1960 AMC 12

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc12/1960

1. If 2 is a solution (root) of $x^3 + hx + 10 = 0$, then h equals:

A 10

B 9

C 2

D -2

E -9

2. It takes 5 seconds for a clock to strike 6 o'clock beginning at $6{:}00$ o'clock precisely. If the strikings are uniformly spaced, how long, in seconds, does it take to strike 12 o'clock?

A $9\frac{1}{3}$

B 10

C 11

D $14\frac{2}{3}$

E none of these

3. Applied to a bill for $\$10,000$ the difference between a discount of 40% and two successive discounts of 36% and 4%, expressed in dollars, is:

A 0

B 144

C 256

D 400

E 416

4. Each of two angles of a triangle is 60° and the included side is 4 inches. The area of the triangle, in square inches, is:

A $8\sqrt{3}$

B 8

C $4\sqrt{3}$

D 4

E $2\sqrt{3}$

5. The number of distinct points common to the graphs of $x^2 + y^2 = 9$ and $y^2 = 9$ is:

A infinitely many

B four

C two

D one

E none

6. The circumference of a circle is 100 inches. The side of a square inscribed in this circle, expressed in inches, is:

A $\frac{25\sqrt{2}}{\pi}$

B $\frac{50\sqrt{2}}{\pi}$

C $\frac{100}{\pi}$

D $\frac{100\sqrt{2}}{\pi}$

E $50\sqrt{2}$

7. Circle I passes through the center of, and is tangent to, circle II. The area of circle I is 4 square inches. Then the area of circle II, in square inches, is:

A 8

B $8\sqrt{2}$

C $8\sqrt{\pi}$

D 16

E $16\sqrt{2}$

8. The number $2.5252525\ldots$ can be written as a fraction. When reduced to lowest terms the sum of the numerator and denominator of this fraction is:

A 7

B 29

C 141

D 349

E none of these

9. The fraction

$$\frac{a^2+b^2-c^2+2ab}{a^2+c^2-b^2+2ac}$$

is (with suitable restrictions on the values of $a, b,$ and c):

A irreducible

B reducible to -1

C reducible to a polynomial of three terms

D reducible to $\dfrac{a-b+c}{a+b-c}$

E reducible to $\dfrac{a+b-c}{a-b+c}$

10. Given the following six statements:

(1) All women are good drivers.

(2) Some women are good drivers.

(3) No men are good drivers.

(4) All men are bad drivers.

(5) At least one man is a bad driver.

(6) All men are good drivers.

The statement that negates statement (6) is:

A (1)

B (2)

C (3)

D (4)

E (5)

11. For a given value of $k,$ the product of the roots of

$$x^2 - 3kx + 2k^2 - 1 = 0$$

is $7.$ The roots may be characterized as:

A integral and positive

B integral and negative

C rational, but not integral

D irrational

E imaginary

12. The locus of the centers of all circles of given radius $a,$ in the same plane, passing through a fixed point, is:

A a point

B a straight line

C two straight lines

D a circle

E two circles

13. The polygon(s) formed by $y = 3x + 2, y = -3x + 2,$ and $y = -2,$ is (are):

- A — an equilateral triangle
- B — an isosceles triangle
- C — a right triangle
- D — a triangle and a trapezoid
- E — a quadrilateral

14. If a and b are real numbers, the equation $3x - 5 + a = bx + 1$ has a unique solution x [the symbol $a \neq 0$ means that a is different from zero]:

- A — for all a and b
- B — if $a \neq 2b$
- C — if $a \neq 6$
- D — if $b \neq 0$
- E — if $b \neq 3$

15. Triangle I is equilateral with side A, perimeter P, area K, and circumradius R (radius of the circumscribed circle). Triangle II is equilateral with side a, perimeter p, area k, and circumradius r. If A is different from a, then:

A $P:p=R:r$ only sometimes

B $P:p=R:r$ always

C $P:p=K:k$ only sometimes

D $R:r=K:k$ always

E $R:r=K:k$ only sometimes

16. In the numeration system with base 5, counting is as follows: $1, 2, 3, 4, 10, 11, 12, 13, 14, 20, \ldots$ The number whose description in the decimal system is 69, when described in the base 5 system, is a number with:

A two consecutive digits

B two non-consecutive digits

C three consecutive digits

D three non-consecutive digits

E four digits

17. The formula $N = 8 \cdot 10^8 \cdot x^{-\frac{3}{2}}$ gives, for a certain group, the number of individuals whose income exceeds x dollars. The lowest income, in dollars, of the wealthiest 800 individuals is at least:

