1961 AMC 12 Solutions

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1. When simplified, $\left(-\frac{1}{125}\right)^{-\frac{2}{3}}$ becomes:

A $\frac{1}{25}$

B $-\frac{1}{25}$

C 25

D -25

E $25\sqrt{-1}$

Solution:

Because the cube root is real,

$$\left(-\frac{1}{125}\right)^{\frac{2}{3}} = \left(-\frac{1}{5}\right)^{2} = \frac{1}{25}.$$

The negative exponent takes the reciprocal, giving 25.

Therefore, the correct answer is **C**.

2. An automobile travels $\frac{a}{6}$ feet in r seconds. If this rate is maintained for 3 minutes, how many yards does it travel in the 3 minutes?

A $\dfrac{a}{1080r}$

B $\dfrac{30r}{a}$

C $\dfrac{30a}{r}$

D $\dfrac{10r}{a}$

E $\dfrac{10a}{r}$

Solution:

The speed is $\frac{a}{6r}$ feet per second. In 180 seconds the automobile travels

$$180 \cdot \frac{a}{6r} = \frac{30a}{r}$$

feet, or $\frac{10a}{r}$ yards.

Thus, the correct answer is **E**.

3. If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are to meet at right angles, the value of a is:

A $\pm\dfrac{2}{3}$

B $-\dfrac{2}{3}$

C $-\dfrac{3}{2}$

D 6

E -6

Solution:

The two slopes are $-\frac{1}{2}$ and $-\frac{a}{3}$. Perpendicularity requires

$$\left(-\frac{1}{2}\right)\left(-\frac{a}{3}\right) = -1,$$

so $a = -6$.

Therefore, the correct answer is **E**.

4. Let the set consisting of the squares of the positive integers be called u; thus u is the set $1, 4, 9, \dots$. If a certain operation on one or more members of the set always yields a member of the set, we say that the set is closed under that operation. Then u is closed under:

- A addition
- B multiplication
- C division
- D extraction of a positive integral root
- E none of these

Solution:

For positive integers m, n,

$$m^2 n^2 = (mn)^2,$$

which is again the square of a positive integer. The other operations fail, for example $1 + 4 = 5$, $\frac{1}{4}$, and $\sqrt{4} = 2$.

Thus, the correct answer is **B**.

5. Let $S = (x-1)^4 + 4(x-1)^3 + 6(x-1)^2 + 4(x-1) + 1$. Then S equals:

A $(x-2)^4$

B $(x-1)^4$

C x^4

D $(x+1)^4$

E $x^4 + 1$

Solution:

The expression is the binomial expansion

$$((x-1)+1)^4 = x^4.$$

Therefore, the correct answer is **C**.

6. When simplified, $\log 8 \div \log \frac{1}{8}$ becomes:

A $6\log 2$

B $\log 2$

C 1

D 0

E -1

Solution:

Since $\log(\frac{1}{8}) = \log(8^{-1}) = -\log 8$,

$$\frac{\log 8}{\log(\frac{1}{8})} = -1.$$

Thus, the correct answer is **E**.

7. When simplified, the third term in the expansion of

$$\left(\frac{a}{\sqrt{x}} - \frac{\sqrt{x}}{a^2}\right)^6$$

is:

A $\frac{15}{x}$

B $-\frac{15}{x}$

C $-\frac{6x^2}{a^9}$

D $\frac{20}{a^3}$

E $-\frac{20}{a^3}$

Solution:

The third term is

$$\binom{6}{2}\left(\frac{a}{\sqrt{x}}\right)^4\left(-\frac{\sqrt{x}}{a^2}\right)^2$$
$$= 15 \cdot \frac{a^4}{x^2} \cdot \frac{x}{a^4} = \frac{15}{x}.$$

Therefore, the correct answer is **A**.

8. Let the two base angles of a triangle be A and B, with B larger than A. The altitude to the base divides the vertex angle C into two parts, C_1 and C_2, with C_2 adjacent to side a. Then:

A $C_1 + C_2 = A + B$

B $C_1 - C_2 = B - A$

C $C_1 - C_2 = A - B$

D $C_1 + C_2 = B - A$

E $C_1 - C_2 = A + B$

Solution:

The two right triangles give

$$A + C_1 = 90^\circ,$$
$$B + C_2 = 90^\circ.$$

Subtracting the second relation from the first gives $C_1 - C_2 = B - A$.

