1950 AMC 12 Solutions

Typeset by: LIVE by Po-Shen Loh
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1. If 64 is divided into three parts proportional to $2, 4,$ and $6,$ the smallest part is:

A $5\frac{1}{3}$

B 11

C $10\frac{2}{3}$

D 5

E None of these answers

Solution:

The ratio contains $2+4+6=12$ equal shares. The smallest part is therefore

$$64\left(\frac{2}{12}\right)=\frac{32}{3}=10\frac{2}{3}.$$

Thus, the correct answer is **C**.

2. Let $R = gS - 4$. When $S = 8, R = 16$. When $S = 10, R$ is equal to:

A 11

B 14

C 20

D 21

E None of these answers

Solution:

Substituting $S = 8$ and $R = 16$ gives $16 = 8g - 4$, so $g = \frac{5}{2}$. Hence, when $S = 10$,

$$R = \frac{5}{2}(10) - 4 = 21.$$

Thus, the correct answer is **D**.

3. The sum of the roots of the equation $4x^2 + 5 - 8x = 0$ is equal to:

A 8

B -5

C $-\dfrac{5}{4}$

D -2

E None of these answers

Solution:

In standard form the equation is $4x^2 - 8x + 5 = 0$. By Vieta's formulas, the sum of its roots is

$$-\frac{-8}{4} = 2.$$

This value is not among choices A through D.

Thus, the correct answer is **E**.

4. Reduced to lowest terms,

$$\frac{a^2 - b^2}{ab} - \frac{ab - b^2}{ab - a^2}$$

is equal to:

A $\frac{a}{b}$

B $\frac{a^2 - 2b^2}{ab}$

C a^2

D $a - 2b$

E None of these answers

Solution:

Factoring the numerators and denominator gives

$$\begin{aligned} &\frac{a^2 - b^2}{ab} - \frac{ab - b^2}{ab - a^2} \\ &= \left(\frac{a}{b} - \frac{b}{a}\right) \\ &\quad - \frac{b(a-b)}{a(b-a)} \\ &= \frac{a}{b} - \frac{b}{a} + \frac{b}{a} \\ &= \frac{a}{b}. \end{aligned}$$

Thus, the correct answer is **A**.

5. If five geometric means are inserted between 8 and 5832, the fifth term in the geometric series is:

A 648

B 832

C 1168

D 1944

E None of these answers

Solution:

There are six common-ratio steps from the first term 8 to the seventh term 5832. Thus

$$q^6 = \frac{5832}{8} = 729 = 3^6,$$

so the positive common ratio is $q = 3$. The fifth term is

$$8q^4 = 8 \cdot 3^4 = 648.$$

Thus, the correct answer is **A**.

6. The values of y which will satisfy the equations

$$\begin{aligned} 2x^2 + 6x + 5y + 1 &= 0, \\ 2x + y + 3 &= 0 \end{aligned}$$

may be found by solving:

A $y^2 + 14y - 7 = 0$

B $y^2 + 8y + 1 = 0$

C $y^2 + 10y - 7 = 0$

D $y^2 + y - 12 = 0$

E None of the above equations

Solution:

From the second equation, $x = -\frac{y+3}{2}$. Substituting into the first equation and multiplying by 2 gives

$$\begin{aligned} (y+3)^2 - 6(y+3) \\ + 10y + 2 &= 0, \\ y^2 + 10y - 7 &= 0. \end{aligned}$$

Thus, the correct answer is **C**.

7. If the digit 1 is placed after a two digit number whose tens' digit is t, and units' digit is u, the new number is:

A $10t + u + 1$

B $100t + 10u + 1$

C $1000t + 10u + 1$

D $t + u + 1$

E None of these answers

Solution:

The two digit number is $10t + u$. Placing 1 after it produces

$$\begin{aligned} &10(10t + u) + 1 \\ &= 100t + 10u + 1. \end{aligned}$$

Thus, the correct answer is **B**.

8. If the radius of a circle is increased 100%, the area is increased:

A 100%

B 200%

C 300%

D 400%

E By none of these

Solution:

Increasing r by 100% changes it to $2r$. The area changes from πr^2 to $\pi(2r)^2 = 4\pi r^2$, an increase of $3\pi r^2$. This is 300% of the original area.

Thus, the correct answer is **C**.