A 10^4

B 10^6

C 10^8

D 10^{12}

E 10^{16}

18. The pair of equations $3^{x+y} = 81$ and $81^{x-y} = 3$ has:

A no common solution

B the solution $x = 2, y = 2$

C the solution $x = 2\frac{1}{2}, y = 1\frac{1}{2}$

D a common solution in positive and negative integers

E none of these

19. Consider equation I: $x + y + z = 46$, where $x, y,$ and z are positive integers, and equation II: $x + y + z + w = 46$, where $x, y, z,$ and w are positive integers. Then:

A I can be solved in consecutive integers

B I can be solved in consecutive even integers

C II can be solved in consecutive integers

D II can be solved in consecutive even integers

E II can be solved in consecutive odd integers

20. The coefficient of x^7 in the expansion of

$$\left(\frac{x^2}{2} - \frac{2}{x}\right)^8$$

is:

A 56

B -56

C 14

D -14

E 0

21. The diagonal of square I is $a+b$. The perimeter of square II with twice the area of I is:

A $(a+b)^2$

B $\sqrt{2}(a+b)^2$

C $2(a+b)$

D $\sqrt{8}(a+b)$

E $4(a+b)$

22. The equality $(x+m)^2-(x+n)^2=(m-n)^2$, where m and n are unequal non-zero constants, is satisfied by $x=am+bn$ where:

A $a=0, b$ has a unique non-zero value

B $a=0, b$ has two non-zero values

C $b=0, a$ has a unique non-zero value

D $b=0, a$ has two non-zero values

E a and b each have a unique non-zero value

23. The radius R of a cylindrical box is 8 inches, the height H is 3 inches. The volume $V = \pi R^2 H$ is to be increased by the same fixed positive amount when R is increased by x inches as when H is increased by x inches. This condition is satisfied by:

A no real value of x

B one integral value of x

C one rational, but not integral, value of x

D one irrational value of x

E two real values of x

24. If $\log_{2x} 216 = x$, where x is real, then x is:

A a non-square, non-cube integer

B a non-square, non-cube, non-integral rational number

C an irrational number

D a perfect square

E a perfect cube

25. Let m and n be any two odd numbers, with n less than m. The largest integer which divides all possible numbers of the form $m^2 - n^2$ is:

A 2

B 4

C 6

D 8

E 16

26. Find the set of x-values satisfying the inequality

$$\left|\frac{5-x}{3}\right| < 2.$$

[The symbol $|a|$ means $+a$ if a is positive, $-a$ if a is negative, 0 if a is zero. The notation $1 < a < 2$ means that a can have any value between 1 and 2, excluding 1 and 2.]

A $1 < x < 11$

B $-1 < x < 11$

C $x < 11$

D x>11

E $|x| < 6$

27. Let S be the sum of the interior angles of a polygon P for which each interior angle is $7\frac{1}{2}$ times the exterior angle at the same vertex. Then:

- A $S = 2660^\circ$ and P may be regular
- B $S = 2660^\circ$ and P is not regular
- C $S = 2700^\circ$ and P is regular
- D $S = 2700^\circ$ and P is not regular
- E $S = 2700^\circ$ and P may or may not be regular

28. The equation

$$x - \frac{7}{x-3} = 3 - \frac{7}{x-3}$$

has:

- A infinitely many integral roots
- B no root
- C one integral root
- D two equal integral roots
- E two equal non-integral roots

29. Five times A's money added to B's money is more than $\$51.00$. Three times A's money minus B's money is $\$21.00$. If a represents A's money in dollars and b represents B's money in dollars, then:

A a>9, b>6

B a>9, $b < 6$

C a>9, $b = 6$

D a>9, but we can put no bounds on b

E $2a = 3b$

30. Given the line $3x + 5y = 15$ and a point on this line equidistant from the coordinate axes. Such a point exists in:

A none of the quadrants

B quadrant I only

C quadrants I, II only

D quadrants $\mathrm{I}, \mathrm{II}, \mathrm{III}$ only

E each of the quadrants

31. For $x^2 + 2x + 5$ to be a factor of $x^4 + px^2 + q$, the values of p and q must be, respectively:

A $-2, 5$

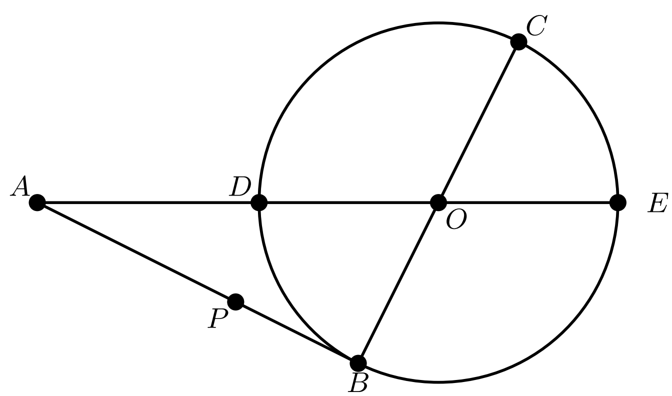
B $5, 25$

C $10, 20$

D $6, 25$

E $14, 25$

32. In this figure the center of the circle is O. $\overline{AB} \perp \overline{BC}$, $ADOE$ is a straight line, $\overline{AP} = \overline{AD}$, and $\overline{AB}$ has a length twice the radius. Then:



A $\overline{AP}^2 = \overline{PB} \cdot \overline{AB}$

B $\overline{AP} \cdot \overline{DO} = \overline{PB} \cdot \overline{AD}$

C $\overline{AB}^2 = \overline{AD} \cdot \overline{DE}$

D $\overline{AB} \cdot \overline{AD} = \overline{OB} \cdot \overline{AO}$

E none of these

33. You are given a sequence of 58 terms; each term has the form $P+n$ where P stands for the product $2 \cdot 3 \cdot 5 \cdots 61$ of all prime numbers (a prime number is a number divisible only by 1 and itself) less than or equal to 61, and n takes, successively, the values $2, 3, 4, \ldots, 59$. Let N be the number of primes appearing in this sequence. Then N is:

A 0

B 16

C 17

D 57

E 58

34. Two swimmers, at opposite ends of a 90-foot pool, start to swim the length of the pool, one at the rate of 3 feet per second, the other at 2 feet per second. They swim back and forth for 12 minutes. Allowing no loss of time at the turns, find the number of times they pass each other.

A 24

B 21

C 20

D 19

E 18

35. From point P outside a circle, with a circumference of 10 units, a tangent is drawn. Also from P a secant is drawn dividing the circle into unequal arcs with lengths m and n. It is found that t, the length of the tangent, is the mean proportional between m and n. If m and t are integers, then t may have the following number of values:

- A zero
- B one
- C two
- D three
- E infinitely many

36. Let s_1, s_2, s_3 be the respective sums of $n, 2n, 3n$ terms of the same arithmetic progression with a as the first term and d as the common difference. Let $R = s_3 - s_2 - s_1$. Then R is dependent on:

- A a and d
- B d and n
- C a and n
- D $a, d,$ and n
- E neither a nor d nor n

37. The base of a triangle is of length b, and the altitude is of length h. A rectangle of height x is inscribed in the triangle with the base of the rectangle in the base of the triangle. The area of the rectangle is:

A $\frac{bx}{h}(h-x)$

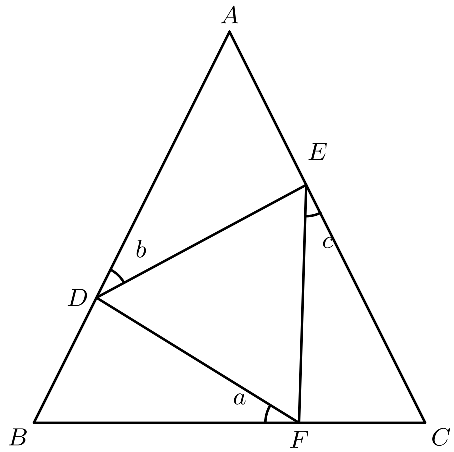
B $\frac{hx}{b}(b-x)$

C $\frac{bx}{h}(h-2x)$

D $x(b-x)$

E $x(h-x)$

38. In this diagram $\overline{AB}$ and $\overline{AC}$ are the equal sides of an isosceles triangle ABC, in which is inscribed equilateral triangle DEF. Designate angle BFD by a, angle ADE by b, and angle FEC by c. Then:



- A $b = \dfrac{a+c}{2}$
- B $b = \dfrac{a-c}{2}$
- C $a = \dfrac{b-c}{2}$
- D $a = \dfrac{b+c}{2}$
- E none of these

39. To satisfy the equation

$$\frac{a+b}{a} = \frac{b}{a+b},$$

a and b must be:

- A both rational
- B both real but not rational
- C both not real
- D one real, one not real
- E one real, one not real or both not real

40. Given right triangle ABC with legs $\overline{BC} = 3, \overline{AC} = 4.$ Find the length of the shorter angle trisector from C to the hypotenuse:

- A $\dfrac{32\sqrt{3}-24}{13}$
- B $\dfrac{12\sqrt{3}-9}{13}$
- C $6\sqrt{3}-8$
- D $\dfrac{5\sqrt{10}}{6}$
- E $\dfrac{25}{6}$

Solutions: https://live.poshenloh.com/past-contests/amc12/1960/solutions