2026 AMC 10A Solutions

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1. Helena's vinyl record collection contains 33 LP records and 45 EP records. An LP record plays for three times as long as an EP record. How many more EP records must Helena buy so that the total playing time of her EP records equals the total playing time of her LP records?

A 45

B 54

C 63

D 78

E 87

Small hint:

33 LP records play as long as 3×33 EP records

Big hint:

Subtract the 45 EP records Helena already has

Solution:

Each LP record plays as long as three EP records, so the 33 LP records together play as long as $3 \times 33 = 99$ EP records. Helena already has 45 EP records, so she needs $99 - 45 = 54$ more.

Thus, the correct answer is **B**.

2. A sign in a parking lot states that the total price includes a 27% tax. To the nearest whole number, what percentage of the amount a customer pays is this tax?

A 21%

B 22%

C 27%

D 37%

E 73%

Small hint:

A $\$100$ price means $\$27$ tax and $\$127$ paid

Big hint:

$\frac{27}{127}$ is close to $\frac{25}{125}$

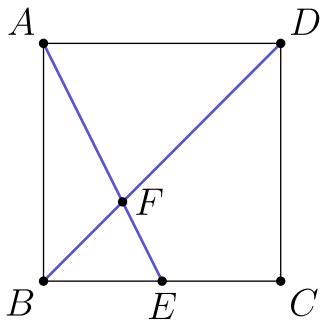
Solution:

Say the price before tax is $\$100$. Then the tax is $\$27$, and the customer pays $\$127$ in all. So the tax is $\frac{27}{127}$ of the amount paid.

This is close to $\frac{25}{125} = \frac{1}{5}$, so the tax is near 20%. Now $0.21 \times 127 = 26.67$ and $0.22 \times 127 = 27.94$, and 27 is closer to 26.67. So to the nearest whole number the tax is 21% of the amount paid.

Thus, the correct answer is **A**.

3. Square $ABCD$ has side length 24. Let E be the midpoint of $\overline{BC}$, and let F be the intersection of $\overline{AE}$ and $\overline{BD}$.



What is the area of $\triangle BEF$?

A 36

B 40

C 45

D 48

E 52

Small hint:

$\overline{BE} \parallel \overline{AD}$, so triangles FBE and FDA are similar

Big hint:

BE is half of AD, so $\triangle FBE$ is half as tall as $\triangle FDA$

Solution:

Since $\overline{BE} \parallel \overline{AD}$, triangles FBE and FDA are similar, and BE is half of AD. So the height of $\triangle FBE$ is half the height of $\triangle FDA$. The two heights add up to the side 24, so F is one third of the way up, at height 8.

Then $\triangle BEF$ has area $\frac{1}{2} \times 12 \times 8 = 48$.

Alternatively, $\overline{AE}$ is a median of $\triangle ABC$. The diagonal $\overline{BD}$ passes through the center of the square, which is the midpoint of $\overline{AC}$, so it contains the median from B. So F is

the centroid of $\triangle ABC$. The three medians cut $\triangle ABC$ into six triangles of equal area, and $\triangle BEF$ is one of them. So its area is $\frac{1}{6}$ of $\frac{1}{2} \times 24 \times 24 = 288$, which is 48.

Thus, the correct answer is **D**.

4. Half of the internal angles of a polygon measure 150° each, and the other half of its internal angles measure 190° each. How many sides does the polygon have?

A 36

B 40

C 60

D 72

E 180

Small hint:

Exterior angles, each 180° minus the interior angle, sum to 360° even when some are negative

Big hint:

The interior angles average 170°

Solution:

The exterior angle at each corner is 180° minus the interior angle: 30° at the 150° angles and -10° at the 190° angles, where the boundary turns back the other way. Going once around any polygon turns through 360° in all, so the exterior angles add up to 360°.

The interior angles average 170°, so the exterior angles average 10°. With n sides, $10n = 360$, so $n = 36$.

Thus, the correct answer is **A**.

5. Let $a_1, a_2, a_3, \ldots$ be a sequence with $a_1 = 94, a_2 = 190$, and a_n equal to the arithmetic mean of the previous $n-1$ terms for $n \geq 3$. What is the value of a_{10}?