9. The area of the largest triangle that can be inscribed in a semicircle whose radius is r is:

A r^2

B r^3

C $2r^2$

D $2r^3$

E $\frac{1}{2}r^2$

Solution:

A triangle in the semicircle has base at most the diameter $2r$ and altitude at most r. Therefore its area is at most

$$\frac{1}{2}(2r)(r) = r^2.$$

This bound is attained by using the diameter as the base and the topmost point of the semicircle as the third vertex.

Thus, the correct answer is **A**.

10. After rationalizing the numerator of $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}$, the denominator in simplest form is:

A $\sqrt{3}(\sqrt{3}+\sqrt{2})$

B $\sqrt{3}(\sqrt{3}-\sqrt{2})$

C $3-\sqrt{3}\sqrt{2}$

D $3+\sqrt{6}$

E None of these answers

Solution:

Multiplying by the conjugate of the numerator gives

$$\begin{aligned} &\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \\ &= \frac{1}{\sqrt{3}(\sqrt{3}+\sqrt{2})} \\ &= \frac{1}{3+\sqrt{6}}. \end{aligned}$$

Thus, the correct answer is **D**.

11. If in the formula $C = \dfrac{en}{R+nr}$, n is increased while $e, R,$ and r are kept constant, then C :

A Decreases

B Increases

C Remains constant

D Increases and then decreases

E Decreases and then increases

Solution:

Rewrite the formula as

$$C = \frac{e}{\frac{R}{n}+r}.$$

As positive n increases, $\frac{R}{n}$ decreases, so the positive denominator decreases while the numerator remains fixed. Therefore C increases.

Thus, the correct answer is **B**.

12. As the number of sides of a polygon increases from 3 to n, the sum of the exterior angles formed by extending each side in succession:

A Increases

B Decreases

C Remains constant

D Cannot be predicted

E Becomes $(n-3)$ straight angles

Solution:

Traversing the polygon and turning through each exterior angle makes one complete turn. Hence the sum is always 360°, independent of the number of sides.

Thus, the correct answer is **C**.

13. The roots of $(x^2 - 3x + 2)(x)(x - 4) = 0$ are:

A 4

B 0 and 4

C 1 and 2

D $0, 1, 2,$ and 4

E $1, 2,$ and 4

Solution:

Since $x^2 - 3x + 2 = (x - 1)(x - 2)$, the equation becomes

$$(x - 1)(x - 2)x(x - 4) = 0.$$

Its roots are $0, 1, 2,$ and 4.

Thus, the correct answer is **D**.

14. For the simultaneous equations

$$2x - 3y = 8,$$
$$6y - 4x = 9,$$

A $x = 4, y = 0$

B $x = 0, y = \frac{3}{2}$

C $x = 0, y = 0$

D There is no solution

E There are an infinite number of solutions

Solution:

The left side of the second equation is $-2(2x - 3y)$. If the first equation holds, that expression must equal $-2 \cdot 8 = -16$, but the second equation says it equals 9. This contradiction means that no ordered pair satisfies both equations.

Thus, the correct answer is **D**.

15. The real factors of $x^2 + 4$ are:

- A $(x^2+2)(x^2+2)$
- B $(x^2+2)(x^2-2)$
- C $x^2(x^2+4)$
- D $(x^2-2x+2)(x^2+2x+2)$
- E Non-existent

Solution:

A real linear factor would give a real root. But $x^2+4=0$ implies $x^2=-4$, which has no real solution. Therefore the polynomial has no real linear factors.

Thus, the correct answer is **E**.

16. The number of terms in the expansion of $\left[(a+3b)^2(a-3b)^2\right]^2$ when simplified is:

A 4

B 5

C 6

D 7

E 8

Solution:

Using the difference of squares,

$$\begin{aligned}&\left[(a+3b)^2(a-3b)^2\right]^2\\&=\left[(a^2-9b^2)^2\right]^2\\&=(a^2-9b^2)^4.\end{aligned}$$

Its binomial expansion has one nonzero term for each exponent choice $0, 1, 2, 3, 4,$ so it has five terms.

Thus, the correct answer is **B**.

17. The formula which expresses the relationship between x and y as shown in the accompanying table is:

x	0	1	2	3	4
y	100	90	70	40	0

A $y = 100 - 10x$

B $y = 100 - 5x^2$

C $y = 100 - 5x - 5x^2$

D $y = 20 - x - x^2$

E None of these

Solution:

At $x = 1$, choices A and C both give 90, while B gives 95 and D gives 18. Testing the two survivors at $x = 2$, choice A gives 80, whereas choice C gives

$$100 - 5(2) - 5(2^2) = 70,$$

matching the table.