Thus, the correct answer is **B**.

9. Let r be the result of doubling both the base and the exponent of $a^b, b \neq 0$. If r equals the product of a^b by x^b, then x equals:

- A a
- B $2a$
- C $4a$
- D 2
- E 4

Solution:

We have

$$r = (2a)^{2b} = (4a^2)^b$$

and also $r = a^b x^b = (ax)^b$. Thus $ax = 4a^2$, so $x = 4a$.

Therefore, the correct answer is **C**.

10. Each side of triangle ABC is 12 units. D is the foot of the perpendicular dropped from A on BC, and E is the midpoint of AD. The length of BE, in the same unit, is:

- A $\sqrt{18}$
- B $\sqrt{28}$
- C 6
- D $\sqrt{63}$
- E $\sqrt{98}$

Solution:

In the equilateral triangle, $BD = 6$ and $AD = 6\sqrt{3}$. Hence $DE = 3\sqrt{3}$, and right triangle BDE gives

$$\begin{aligned} BE^2 &= 6^2 + (3\sqrt{3})^2 \\ &= 36 + 27 = 63. \end{aligned}$$

Therefore $BE = \sqrt{63}$.

Thus, the correct answer is **D**.

11. Two tangents are drawn to a circle from an exterior point A; they touch the circle at points B and C, respectively. A third tangent intersects segment AB in P and AC in R, and touches the circle at Q. If $AB = 20$, then the perimeter of triangle APR is:

A 42

B 40.5

C 40

D $39\frac{7}{8}$

E not determined by the given information

Solution:

Equal tangent segments give $PQ = PB$ and $RQ = RC$. Thus

$$\begin{aligned} AP + PR + RA &= AP + PQ \\ &\quad + RQ + RA \\ &= AP + PB \\ &\quad + RC + RA \\ &= AB + AC. \end{aligned}$$

Also $AB = AC = 20$, so the perimeter is 40.

Therefore, the correct answer is **C**.

12. The first three terms of a geometric progression are $\sqrt{2}, \sqrt[3]{2}, \sqrt[6]{2}$. Find the fourth term.

A 1

B $\sqrt[7]{2}$

C $\sqrt[8]{2}$

D $\sqrt[9]{2}$

E $\sqrt[10]{2}$

Solution:

The exponents are $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$. Each term is obtained by multiplying by $2^{-\frac{1}{6}}$, so the next exponent is $\frac{1}{6} - \frac{1}{6} = 0$. The fourth term is $2^0 = 1$.

Thus, the correct answer is **A**.

13. The symbol $|a|$ means a if a is a positive number or zero, and $-a$ if a is a negative number. For all real values of t the expression $\sqrt{t^4+t^2}$ is equal to:

A t^3

B t^2+t

C $|t^2+t|$

D $t\sqrt{t^2+1}$

E $|t|\sqrt{1+t^2}$

Solution:

Factoring inside the radical gives

$$\begin{aligned}\sqrt{t^2(t^2+1)} &= \sqrt{t^2}\sqrt{t^2+1}\\ &= |t|\sqrt{1+t^2}.\end{aligned}$$

Therefore, the correct answer is **E**.

14. A rhombus is given with one diagonal twice the length of the other diagonal. Express the side of the rhombus in terms of K, where K is the area of the rhombus in square inches.

A $\sqrt{K}$

B $\frac{1}{2}\sqrt{2K}$

C $\frac{1}{3}\sqrt{3K}$

D $\frac{1}{4}\sqrt{4K}$

E none of these are correct

Solution:

Let the diagonals be d and $2d$. Then $K = \frac{d(2d)}{2} = d^2$. A side has length

$$\begin{aligned} \sqrt{\left(\frac{d}{2}\right)^2 + d^2} &= \frac{\sqrt{5}}{2}d \\ &= \frac{1}{2}\sqrt{5K}. \end{aligned}$$

This is not among the first four choices.

Thus, the correct answer is **E**.

15. If x men working x hours a day for x days produce x articles, then the number of articles (not necessarily an integer) produced by y men working y hours a day for y days is:

A $\frac{x^3}{y^2}$

B $\frac{y^3}{x^2}$

C $\frac{x^2}{y^3}$

D $\frac{y^2}{x^3}$

E y

Solution:

The first group uses x^3 man-hours to make x articles, so the rate is $\frac{1}{x^2}$ article per man-hour. The second group supplies y^3 man-hours and therefore makes $\frac{y^3}{x^2}$ articles.