A	47
B	95
C	142
D	284
E	564

Small hint:

a_3 is the mean of 94 and 190

Big hint:

Appending the current mean leaves the mean unchanged

Solution:

First, $a_3 = \frac{94+190}{2} = 142$. Now the first three terms have mean 142, because a_3 equals the mean of the two terms before it. So $a_4 = 142$.

Each time we append a term equal to the current mean, the mean stays the same. Therefore every term from a_3 on equals 142, and $a_{10} = 142$.

Thus, the correct answer is **C**.

6. A triangle with sides $16, 30,$ and 34 is inscribed in a circle, dividing the interior of the circle into four regions. Let $A, B,$ and C be the areas of the non-triangular regions, with C being the greatest. What is the value of $A + B$?

A $\frac{289}{2}\pi - 240$

B $\frac{289}{2}\pi - 210$

C $289\pi - 480$

D $289\pi - 360$

E $289\pi - 240$

Small hint:

Halve the sides: $8, 15, 17$

Big hint:

A right triangle's hypotenuse is a diameter of its circle

Solution:

Halving the sides gives $8, 15, 17$, a Pythagorean triple, so the triangle is right. Its hypotenuse 34 is a diameter, and the radius is 17.

The diameter cuts the disk into two halves of area $\frac{289}{2}\pi$ each. One half contains no part of the triangle, so it is the largest region C. The other half is the triangle together with regions A and B, so $A + B = \frac{289}{2}\pi - \frac{1}{2} \times 16 \times 30 = \frac{289}{2}\pi - 240$.

Thus, the correct answer is **A**.

7. How many ordered triples of positive integers (x, y, z) satisfy both of the following equations?

$$x + y + z = 31$$
$$x + 3y + 3z = 41$$

A 4

B 5

C 6

D 7

E 8

Small hint:

Subtract the first equation from the second

Big hint:

That fixes $y + z$ and x. Count the ways to split $y + z$ into two positive integers.

Solution:

Subtracting the first equation from the second gives $2y + 2z = 10$, so $y + z = 5$ and $x = 31 - 5 = 26$. The pair (y, z) can be $(1, 4), (2, 3), (3, 2)$, or $(4, 1)$, so there are 4 triples.

Thus, the correct answer is **A**.

8. A driver sees an electronic highway sign stating that the next exit is 7 miles away and it will take 5 minutes to reach it. Assume both numbers have been rounded to the nearest integer. Among the following options, what is the lowest speed limit (in miles per hour) that would possibly allow the driver to legally reach the exit within the given time frame?

A 65

B 70

C 75

D 80

E 85

Small hint:

The distance is at least 6.5 miles, and the time is under 5.5 minutes

Big hint:

Use the shortest distance and the longest time, in miles per hour

Solution:

The exit is at least 6.5 miles away, and the trip takes less than 5.5 minutes, since 5.5 would round up to 6. So the speed must be more than

$$\frac{6.5}{5.5/60} = \frac{6.5 \times 60}{5.5}$$
$$= \frac{13 \times 60}{11} = \frac{780}{11} \approx 70.9$$

miles per hour, and the lowest choice that can work is 75. (At 75 mph, 6.5 miles takes 5.2 minutes.)

Thus, the correct answer is **C**.

9. Two similar but not congruent scalene triangles each have a side with length 9 and a side with length 12. What is the length of the longest side of the larger triangle?

A 12

B 14

C 15

D 16

E 17

Small hint:

Scaling by $k > 1$ lengthens every side, so the smaller triangle's 9 must become the larger triangle's 12

Big hint:

The larger triangle's 9 must come from the smaller triangle's third side

Solution:

Let the smaller triangle have sides $x, 9$, and 12, and scale it by $k > 1$. The larger triangle has sides $kx, 9k$, and $12k$, two of which are 9 and 12. Since $9k > 9$ and $12k > 12$, the only possibility is $kx = 9$ and $9k = 12$.

So $k = \frac{4}{3}$, and the larger triangle has sides $9, 12$, and $12 \times \frac{4}{3} = 16$. It comes from the smaller triangle with sides $\frac{27}{4}, 9$, and 12, which is a genuine triangle since $\frac{27}{4} + 9 > 12$. The answer is 16.

Thus, the correct answer is **D**.