Thus, the correct answer is **C**.

18. Of the following

$$(1) \quad a(x-y) = ax - ay$$

$$(2) \quad a^{x-y} = a^x - a^y$$

$$(3) \quad \log(x-y) = \log x - \log y$$

$$(4) \quad \frac{\log x}{\log y} = \log x - \log y$$

$$(5) \quad a(xy) = ax \times ay$$

A Only 1 and 4 are true

B Only 1 and 5 are true

C Only 1 and 3 are true

D Only 1 and 2 are true

E Only 1 is true

Solution:

Statement 1 is the distributive property. Statement 2 is false because $a^{x-y} = \frac{a^x}{a^y}$. Statement 3 is false because $\log x - \log y = \log(\frac{x}{y})$. Statement 4 confuses the change-of-base quotient with a difference, and statement 5 has right side a^2xy, not axy.

Thus, only statement 1 is true, so the correct answer is **E**.

19. If m men can do a job in d days, then $m+r$ men can do the job in:

A $d+r$ days

B $d-r$ days

C $\frac{md}{m+r}$ days

D $\frac{d}{m+r}$ days

E None of these

Solution:

The job requires $m \cdot d = md$ man-days. With $m+r$ men working at the same individual rate, the required number of days is

$$\frac{md}{m+r}.$$

Thus, the correct answer is **C**.

20. When $x^{13}+1$ is divided by $x-1$, the remainder is:

- A 1
- B -1
- C 0
- D 2
- E None of these answers

Solution:

By the Remainder Theorem, the remainder upon division by $x-1$ is the value of the polynomial at $x=1$. This value is

$$1^{13}+1=2.$$

Thus, the correct answer is **D**.

21. The volume of a rectangular solid each of whose side, front, and bottom faces are 12 square inches, 8 square inches, and 6 square inches respectively is:

A 576 cubic inches

B 24 cubic inches

C 9 cubic inches

D 104 cubic inches

E None of these

Solution:

If the edge lengths are x, y, and z, then the three face areas are xy, yz, and xz. Therefore

$$\begin{aligned}(xyz)^2 &= (xy)(yz)(xz)\\ &= 12 \cdot 8 \cdot 6 = 576.\end{aligned}$$

Since the volume is positive, $xyz = \sqrt{576} = 24$ cubic inches.

Thus, the correct answer is **B**.

22. Successive discounts of 10% and 20% are equivalent to a single discount of:

A 30%

B 15%

C 72%

D 28%

E None of these

Solution:

Starting from a price of $100,$ the first discount leaves $90.$ The second discount leaves $0.8(90) = 72.$ The total reduction is therefore $100 - 72 = 28,$ or $28\%.$

Thus, the correct answer is **D**.

23. A man buys a house for $\$10{,}000$ and rents it. He puts $12\frac{1}{2}\%$ of each month's rent aside for repairs and upkeep; pays $\$325$ a year taxes and realizes $5\frac{1}{2}\%$ on his investment. The monthly rent is:

A $\$64.82$

B $\$83.33$

C $\$72.08$

D $\$45.83$

E $\$177.08$

Solution:

Let the monthly rent be r dollars. The yearly rent is $12r$, of which $1 - 0.125 = \frac{7}{8}$ remains after setting aside the upkeep money. The desired yearly return is $0.055(10,000) = 550$ dollars, so

$$\frac{7}{8}(12r) - 325 = 550.$$

Hence $10.5r = 875$, giving $r = 83.\overline{3}$, which is $\$83.33$ to the nearest cent.

Thus, the correct answer is **B**.

24. The equation $x + \sqrt{x-2} = 4$ has:

A 2 real roots

B 1 real and 1 imaginary root

C 2 imaginary roots

D No roots

E 1 real root

Solution:

Isolating and squaring gives

$$\begin{aligned} \sqrt{x-2} &= 4 - x, \\ x - 2 &= (4-x)^2, \\ x^2 - 9x + 18 &= 0. \end{aligned}$$

The quadratic roots are 3 and 6. The value 3 satisfies the original equation, but 6 gives $6 + \sqrt{4} = 8$, so it is extraneous.

