1954 AMC 12

Time limit: 75 minutes

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1. The square of $5-\sqrt{y^2-25}$ is:

A $y^2-5\sqrt{y^2-25}$

B $-y^2$

C y^2

D $(5-y)^2$

E $y^2-10\sqrt{y^2-25}$

2. The equation

$$\frac{2x^2}{x-1} - \frac{2x+7}{3} + \frac{4-6x}{x-1} + 1 = 0$$

can be transformed by eliminating fractions to the equation $x^2 - 5x + 4 = 0$. The roots of the latter equation are 4 and 1. Then the roots of the first equation are:

A 4 and 1

B only 1

C only 4

D neither 4 nor 1

E 4 and some other root

3. If x varies as the cube of y, and y varies as the fifth root of z, then x varies as the nth power of z, where n is:

A $\frac{1}{15}$

B $\frac{5}{3}$

C $\frac{3}{5}$

D 15

E 8

4. If the Highest Common Divisor of 6432 and 132 is diminished by 8, it will equal:

A -6

B 6

C -2

D 3

E 4

5. A regular hexagon is inscribed in a circle of radius 10 inches. Its area is:

A $150\sqrt{3}$ sq. in.

B 150 sq. in.

C $25\sqrt{3}$ sq. in.

D 600 sq. in.

E $300\sqrt{3}$ sq. in.

6. The value of $\left(\frac{1}{16}\right)^{a^0} + \left(\frac{1}{16a}\right)^0 - 64^{-\frac{1}{2}} - (-32)^{-\frac{4}{5}}$ is:

A $1\frac{13}{16}$

B $1\frac{3}{16}$

C 1

D $\frac{7}{8}$

E $\frac{1}{16}$

7. A housewife saved $\$2.50$ in buying a dress on sale. If she spent $\$25$ for the dress, she saved about:

A 8%

B 9%

C 10%

D 11%

E 12%

8. The base of a triangle is twice as long as a side of a square and their areas are the same. Then the ratio of the altitude of the triangle to the side of the square is:

A $\frac{1}{4}$

B $\frac{1}{2}$

C 1

D 2

E 4

9. A point P is outside a circle and is 13 inches from the center. A secant from P cuts the circle at Q and R so that the external segment of the secant $\overline{PQ}$ is 9 inches and $\overline{QR}$ is 7 inches. The radius of the circle is:

A 3 in.

B 4 in.

C 5 in.

D 6 in.

E 7 in.

10. The sum of the numerical coefficients in the expansion of the binomial $(a+b)^6$ is:

A 32

B 16

C 64

D 48

E 7

11. A merchant placed on display some dresses, each with a marked price. He then posted a sign "$\frac{1}{3}$ off on these dresses." The cost of the dresses was $\frac{3}{4}$ of the price at which he actually sold them. Then the ratio of the cost to the marked price was:

A $\frac{1}{2}$

B $\frac{1}{3}$

C $\frac{1}{4}$

D $\frac{2}{3}$

E $\frac{3}{4}$

12. The solution of the equations

$$2x - 3y = 7,$$
$$4x - 6y = 20$$

is:

A $x = 18, y = 12$

B $x = 0, y = 0$

C There is no solution

D There are an unlimited number of solutions

E $x = 8, y = 5$

13. A quadrilateral is inscribed in a circle. If angles are inscribed in the four arcs cut off by the sides of the quadrilateral, without intersecting the quadrilateral, their sum will be:

A 180°

B 540°

C 360°

D 450°

E 1080°

14. When simplified $\sqrt{1+\left(\frac{x^4-1}{2x^2}\right)^2}$ equals:

A $\frac{x^4+2x^2-1}{2x^2}$

B $\frac{x^4-1}{2x^2}$

C $\frac{\sqrt{x^2+1}}{2}$

D $\frac{x^2}{\sqrt{2}}$

E $\frac{x^2}{2}+\frac{1}{2x^2}$

15. $\log 125$ equals:

A $100 \log 1.25$

B $5 \log 3$

C $3 \log 25$

D $3-3 \log 2$

E $(\log 25)(\log 5)$

16. If $f(x) = 5x^2 - 2x - 1$, then $f(x+h) - f(x)$ equals:

A $5h^2 - 2h$

B $10xh - 4x + 2$

C $10xh - 2x - 2$

D $h(10x + 5h - 2)$

E $3h$

17. The graph of the function $f(x) = 2x^3 - 7$ goes:

A up to the right and down to the left

B down to the right and up to the left

C up to the right and up to the left

D down to the right and down to the left

E none of these ways

18. Of the following sets, the one that includes all values of x which will satisfy $2x - 3 > 7 - x$ is:

A $x > 4$

B $x < \frac{10}{3}$

C $x = \frac{10}{3}$

D $x > \frac{10}{3}$

E $x < 0$

19. If the three points of contact of a circle inscribed in a triangle are joined, the angles of the resulting triangle:

A are always equal to 60°

B are always one obtuse angle and two unequal acute angles

C are always one obtuse angle and two equal acute angles

D are always acute angles

E are always unequal to each other

20. The equation $x^3 + 6x^2 + 11x + 6 = 0$ has:

A no negative real roots

B no positive real roots

C no real roots

D 1 positive and 2 negative roots

E 2 positive and 1 negative root

21. The roots of the equation $2\sqrt{x} + 2x^{-\frac{1}{2}} = 5$ can be found by solving:

A $16x^2 - 92x + 1 = 0$

B $4x^2 - 25x + 4 = 0$

C $4x^2 - 17x + 4 = 0$

D $2x^2 - 21x + 2 = 0$

E $4x^2 - 25x - 4 = 0$

22. The expression $\dfrac{2x^2 - x}{(x+1)(x-2)} - \dfrac{4+x}{(x+1)(x-2)}$ cannot be evaluated for $x = -1$ or $x = 2$, since division by zero is not allowed. For other values of x :

A The expression takes on many different values.

B The expression has only the value $(2.)$

C The expression has only the value $(1.)$

D The expression always has a value between -1 and $2.$

E The expression has a value greater than 2 or less than $-1.$

23. If the margin made on an article costing C dollars and selling for S dollars is $M = \dfrac{1}{n}C$, then the margin is given by:

A $M = \dfrac{1}{n-1}S$

B $M = \dfrac{1}{n}S$

C $M = \dfrac{n}{n+1}S$

D $M = \dfrac{1}{n+1}S$

E $M = \dfrac{n}{n-1}S$

24. The values of k for which the equation $2x^2 - kx + x + 8 = 0$ will have real and equal roots are:

A 9 and -7

B only -7

C 9 and 7

D -9 and -7

E only 9

25. The two roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are 1 and:

A $\dfrac{b(c-a)}{a(b-c)}$

B $\dfrac{a(b-c)}{c(a-b)}$

C $\dfrac{a(b-c)}{b(c-a)}$

D $\dfrac{c(a-b)}{a(b-c)}$

E $\dfrac{c(a-b)}{b(c-a)}$

26. The straight line AB is divided at C so that $\overline{AC} = 3\overline{CB}$. Circles are described on $\overline{AC}$ and $\overline{CB}$ as diameters and a common tangent meets $\overline{AB}$ produced at D. Then $\overline{BD}$ equals:

A the diameter of the smaller circle

B the radius of the smaller circle

C the radius of the larger circle

D $\overline{CB}\sqrt{3}$

E the difference of the two radii

27. A right circular cone has for its base a circle having the same radius as a given sphere. The volume of the cone is one-half that of the sphere. The ratio of the altitude of the cone to the radius of its base is:

A $\frac{1}{1}$

B $\frac{1}{2}$

C $\frac{2}{3}$

D $\frac{2}{1}$

E $\sqrt{\frac{5}{4}}$

28. If $\frac{m}{n} = \frac{4}{3}$ and $\frac{r}{t} = \frac{9}{14}$, the value of $\frac{3mr - nt}{4nt - 7mr}$ is:

A $-5\frac{1}{2}$

B $-\frac{11}{14}$

C $-1\frac{1}{4}$

D $\frac{11}{14}$

E $-\frac{2}{3}$

29. If the ratio of the legs of a right triangle is $1 : 2$, then the ratio of the corresponding segments of the hypotenuse made by a perpendicular upon it from the vertex is:

A $1 : 4$

B $1 : \sqrt{2}$

C $1 : 2$

D $1 : \sqrt{5}$

E $1 : 5$

30. A and B together can do a job in 2 days; B and C can do it in four days; and A and C in $2\frac{2}{5}$ days. The number of days required for A to do the job alone is:

A 1

B 3

C 6

D 12

E 2.8

31. In triangle $ABC, \overline{AB} = \overline{AC}, \angle A = 40^\circ$. Point O is within the triangle with $\angle OBC \cong \angle OCA$. The number of degrees in angle BOC is:

A 110

B 35

C 140

D 55

E 70

32. The factors of $x^4 + 64$ are:

A $(x^2 + 8)^2$

B $(x^2 + 8)(x^2 - 8)$

C $(x^2 + 2x + 4)(x^2 - 8x + 16)$

D $(x^2 - 4x + 8)(x^2 - 4x - 8)$

E $(x^2 - 4x + 8)(x^2 + 4x + 8)$

33. A bank charges $\$6$ for a loan of $\$120$. The borrower receives $\$114$ and repays the loan in 12 installments of $\$10$ a month. The interest rate is approximately:

A 5%

B 6%

C 7%

D 9%

E 15%

34. The fraction $\frac{1}{3}$:

A equals 0.33333333

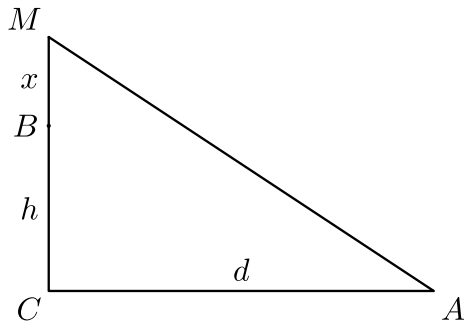
B is less than 0.33333333 by $\frac{1}{3 \cdot 10^8}$

C is less than 0.33333333 by $\frac{1}{3 \cdot 10^9}$

D is greater than 0.33333333 by $\frac{1}{3 \cdot 10^8}$

E is greater than 0.33333333 by $\frac{1}{3 \cdot 10^9}$

35. In the right triangle shown the sum of the distances $\overline{BM}$ and $\overline{MA}$ is equal to the sum of the distances $\overline{BC}$ and $\overline{CA}$. If $\overline{MB} = x, \overline{CB} = h$, and $\overline{CA} = d$, then x equals:



A $\frac{hd}{2h + d}$

B $d - h$

C $\frac{1}{2}d$

D $h + d - \sqrt{2d}$

E $\sqrt{h^2 + d^2} - h$

36. A boat has a speed of 15 mph in still water. In a stream that has a current of 5 mph it travels a certain distance downstream and returns. The ratio of the average speed for the round trip to the speed in still water is:

A $\frac{5}{4}$

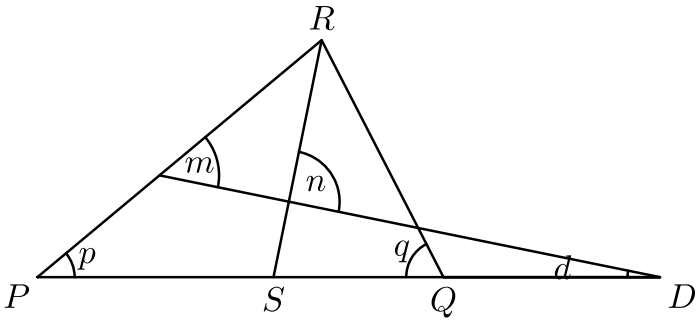
B $\frac{1}{1}$

C $\frac{8}{9}$

D $\frac{7}{8}$

E $\frac{9}{8}$

37. Given triangle PQR with $\overline{RS}$ bisecting $\angle R$, $\overline{PQ}$ extended to D, and $\angle n$ a right angle, then:



A $\angle m = \frac{1}{2}(\angle p - \angle q)$

B $\angle m = \frac{1}{2}(\angle p + \angle q)$

C $\angle d = \frac{1}{2}(\angle q + \angle p)$

D $\angle d = \frac{1}{2}\angle m$

E none of these is correct

38. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, the value of x when $3^{x+3} = 135$ is approximately:

A 5

B 1.47

C 1.67

D 1.78

E 1.63

39. The locus of the midpoint of a line segment that is drawn from a given external point P to a given circle with center O and radius r, is:

A a straight line perpendicular to $\overline{PO}$

B a straight line parallel to $\overline{PO}$

C a circle with center P and radius r

D a circle with center at the midpoint of $\overline{PO}$ and radius $2r$

E a circle with center at the midpoint of $\overline{PO}$ and radius $\frac{1}{2}r$

40. If $\left(a+\frac{1}{a}\right)^2 = 3$, then $a^3 + \frac{1}{a^3}$ equals:

A $\frac{10\sqrt{3}}{3}$

B $3\sqrt{3}$

C 0

D $7\sqrt{7}$

E $6\sqrt{3}$

41. The sum of all the roots of $4x^3 - 8x^2 - 63x - 9 = 0$ is:

A 8

B 2

C -8

D -2

E 0

42. Consider the graphs of $(1)\ y = x^2 - \frac{1}{2}x + 2$ and $(2)\ y = x^2 + \frac{1}{2}x + 2$ on the same set of axes. These parabolas have exactly the same shape. Then:

A the graphs coincide.

B the graph of (1) is lower than the graph of (2).

