angles at B and C of triangle ABC meet at D. Then, if all measurements are in degrees, angle BDC equals:

A $\frac{1}{2}(90 - A)$

B $90 - A$

C $\frac{1}{2}(180 - A)$

D $180 - A$

E $180 - 2A$

8. Given the set of n numbers, $n > 1$, of which one is $1 - \frac{1}{n}$ and all the others are 1. The arithmetic mean of the n numbers is:

A 1

B $n - \frac{1}{n}$

C $n - \frac{1}{n^2}$

D $1 - \frac{1}{n^2}$

E $1 - \frac{1}{n} - \frac{1}{n^2}$

9. When $x^9 - x$ is factored as completely as possible into polynomials and monomials with real integral coefficients, the number of factors is:

A more than 5

B 5

C 4

D 3

E 2

10. A man drives 150 miles to the seashore in 3 hours and 20 minutes. He returns from the shore to the starting point in 4 hours and 10 minutes. Let r be the average rate for the entire trip. Then the average rate for the trip going exceeds r, in miles per hour, by:

A 5

B $4\frac{1}{2}$

C 4

D 2

E 1

11. The difference between the larger root and the smaller root of

$$x^2 - px + \frac{p^2 - 1}{4} = 0$$

is:

A 0

B 1

C 2

D p

E $p + 1$

12. When $\left(1-\frac{1}{a}\right)^6$ is expanded the sum of the last three coefficients is:

A 22

B 11

C 10

D -10

E -11

13. R varies directly as S and inversely as T. When $R=\frac{4}{3}$ and $T=\frac{9}{14}, S=\frac{3}{7}$. Find S when $R=\sqrt{48}$ and $T=\sqrt{75}$.

A 28

B 30

C 40

D 42

E 60

14. Let s be the limiting sum of the geometric series $4 - \frac{8}{3} + \frac{16}{9} - \cdots$, as the number of terms increases without bound. Then s equals:

A a number between 0 and 1

B 2.4

C 2.5

D 3.6

E 12

15. Given triangle ABC with base AB fixed in length and position. As the vertex C moves on a straight line, the intersection point of the three medians moves on:

A a circle

B a parabola

C an ellipse

D a straight line

E a curve here not listed

16. Given rectangle R_1 with one side 2 inches and area 12 square inches. Rectangle R_2 with diagonal 15 inches is similar to R_1. Expressed in square inches the area of R_2 is:

A $\frac{9}{2}$

B 36

C $\frac{135}{2}$

D $9\sqrt{10}$

E $\frac{27\sqrt{10}}{4}$

17. If $a = \log_8 225$ and $b = \log_2 15$, then a, in terms of b, is:

A $\frac{b}{2}$

B $\frac{2b}{3}$

C b

D $\frac{3b}{2}$

E $2b$

18. A regular dodecagon (12 sides) is inscribed in a circle with radius r inches. The area of the dodecagon, in square inches, is:

A $3r^2$

B $2r^2$

C $\dfrac{3r^2\sqrt{3}}{4}$

D $r^2\sqrt{3}$

E $3r^2\sqrt{3}$

19. If the parabola $y = ax^2 + bx + c$ passes through the points $(-1, 12)$, $(0, 5)$, and $(2, -3)$, the value of $a + b + c$ is:

A -4

B -2

C 0

D 1

E 2

20. The angles of a pentagon are in arithmetic progression. One of the angles, in degrees, must be:

A 108

B 90

C 72

D 54

E 36

21. It is given that one root of $2x^2 + rx + s = 0$, with r and s real numbers, is $3 + 2i\ (i = \sqrt{-1})$. The value of s is:

A undetermined

B 5

C 6

D -13

E 26

22. The number 121_b, written in the integral base b, is the square of an integer, for:

A $b = 10$, only

B $b = 10$ and $b = 5$, only

C $2 \leq b \leq 10$

D $b > 2$

E no value of (b)

23. In triangle ABC, CD is the altitude to AB and AE is the altitude to BC. If the lengths of AB, CD, and AE are known, the length of DB is:

A not determined by the information given

B determined only if A is an acute angle

C determined only if B is an acute angle

D determined only if ABC is an acute triangle

E none of these is correct

24. Three machines P, Q, and R, working together, can do a job in x hours. When working alone, P needs an additional 6 hours to do the job; Q, one additional hour; and R, x additional hours. The value of x is:

A $\frac{2}{3}$

B $\frac{11}{12}$

C $\frac{3}{2}$

D 2

E 3

25. Given square $ABCD$ with side 8 feet. A circle is drawn through vertices A and D and tangent to side BC. The radius of the circle, in feet, is:

A 4

B $4\sqrt{2}$

C 5

D $5\sqrt{2}$

E 6

26. For any real value of x the maximum value of $8x - 3x^2$ is:

A 0

B $\frac{8}{3}$

C 4

D 5

E $\frac{16}{3}$

27. Let $a \mathbin{@} b$ represent the operation on two numbers, a and b, which selects the larger of the two numbers, with $a \mathbin{@} a = a$. Let $a \mathbin{!} b$ represent the operation which selects the smaller of the two numbers, with $a \mathbin{!} a = a$. Which of the following three rules is (are) correct?

$$\begin{aligned} &(1) \quad a \mathbin{@} b = b \mathbin{@} a, \\ &(2) \quad a \mathbin{@} (b \mathbin{@} c) = (a \mathbin{@} b) \mathbin{@} c, \\ &(3) \quad a \mathbin{!} (b \mathbin{@} c) \\ &\qquad\qquad = (a \mathbin{!} b) \mathbin{@} (a \mathbin{!} c). \end{aligned}$$

A (1) only

B (2) only

C (1) and (2) only

D (1) and (3) only

E all three

28. The set of x-values satisfying the equation

$$x^{\log_{10} x} = \frac{x^3}{100}$$

consists of:

A $\frac{1}{10}$, only

B 10, only

C 100, only

D 10 or 100, only

E more than two real numbers

29. Which of the following sets of x-values satisfy the inequality $2x^2 + x < 6$?

A $-2 < x < \frac{3}{2}$

B $x > \frac{3}{2}$ or $x < -2$

C $x < \frac{3}{2}$

D $\frac{3}{2} < x < 2$

E $x < -2$

30. Consider the statements:

(1) p and q are both true
(2) p is true and q is false
(3) p is false and q is true
(4) p is false and q is false.

How many of these imply the negation of the statement "p and q are both true"?

A 0

B 1

C 2

D 3

E 4

31. The ratio of the interior angles of two regular polygons with sides of unit length is $3:2$. How many such pairs are there?

A 1

B 2

C 3

D 4

E infinitely many

32. If $x_{k+1} = x_k + \frac{1}{2}$ for $k = 1, 2, \ldots, n-1$ and $x_1 = 1$, find $x_1 + x_2 + \cdots + x_n$.

A $\frac{n+1}{2}$

B $\frac{n+3}{2}$

C $\frac{n^2-1}{2}$

D $\frac{n^2+n}{4}$

E $\frac{n^2+3n}{4}$

33. The set of x-values satisfying the inequality $2 \leq |x-1| \leq 5$ is:

A $-4 \leq x \leq -1$ or $3 \leq x \leq 6$

B $3 \leq x \leq 6$ or $-6 \leq x \leq -3$

C $x \leq -1$ or $x \geq 3$

D $-1 \leq x \leq 3$

E $-4 \leq x \leq 6$

34. For what real values of K does $x = K^2(x-1)(x-2)$ have real roots?

A none

B $-2 < K < 1$

C $-2\sqrt{2} < K < 2\sqrt{2}$

D $K > 1$ or $K < -2$

E all

35. A man on his way to dinner shortly after $6{:}00$ p.m. observes that the hands of his watch form an angle of 110°. Returning before $7{:}00$ p.m. he notices that again the hands of his watch form an angle of 110°. The number of minutes that he has been away is:

A $36\frac{2}{3}$

B 40

C 42

D 42.4

E 45

36. If both x and y are integers, how many pairs of solutions are there of the equation $(x-8)(x-10) = 2^y$?

A 0

B 1

C 2

D 3

E more than 3

37. $ABCD$ is a square with side of unit length. Points E and F are taken respectively on sides AB and AD so that $AE = AF$ and the quadrilateral $CDFE$ has maximum area. In square units this maximum area is:

A $\frac{1}{2}$

B $\frac{9}{16}$

C $\frac{19}{32}$

D $\frac{5}{8}$

E $\frac{2}{3}$

38. The population of Nosuch Junction at one time was a perfect square. Later, with an increase of 100, the population was one more than a perfect square. Now, with an additional increase of 100, the population is again a perfect square.

The original population is a multiple of:

A 3

B 7

C 9

D 11

E 17

39. The medians AN and BP of a triangle with unequal sides are, respectively, 3 inches and 6 inches long. Its area is $3\sqrt{15}$ square inches. The length of the third median, in inches, is:

A 4

B $3\sqrt{3}$

C $3\sqrt{6}$

D $6\sqrt{3}$

E $6\sqrt{6}$

40. The limiting sum of the infinite series

$$\frac{1}{10}+\frac{2}{10^2}+\frac{3}{10^3}+\cdots,$$

whose nth term is $\frac{n}{10^n}$, is:

A $\frac{1}{9}$

B $\frac{10}{81}$

C $\frac{1}{8}$

D $\frac{17}{72}$

E larger than any finite quantity

Solutions: https://live.poshenloh.com/past-contests/amc12/1962/solutions