Thus, the correct answer is **E**.

25. The value of $\log_5 \dfrac{(125)(625)}{25}$ is equal to:

A 725

B 6

C 3125

D 5

E None of these answers

Solution:

Since $125 = 5^3, 625 = 5^4$, and $25 = 5^2$,

$$\log_5 \frac{(125)(625)}{25}$$
$$= \log_5 \left(5^{3+4-2}\right)$$
$$= \log_5(5^5) = 5.$$

Thus, the correct answer is **D**.

26. If $\log_{10} m = b - \log_{10} n$, then $m =$

A $\dfrac{b}{n}$

B bn

C $10^b n$

D $b - 10^n$

E $\dfrac{10^b}{n}$

Solution:

Rearranging and using the product rule for logarithms gives

$$\log_{10} m + \log_{10} n$$
$$= \log_{10}(mn) = b.$$

Therefore $mn = 10^b$, so $m = \dfrac{10^b}{n}$.

Thus, the correct answer is **E**.

27. A car travels 120 miles from A to B at 30 miles per hour but returns the same distance at 40 miles per hour. The average speed for the round trip is closest to:

- A 33 mph
- B 34 mph
- C 35 mph
- D 36 mph
- E 37 mph

Solution:

The total distance is 240 miles. The travel times are 4 hours and 3 hours, so the average speed is

$$\frac{240}{4+3} = \frac{240}{7} \approx 34.29 \text{ mph.}$$

This is closest to 34 mph.

Thus, the correct answer is **B**.

28. Two boys A and B start at the same time to ride from Port Jervis to Poughkeepsie, 60 miles away. A travels 4 miles an hour slower than B. B reaches Poughkeepsie and at once turns back meeting A 12 miles from Poughkeepsie. The rate of A was:

- A 4 mph
- B 8 mph
- C 12 mph
- D 16 mph
- E 20 mph

Solution:

At the meeting point, A has traveled $60 - 12 = 48$ miles and B has traveled $60 + 12 = 72$ miles. Since their travel times are equal, their speeds are in the ratio $48 : 72 = 2 : 3$. If A's speed is v, then B's is $v + 4$, so

$$\frac{v}{v+4} = \frac{2}{3}.$$

Thus $3v = 2v + 8$, and $v = 8$ mph.

Thus, the correct answer is **B**.

29. A manufacturer built a machine which will address 500 envelopes in 8 minutes. He wishes to build another machine so that when both are operating together they will address 500 envelopes in 2 minutes. The equation used to find how many minutes x it would require the second machine to address 500 envelopes alone is:

A $8 - x = 2$

B $\frac{1}{8} + \frac{1}{x} = \frac{1}{2}$

C $\frac{500}{8} + \frac{500}{x} = 500$

D $\frac{x}{2} + \frac{x}{8} = 1$

E None of these answers

Solution:

The first machine completes $\frac{1}{8}$ of a 500-envelope batch per minute, and the second completes $\frac{1}{x}$ of a batch per minute. Together they must complete $\frac{1}{2}$ of a batch per minute. Therefore the required equation is

$$\frac{1}{8} + \frac{1}{x} = \frac{1}{2}.$$

Thus, the correct answer is **B**.

30. From a group of boys and girls, 15 girls leave. There are then left two boys for each girl. After this 45 boys leave. There are then 5 girls for each boy. The number of girls in the beginning was:

A 40

B 43

C 29

D 50

E None of these

Solution:

Let the original counts be G girls and B boys. The two conditions give

$$\begin{aligned} B &= 2(G-15), \\ G-15 &= 5(B-45). \end{aligned}$$

Substituting the first into the second yields

$$\begin{aligned} G-15 &= 5\big(2(G-15)-45\big) \\ &= 10G-375. \end{aligned}$$

Hence $9G = 360$, so $G = 40$.

Thus, the correct answer is **A**.

31. John ordered 4 pairs of black socks and some additional pairs of blue socks. The price of the black socks per pair was twice that of the blue. When the order was filled, it was found that the number of pairs of the two colors had been interchanged. This increased the bill by 50%. The ratio of the number of pairs of black socks to the number of pairs of blue socks in the original order was:

- A $4:1$
- B $2:1$
- C $1:4$
- D $1:2$
- E $1:8$

Solution:

Let a blue pair cost p, so a black pair costs $2p$, and let n be the original number of blue pairs. The original bill is $(8+n)p$. After the quantities are interchanged, the bill is $(2n+4)p$. The latter is 50% greater, so

$$2n+4=\frac{3}{2}(n+8).$$

Thus $n=16$, and the original black-to-blue ratio is $4:16=1:4$.