C the graph of (1) is to the left of the graph of (2).

D the graph of (1) is to the right of the graph of (2).

E the graph of (1) is higher than the graph of (2).

43. The hypotenuse of a right triangle is 10 inches and the radius of the inscribed circle is 1 inch. The perimeter of the triangle in inches is:

A 15

B 22

C 24

D 26

E 30

44. A man born in the first half of the nineteenth century was x years old in the year x^2. He was born in:

A 1849

B 1825

C 1812

D 1836

E 1806

45. In a rhombus $ABCD$, line segments are drawn within the rhombus, parallel to diagonal $\overline{BD}$, and terminated in the sides of the rhombus. A graph is drawn showing the length of a segment as a function of its distance from vertex A. The graph is:

A a straight line passing through the origin.

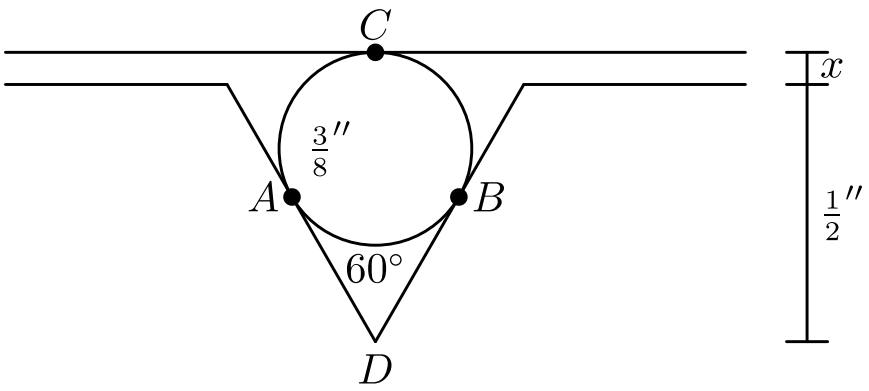
B a straight line cutting across the upper right quadrant.

C two line segments forming an upright $(\mathrm{V}.)$

D two line segments forming an inverted $(\mathrm{V}.)$

E none of these.

46. In the diagram, if points A, B, and C are points of tangency, then x equals:



A $\frac{3}{16}$ in.

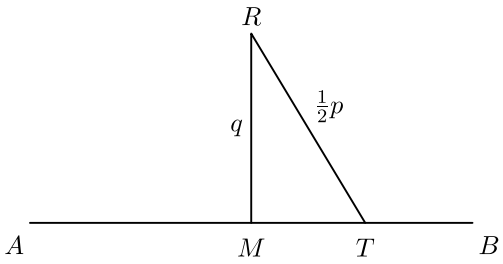
B $\frac{1}{8}$ in.

C $\frac{1}{32}$ in.

D $\frac{3}{32}$ in.

E $\frac{1}{16}$ in.

47. At the midpoint of line segment $\overline{AB}$ which is p units long, a perpendicular $\overline{MR}$ is erected with length q units. An arc is described from R with a radius equal to $\frac{1}{2}\overline{AB}$, meeting $\overline{AB}$ at T. Then $\overline{AT}$ and $\overline{TB}$ are the roots of:



A $x^2 + px + q^2 = 0$

B $x^2 - px + q^2 = 0$

C $x^2 + px - q^2 = 0$

D $x^2 - px - q^2 = 0$

E $x^2 - px + q = 0$

48. A train, an hour after starting, meets with an accident which detains it a half hour, after which it proceeds at $\frac{3}{4}$ of its former rate and arrives $3\frac{1}{2}$ hours late. Had the accident happened 90 miles farther along the line, it would have arrived only 3 hours late. The length of the trip in miles was:

A 400

B 465

C 600

D 640

E 550

49. The difference of the squares of two odd numbers is always divisible by 8. If $a > b$, and $2a + 1$ and $2b + 1$ are the odd numbers, to prove the given statement we put the difference of the squares in the form:

A $(2a + 1)^2 - (2b + 1)^2$

B $4a^2 - 4b^2 + 4a - 4b$

C $4[a(a + 1) - b(b + 1)]$

D $4(a - b)(a + b + 1)$

E $4(a^2 + a - b^2 - b)$

50. The times between 7 and 8 o'clock, correct to the nearest minute, when the hands of a clock will form an angle of 84 degrees are:

A $7{:}23$ and $7{:}53$

B $7{:}20$ and $7{:}50$

C $7{:}22$ and $7{:}53$

D $7{:}23$ and $7{:}52$

E $7{:}21$ and $7{:}49$

Solutions: https://live.poshenloh.com/past-contests/amc12/1954/solutions