Thus, the correct answer is **B**.

16. An altitude h of a triangle is increased by a length m. How much must be taken from the corresponding base b so that the area of the new triangle is one-half that of the original triangle?

A $\dfrac{bm}{h+m}$

B $\dfrac{bh}{2h+2m}$

C $\dfrac{b(2m+h)}{m+h}$

D $\dfrac{b(m+h)}{2m+h}$

E $\dfrac{b(2m+h)}{2(h+m)}$

Solution:

If t is removed, the new area condition is

$$\frac{1}{2}(b-t)(h+m) = \frac{1}{2}\left(\frac{1}{2}bh\right).$$

Hence $b - t = \frac{bh}{[2(h+m)]}$, so

$$\begin{aligned} t &= b - \frac{bh}{2(h+m)} \\ &= \frac{b(2m+h)}{2(h+m)}. \end{aligned}$$

Therefore, the correct answer is **E**.

17. In the base ten number system the number 526 means $5 \cdot 10^2 + 2 \cdot 10 + 6$. In the Land of Mathesis, however, numbers are written in the base r. Jones purchases an automobile there for 440 monetary units (abbreviated m.u.). He gives the salesman a 1000 m.u. bill, and receives, in change, 340 m.u. The base r is:

A 2

B 5

C 7

D 8

E 12

Solution:

In ordinary notation the transaction says

$$r^3 - (4r^2 + 4r) = 3r^2 + 4r.$$

Thus $r(r^2 - 7r - 8) = 0$, so $r = 8$ or $r = -1$. A numeral base must be positive and exceed 4, hence $r = 8$.

Therefore, the correct answer is **D**.

18. The yearly changes in the population census of a town for four consecutive years are, respectively, 25% increase, 25% increase, 25% decrease, 25% decrease. The net change over the four years, to the nearest percent, is:

A -12

B -1

C 0

D 1

E 12

Solution:

The final population is multiplied by

$$(1.25)^2(0.75)^2 = \frac{225}{256} \approx 0.879.$$

This is a decrease of about 12.1%, which rounds to 12%.

Thus, the correct answer is **A**.

19. Consider the graphs of $y = 2\log x$ and $y = \log 2x$. We may say that:

- A they do not intersect
- B they intersect at 1 point only
- C they intersect at 2 points only
- D they intersect at a finite number of points but greater than 2
- E they coincide

Solution:

On the domain $x > 0$, equality requires

$$\log(x^2) = \log(2x),$$

so $x^2 = 2x$. The only positive solution is $x = 2$, giving exactly one intersection.

Therefore, the correct answer is **B**.

20. The set of points satisfying the pair of inequalities $y > 2x$ and $y > 4 - x$ is contained entirely in quadrants:

A I and II

B II and III

C I and III

D III and IV

E I and IV

Solution:

If $x \geq 0$, then $y > 2x \geq 0$. If $x < 0$, then $y > 4 - x > 0$. Thus every feasible point is above the x-axis, while both positive and negative x-values occur. The region lies in quadrants I and II.

Therefore, the correct answer is **A**.

21. Medians AD and CE of triangle ABC intersect in M. The midpoint of AE is N. Let the area of triangle MNE be k times the area of triangle ABC. Then k equals:

A $\frac{1}{6}$

B $\frac{1}{8}$

C $\frac{1}{9}$

D $\frac{1}{12}$

E $\frac{1}{16}$

Solution:

Set $A = \mathbf{0},\ B = \mathbf{b},\ C = \mathbf{c}.$ Then

$$E = \frac{\mathbf{b}}{2}, \qquad N = \frac{\mathbf{b}}{4},$$
$$M = \frac{\mathbf{b}+\mathbf{c}}{3}.$$

A determinant calculation gives

$$[MNE] = \frac{1}{24}\left|\det(\mathbf{b},\mathbf{c})\right|,$$

while

$$[ABC] = \frac{1}{2}\left|\det(\mathbf{b},\mathbf{c})\right|.$$

Their ratio is $\frac{1}{12}$.

Thus, the correct answer is **D**.