Thus, the correct answer is **C**.

32. A 25 foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips 4 feet, then the foot of the ladder will slide:

A 9 ft

B 15 ft

C 5 ft

D 8 ft

E 4 ft

Solution:

The initial height is

$$\sqrt{25^2 - 7^2} = \sqrt{576} = 24 \text{ feet.}$$

After the top slips, the height is 20 feet, so the new horizontal distance is $\sqrt{25^2 - 20^2} = 15$ feet. The foot therefore slides $15 - 7 = 8$ feet.

Thus, the correct answer is **D**.

33. The number of circular pipes with an inside diameter of 1 inch which will carry the same amount of water as a pipe with an inside diameter of 6 inches is:

A 6π

B 6

C 12

D 36

E 36π

Solution:

The ratio of the diameters is $6:1$, so the ratio of cross-sectional areas is

$$6^2 : 1^2 = 36 : 1.$$

Thus 36 of the smaller pipes have the same total cross-sectional area as the larger pipe.

Thus, the correct answer is **D**.

34. When the circumference of a toy balloon is increased from 20 inches to 25 inches, the radius is increased by:

A 5 in

B $2\frac{1}{2}$ in

C $\frac{5}{\pi}$ in

D $\frac{5}{2\pi}$ in

E $\frac{\pi}{5}$ in

Solution:

Because $C = 2\pi r$, the changes satisfy

$$25 - 20 = 2\pi(r_{\text{new}} - r_{\text{old}}).$$

Hence the radius increases by $\frac{5}{2\pi}$ inches.

Thus, the correct answer is **D**.

35. In triangle ABC, $AC = 24$ inches, $BC = 10$ inches, $AB = 26$ inches. The radius of the inscribed circle is:

A 26 in

B 4 in

C 13 in

D 8 in

E None of these

Solution:

Since $10^2 + 24^2 = 26^2$, the triangle is right. Its area is $\frac{1}{2}(10)(24) = 120$, and its semiperimeter is $\frac{10+24+26}{2} = 30$. Using $K = rs$, the inradius is

$$r = \frac{K}{s} = \frac{120}{30} = 4 \text{ inches.}$$

Thus, the correct answer is **B**.

36. A merchant buys goods at 25% off the list price. He desires to mark the goods so that he can give a discount of 20% on the marked price and still clear a profit of 25% on the selling price. What per cent of the list price must he mark the goods?

A 125%

B 100%

C 120%

D 80%

E 75%

Solution:

Let the list and marked prices be L and M. The merchant's cost is $0.75L$, while the selling price after the discount is $0.8M$. A profit equal to 25% of the selling price means that the cost is the remaining 75% of that price. Therefore

$$0.75L = 0.75(0.8M) = 0.6M.$$

Thus $M = 1.25L$, or 125% of the list price.

Thus, the correct answer is **A**.

37. If $y = \log_a x, a > 1,$ which of the following statements is incorrect?

A. If $x = 1, y = 0$

B. If $x = a, y = 1$

C. If $x = -1, y$ is imaginary (complex)

D. If 0<x<1, y is always less than 0 and decreases without limit as x approaches zero

E. Only some of the above statements are correct

Solution:

We have $\log_a 1 = 0$ and $\log_a a = 1.$ A real logarithm is not defined at $-1,$ though its complex values are nonreal. For $a > 1, \log_a x < 0$ on $0 < x < 1,$ and it tends to $-\infty$ as x approaches 0 from the right. Thus statements A through D are all correct, making the claim that only some are correct the incorrect statement.

Thus, the correct answer is **E**.

38. If the expression

$$\begin{vmatrix} a & c \\ d & b \end{vmatrix}$$

has the value $ab - cd$ for all values of a, b, c, and d, then the equation

$$\begin{vmatrix} 2x & 1 \\ x & x \end{vmatrix} = 3$$

A Is satisfied for only 1 value of x

B Is satisfied for 2 values of x

C Is satisfied for no values of x

D Is satisfied for an infinite number of values of x

E None of these

Solution:

The given rule turns the equation into

$$(2x)(x) - (1)(x) = 3,$$

or $2x^2 - x - 3 = 0$. Factoring gives $(2x-3)(x+1) = 0$, whose two distinct solutions are $x = \frac{3}{2}$ and $x = -1$.

Thus, the correct answer is **B**.

39. Given the series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$ and the following five statements:

(1) the sum increases without limit.

(2) the sum decreases without limit.

(3) the difference between any term of the sequence and zero can be made less than any positive quantity no matter how small.

(4) the difference between the sum and 4 can be made less than any positive quantity no matter how small.

(5) the sum approaches a limit.

Of these statements, the correct ones are:

- A Only 3 and 4
- B Only 5
- C Only 2 and 4
- D Only $2, 3,$ and 4
- E Only 4 and 5

Solution:

The partial sums increase toward

$$\frac{2}{1 - \frac{1}{2}} = 4,$$

so they neither increase nor decrease without limit. This makes statements 1 and 2 false, while statements 4 and 5 are true.

In statement $3,$ “any term” refers to an already selected term, whose distance from zero is fixed and cannot be made smaller; one can instead choose a sufficiently late term below any prescribed positive bound. Thus statement 3, as worded, is false.

Therefore only statements 4 and 5 are correct, so the correct answer is **E**.

40. The limit of $\frac{x^2-1}{x-1}$ as x approaches 1 as a limit is:

A 0

B Indeterminate

C $x-1$

D 2

E 1

Solution:

For $x \neq 1$,

$$\begin{aligned} \frac{x^2-1}{x-1} &= \frac{(x-1)(x+1)}{x-1} \\ &= x+1. \end{aligned}$$

Therefore the limit as x approaches 1 is $1+1=2$.

Thus, the correct answer is **D**.

41. The least value of the function $ax^2 + bx + c$ with $a > 0$ is:

A $-\dfrac{b}{a}$

B $-\dfrac{b}{2a}$

C $b^2 - 4ac$

D $\dfrac{4ac - b^2}{4a}$

E None of these

Solution:

Completing the square gives

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}.$$

Because $a > 0$, the squared term has minimum 0. The least value is therefore

$$c - \frac{b^2}{4a} = \frac{4ac - b^2}{4a}.$$

Thus, the correct answer is **D**.

42. The equation $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 2$ is satisfied when x is equal to:

A Infinity

B 2

C $\sqrt[4]{2}$

D $\sqrt{2}$

E None of these

Solution:

Let the value of the infinite tower be T. Removing its bottom x leaves the same tower as the exponent, so $T = x^T$. Since $T = 2$, we obtain

$$2 = x^2.$$

The positive base is therefore $x = \sqrt{2}$, for which the tower is convergent.

Thus, the correct answer is **D**.

43. The sum to infinity of $\frac{1}{7} + \frac{2}{7^2} + \frac{1}{7^3} + \frac{2}{7^4} + \cdots$ is:

A $\frac{1}{5}$

B $\frac{1}{24}$

C $\frac{5}{48}$

D $\frac{1}{16}$

E None of these

Solution:

Grouping consecutive terms gives a geometric series whose first grouped term is

$$\frac{1}{7} + \frac{2}{49} = \frac{9}{49}$$

and whose ratio is $\frac{1}{49}$. Hence the sum is

$$\frac{\frac{9}{49}}{1 - \frac{1}{49}} = \frac{9}{48} = \frac{3}{16}.$$

This is not among choices A through D.

Thus, the correct answer is **E**.

44. The graph of $y = \log x$

A Cuts the y-axis

B Cuts all lines perpendicular to the x-axis

C Cuts the x-axis

D Cuts neither axis

E Cuts all circles whose center is at the origin

Solution:

Since $\log 1 = 0$, the graph passes through $(1, 0)$ and therefore cuts the x-axis. It cannot cut the y-axis because $x = 0$ is outside its domain.

Thus, the correct answer is **C**.

45. The number of diagonals that can be drawn in a polygon of 100 sides is:

A 4850

B 4950

C 9900

D 98

E 8800

Solution:

There are $\binom{100}{2}$ segments joining pairs of vertices. Exactly 100 of these are sides, so the number of diagonals is

$$\begin{aligned}\binom{100}{2} - 100 &= 4950 - 100 \\ &= 4850.\end{aligned}$$

Thus, the correct answer is **A**.