22. If $3x^3 - 9x^2 + kx - 12$ is divisible by $x - 3$, then it is also divisible by:

A $3x^2 - x + 4$

B $3x^2 - 4$

C $3x^2 + 4$

D $3x - 4$

E $3x + 4$

Solution:

Let $P(x) = 3x^3 - 9x^2 + kx - 12$. Since $x - 3$ is a factor,

$$P(3) = 81 - 81 + 3k - 12 = 0,$$

so $k = 4$. Direct division gives

$$P(x) = (x - 3)(3x^2 + 4).$$

Therefore, the correct answer is **C**.

23. Points P and Q are both in the line segment AB and on the same side of its midpoint. P divides AB in the ratio $2 : 3$, and Q divides AB in the ratio $3 : 4$. If $PQ = 2$, then the length of AB is:

- A 60
- B 70
- C 75
- D 80
- E 85

Solution:

The division ratios give

$$\frac{AP}{AB} = \frac{2}{5}, \qquad \frac{AQ}{AB} = \frac{3}{7}.$$

Hence $PQ = (\frac{3}{7} - \frac{2}{5})AB = \frac{AB}{35}$. Since $PQ = 2$, we obtain $AB = 70$.

Thus, the correct answer is **B**.

24. Thirty-one books are arranged from left to right in order of increasing prices. The price of each book differs by $\$2$ from that of each adjacent book. For the price of the book at the extreme right a customer can buy the middle book and an adjacent one. Then:

A the adjacent book referred to is at the left of the middle book

B the middle book sells for $\$36$

C the cheapest book sells for $\$4$

D the most expensive book sells for $\$64$

E none of these are correct

Solution:

The prices are $p, p+2, \ldots, p+60$, and the middle price is $p+30$. If the right neighbor is used, then

$$(p+30)+(p+32)=p+60,$$

which gives $p=-2$, impossible for a selling price. The left neighbor gives

$$(p+30)+(p+28)=p+60,$$

so $p=2$. Thus the adjacent book must be the one to the left.

Therefore, the correct answer is **A**.

25. Triangle ABC is isosceles with base AC. Points P and Q are respectively in CB and AB and such that $AC = AP = PQ = QB$. The number of degrees in angle B is:

A $25\frac{5}{7}$

B $26\frac{1}{3}$

C 30

D 40

E not determined by the information given

Solution:

Let $\angle B = m$. Since $PQ = QB$, triangle BQP gives $\angle QPB = m$ and $\angle AQP = 2m$. Since $AP = PQ$, triangle AQP gives $\angle QAP = 2m$, so $\angle QPA = 180^\circ - 4m$. As B, P, C are collinear, $\angle APC = 3m$. Now $AP = AC$ gives $\angle ACP = 3m$. Finally $AB = BC$ makes both base angles of ABC equal to $3m$, and

$$m + 3m + 3m = 180^\circ.$$

Therefore $m = \frac{180^\circ}{7} = 25\frac{5}{7}^\circ$.

Thus, the correct answer is **A**.

26. For a given arithmetic series the sum of the first 50 terms is 200, and the sum of the next 50 terms is 2700. The first term in the series is:

A -1221

B -21.5

C -20.5

D 3

E 3.5

Solution:

The two sums give

$$25(2a + 49d) = 200,$$
$$25(2a + 149d) = 2700.$$

Thus $2a + 49d = 8$ and $2a + 149d = 108$. Subtraction gives $d = 1$, and then $2a = -41$, so $a = -20.5$.

Therefore, the correct answer is **C**.

27. Given two equiangular polygons P_1 and P_2 with different numbers of sides; each angle of P_1 is x degrees and each angle of P_2 is kx degrees, where k is an integer greater than 1. The number of possibilities for the pair (x, k) is:

A infinite

B finite, but greater than two

C two

D one

E zero

Solution:

Every polygon angle is at least $60°$, and every convex polygon angle is less than $180°$. Since $k \geq 2$ and $kx < 180°$, we need $x < 90°$. The only possible equiangular polygon angle in $[60°, 90°)$ is the triangle angle $x = 60°$. Then $k = 2$ gives $kx = 120°$, the angle of a regular hexagon; larger k is impossible. Hence there is one pair.

Thus, the correct answer is **D**.

28. If 2137^{753} is multiplied out, the units' digit in the final product is:

A 1

B 3

C 5

D 7

E 9

Solution:

Powers of 7 have units digits $7, 9, 3, 1$ in a cycle of length 4. Since $753 \equiv 1 \pmod 4$, the units digit of 2137^{753} is 7.

Therefore, the correct answer is **D**.