46. In triangle $ABC, AB = 12, AC = 7,$ and $BC = 10.$ If sides AB and AC are doubled while BC remains the same, then:

A The area is doubled

B The altitude is doubled

C The area is four times the original area

D The median is unchanged

E The area of the triangle is 0

Solution:

The new side lengths are $24, 14,$ and $10.$ Because

$$14 + 10 = 24,$$

they form a degenerate triangle: all three vertices are collinear. Its area is therefore $0.$

Thus, the correct answer is **E**.

47. A rectangle inscribed in a triangle has its base coinciding with the base b of the triangle. If the altitude of the triangle is h, and the altitude x of the rectangle is half the base of the rectangle, then:

A $x = \frac{1}{2}h$

B $x = \frac{bh}{b+h}$

C $x = \frac{bh}{2h+b}$

D $x = \sqrt{\frac{hb}{2}}$

E $x = \frac{1}{2}b$

Solution:

By similarity, the width of the triangle at height x above its base is $b(1 - \frac{x}{h})$. This is the base of the inscribed rectangle. Since the rectangle's altitude is half its base, its base is $2x$. Hence

$$2x = b\left(1 - \frac{x}{h}\right).$$

Multiplying by h and solving gives $x(2h+b) = bh$, so $x = \frac{bh}{2h+b}$.

Thus, the correct answer is **C**.

48. A point is selected at random inside an equilateral triangle. From this point perpendiculars are dropped to each side. The sum of these perpendiculars is:

A Least when the point is the center of gravity of the triangle

B Greater than the altitude of the triangle

C Equal to the altitude of the triangle

D One-half the sum of the sides of the triangle

E Greatest when the point is the center of gravity

Solution:

Let the equilateral triangle have side length s and altitude h, and let the three perpendicular distances be d_1, d_2, d_3. Splitting the triangle at the interior point gives

$$\frac{1}{2}s(d_1 + d_2 + d_3) = \frac{1}{2}sh.$$

Therefore $d_1 + d_2 + d_3 = h$, independent of the selected point.

Thus, the correct answer is **C**.

49. A triangle has a fixed base AB that is 2 inches long. The median from A to side BC is $1\dfrac{1}{2}$ inches long and can have any position emanating from A. The locus of the vertex C of the triangle is:

A A straight line $AB, 1\dfrac{1}{2}$ inches from A

B A circle with A as center and radius 2 inches

C A circle with A as center and radius 3 inches

D A circle with radius 3 inches and center 4 inches from B along BA

E An ellipse with A as focus

Solution:

Put A at the origin and regard B, C, and the midpoint M of BC as vectors. Since $AM = \frac{3}{2}$, the point M moves on a circle of radius $\frac{3}{2}$ centered at A. The midpoint relation gives

$$C = 2M - B.$$

Thus the locus of C is the image of that circle under a dilation by 2 followed by translation by $-B$. It is a circle of radius 3 centered at $-B$.

Because $AB = 2$, the point $-B$ is 4 inches from B along the ray BA. Hence the correct answer is **D**.

50. A privateer discovers a merchantman 10 miles to leeward at $11{:}45$ a.m. and with a good breeze bears down upon her at 11 mph, while the merchantman can only make 8 mph in her attempt to escape. After a two hour chase, the top sail of the privateer is carried away: she can now make only 17 miles while the merchantman makes 15. The privateer will overtake the merchantman at:

- A $3{:}45$ p.m.
- B $3{:}30$ p.m.
- C $5{:}00$ p.m.
- D $2{:}45$ p.m.
- E $5{:}30$ p.m.

Solution:

During the first two hours, the privateer gains

$$2(11-8)=6 \text{ miles},$$

reducing the gap from 10 miles to 4 miles at $1{:}45$ p.m.

After the damage, the ships' speeds are in the ratio $17:15$. Since the merchantman still travels at 8 mph, the privateer's new speed is $8\cdot\frac{17}{15}=\frac{136}{15}$ mph. The closing speed is therefore $\frac{136}{15}-8=\frac{16}{15}$ mph, so closing the remaining gap takes

$$\frac{4}{\frac{16}{15}}=\frac{15}{4}=3\frac{3}{4} \text{ hours.}$$

Adding 3 hours 45 minutes to $1{:}45$ p.m. gives $5{:}30$ p.m.

Thus, the correct answer is **E**.

Problems: https://live.poshenloh.com/past-contests/amc12/1950