29. Let the roots of $ax^2 + bx + c = 0$ be r and s. The equation with roots $ar + b$ and $as + b$ is:

A $x^2 - bx - ac = 0$

B $x^2 - bx + ac = 0$

C $x^2 + 3bx + ca + 2b^2 = 0$

D $x^2 + 3bx - ca + 2b^2 = 0$

E $$x^2 + bx(2-a) \\ + a^2c + b^2(a+1) = 0$$

Solution:

The transformed roots have sum

$$a(r+s) + 2b = -b + 2b = b$$

and product

$$\begin{aligned} &(ar+b)(as+b) \\ &= a^2rs + ab(r+s) + b^2 \\ &= ac. \end{aligned}$$

Therefore their monic equation is $x^2 - bx + ac = 0$.

Thus, the correct answer is **B**.

30. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, then $\log_5 12 =$

A $\dfrac{a+b}{a+1}$

B $\dfrac{2a+b}{a+1}$

C $\dfrac{a+2b}{1+a}$

D $\dfrac{2a+b}{1-a}$

E $\dfrac{a+2b}{1-a}$

Solution:

By change of base,

$$\begin{aligned}\log_5 12 &= \frac{\log 12}{\log 5} \\ &= \frac{2\log 2 + \log 3}{\log 10 - \log 2} \\ &= \frac{2a+b}{1-a}.\end{aligned}$$

Therefore, the correct answer is **D**.

31. In triangle ABC the ratio $AC : CB$ is $3 : 4$. The bisector of the exterior angle at C intersects BA extended at P (A is between P and B). The ratio $PA : AB$ is:

A $1 : 3$

B $3 : 4$

C $4 : 3$

D $3 : 1$

E $7 : 1$

Solution:

The exterior angle bisector theorem gives

$$\frac{PA}{PB} = \frac{AC}{CB} = \frac{3}{4}.$$

Since $PB = PA + AB$, write $PA = 3t,\ PB = 4t$. Then $AB = t$, so $PA : AB = 3 : 1$.

Thus, the correct answer is **D**.

32. A regular polygon of n sides is inscribed in a circle of radius R. The area of the polygon is $3R^2$. Then n equals:

- A 8
- B 10
- C 12
- D 15
- E 18

Solution:

The polygon's area is

$$\frac{n}{2} R^2 \sin \frac{360^\circ}{n}.$$

For $n = 12$, this becomes

$$6R^2 \sin 30^\circ = 3R^2,$$

as required.

Therefore, the correct answer is **C**.

33. The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is:

A 0

B 1

C 2

D 3

E more than three, but finite

Solution:

Factor:

$$(2^x - 3^y)(2^x + 3^y) = 55.$$

Both factors are positive odd divisors. The pair $5, 11$ gives $2^{x+1} = 16$ and $2 \cdot 3^y = 6$, so $(x, y) = (3, 1)$. The pair $1, 55$ would require $2^x = 28$, impossible. Hence there is exactly one solution.

Thus, the correct answer is **B**.

34. Let S be the set of values assumed by the fraction

$$\frac{2x+3}{x+2}$$

when x is any member of the interval $x \geq 0$. If there exists a number M such that no number of the set S is greater than M, then M is an upper bound of S. If there exists a number m such that no number of the set S is less than m, then m is a lower bound of S. We may then say:

A m is in S, but M is not in S

B M is in S, but m is not in S

C both m and M are in S

D neither m nor M is in S

E M does not exist either in or outside S

Solution:

We have

$$\frac{2x+3}{x+2} = 2 - \frac{1}{x+2}.$$

For $x \geq 0$, this increases from $\frac{3}{2}$ toward 2 without reaching 2. Thus $S = [3/2, 2)$, so its lower bound $m = \frac{3}{2}$ belongs to S, while its least upper bound $M = 2$ does not.

Therefore, the correct answer is **A**.

35. The number 695 is to be written with a factorial base of numeration, that is,

$$695 = a_1 + a_2 \cdot 2! + a_3 \cdot 3! \\ + \cdots + a_n \cdot n!,$$

where $a_1, a_2, a_3, \ldots, a_n$ are integers such that $0 \leq a_k \leq k$, and $n!$ means $n(n-1)(n-2)\cdots 2 \cdot 1$. Find a_4.

A 0

B 1

C 2

D 3

E 4

Solution:

Since $5! = 120$,

$$695 = 5 \cdot 120 + 95.$$

Next $4! = 24$, and $95 = 3 \cdot 24 + 23$. Therefore the coefficient a_4 is 3.

Thus, the correct answer is **D**.

36. In triangle ABC the median from A is perpendicular to the median from B. If $BC = 7$ and $AC = 6$, find the length of AB.

A 4

B $\sqrt{17}$

C 4.25

D $2\sqrt{5}$

E 4.5

Solution:

Let $A = \mathbf{0},\ B = \mathbf{b},\ C = \mathbf{c}$, with $|\mathbf{b}| = \ell,\ |\mathbf{c}| = 6$, and $|\mathbf{c} - \mathbf{b}| = 7$. The median directions are $\frac{\mathbf{b}+\mathbf{c}}{2}$ from A and $\frac{\mathbf{c}}{2} - \mathbf{b}$ from B. Their dot product is zero, so

$$(\mathbf{b} + \mathbf{c}) \cdot (\mathbf{c} - 2\mathbf{b}) = 0.$$

Using $\mathbf{b} \cdot \mathbf{c} = \frac{\ell^2+36-49}{2}$, this simplifies to $\ell^2 = 17$. Hence $AB = \sqrt{17}$.

Therefore, the correct answer is **B**.

37. In racing over a distance d at uniform speed, A can beat B by 20 yards, B can beat C by 10 yards, and A can beat C by 28 yards. Then d, in yards, equals:

A not determined by the given information

B 58

C 100

D 116

E 120

Solution:

The margins give

$$\frac{v_B}{v_A} = \frac{d-20}{d},$$
$$\frac{v_C}{v_B} = \frac{d-10}{d},$$
$$\frac{v_C}{v_A} = \frac{d-28}{d}.$$

Therefore

$$\frac{(d-20)(d-10)}{d^2} = \frac{d-28}{d}.$$

Expanding and simplifying yields $2d = 200$, so $d = 100.$

Thus, the correct answer is **C**.

38. Triangle ABC is inscribed in a semicircle of radius r so that its base AB coincides with diameter AB. Point C does not coincide with either A or B. Let $s = AC + BC$. Then, for all permissible positions of C :

A $s^2 \leq 8r^2$

B $s^2 = 8r^2$

C $s^2 \geq 8r^2$

D $s^2 \leq 4r^2$

E $s^2 = 4r^2$

Solution:

By Thales' theorem, ABC is right at C. Put $p = AC,\ q = BC$. Then

$$p^2 + q^2 = AB^2 = 4r^2.$$

Since $(p-q)^2 \geq 0$, we have $2pq \leq p^2 + q^2$. Therefore

$$s^2 = (p+q)^2 \leq 2(p^2+q^2) = 8r^2.$$

Thus, the correct answer is **A**.

39. Any five points are taken inside or on a square with side length 1. Let a be the smallest possible number with the property that it is always possible to select one pair of points from these five such that the distance between them is equal to or less than a. Then a is:

A $\frac{\sqrt{3}}{3}$

B $\frac{\sqrt{2}}{2}$

C $\frac{2\sqrt{2}}{3}$

D 1

E $\sqrt{2}$

Solution:

Divide the square into four squares of side $\frac{1}{2}$. Two of the five points lie in the same small square, so their distance is at most its diagonal, $\frac{\sqrt{2}}{2}$. This bound is attainable by placing points at the four corners and the center: the shortest distance is then $\frac{\sqrt{2}}{2}$. Hence the least guaranteed value is $\frac{\sqrt{2}}{2}$.

Therefore, the correct answer is **B**.

40. Find the minimum value of $\sqrt{x^2+y^2}$ if $5x+12y=60$.

A $\frac{60}{13}$

B $\frac{13}{5}$

C $\frac{13}{12}$

D 1

E 0

Solution:

By Cauchy-Schwarz,

$$\begin{aligned} 60 &= 5x+12y \\ &\leq \sqrt{5^2+12^2} \\ &\quad \cdot \sqrt{x^2+y^2} \\ &= 13\sqrt{x^2+y^2}. \end{aligned}$$

Thus the distance is at least $\frac{60}{13}$. Equality occurs when (x,y) is proportional to $(5,12)$, so the minimum is $\frac{60}{13}$.

Therefore, the correct answer is **A**.

Problems: https://live.poshenloh.com/past-contests/amc12/1961