10. For what odd positive integer n is the mean of $1, 4, 9, 16, \ldots, n^2$ exactly 32% greater than its median?

A 19

B 39

C 49

D 89

E 169

Small hint:

The mean is $\frac{(n+1)(2n+1)}{6}$ and the median is $\left(\frac{n+1}{2}\right)^2$

Big hint:

Their ratio is $\frac{2(2n+1)}{3(n+1)}$. Set it to $\frac{33}{25}$.

Solution:

The mean of the n squares is $\frac{(n+1)(2n+1)}{6}$, and since n is odd, the median is the middle square, $\left(\frac{n+1}{2}\right)^2$. The factor $n+1$ cancels in their ratio:

$$\frac{(n+1)(2n+1)}{6} \cdot \frac{4}{(n+1)^2} = \frac{2(2n+1)}{3(n+1)}.$$

Setting this equal to $\frac{33}{25}$ gives $50(2n+1) = 99(n+1)$, so $n = 49$.

Thus, the correct answer is **C**.

11. Gary writes down six distinct positive integers: $2, 5, 24, 60, 240$, and a three-digit number n. One of those six numbers is the least common multiple of all six of them. What is the sum of all possible values of n?

A 960

B 1680

C 2160

D 2280

E 2400

Small hint:

$2, 5, 24$, and 60 all divide 240

Big hint:

The least common multiple of all six is 240, so $n \mid 240$, or it is n, so $240 \mid n$

Solution:

Each of $2, 5, 24$, and 60 divides 240, so the least common multiple of all six numbers is a multiple of both 240 and n. It is at least 240, and $2, 5, 24$, and 60 are all smaller, so it must be 240 or n.

If it is 240, then n is a three-digit divisor of 240 other than 240 itself, so $n = 120$. If it is n, then n is a three-digit multiple of 240 other than 240, so n is $480, 720$, or 960. The sum is $120 + 480 + 720 + 960 = 2280$.

Thus, the correct answer is **D**.

12. A drawer has 100 red socks and 100 blue socks. Pippi randomly removes one sock at a time, without replacement, until all the socks of one color have been removed. What is the probability that there are exactly 2 socks left in the drawer once that happens?

A $\frac{27}{196}$

B $\frac{49}{198}$

C $\frac{247}{990}$

D $\frac{50}{199}$

E $\frac{25}{99}$

Small hint:

Imagine all 200 socks drawn. The socks left are the final run of one color.

Big hint:

Exactly 2 are left when the last two match and the third-to-last does not

Solution:

Picture all 200 socks coming out in a random order. Pippi stops when one color runs out, and the socks still in the drawer are the final run of one color at the end of the order. So exactly 2 are left when the last two socks match and the third-to-last does not.

Whatever the color of the last sock, the second-to-last matches it with probability $\frac{99}{199}$, and then the third-to-last has the other color with probability $\frac{100}{198}$. The answer is $\frac{99}{199} \times \frac{100}{198} = \frac{50}{199}$.

Thus, the correct answer is **D**.

13. Two bicycle clubs, with disjoint memberships, go on a tour together. On the morning of the tour, members greet each other with a handshake. Everyone shakes hands with everyone else exactly once. There were 119 handshakes between members of the same club and 112 handshakes between members of different clubs. How many more people came from the larger club than from the smaller club?

A 0

B 4

C 6

D 7

E 8

Small hint:

$119 + 112 = 231$ handshakes, and n people make $\frac{n(n-1)}{2}$

Big hint:

Clubs of a and b people make ab handshakes between clubs. Compare $(a+b)^2$ and $(a-b)^2$.

Solution:

Everyone shook hands once with everyone else, so if n people came, $\frac{n(n-1)}{2} = 119 + 112 = 231$. Then $n(n-1) = 462 = 22 \times 21$, so $n = 22$.

If the clubs have a and b people, then $a + b = 22$, and the handshakes between the clubs number $ab = 112$. So $(a-b)^2 = (a+b)^2 - 4ab = 484 - 448 = 36$, and the larger club has 6 more people.

Thus, the correct answer is **C**.

14. Consider a triangle with positive area whose side lengths are functions of t given by $a = t^2 + 1, b = 5t + 7$, and $c = 4t + 4$. How many different integer values of t are possible?

A 0

B 7

C 9

D 10

E infinitely many

Small hint:

Compute $a + c - b$ and $b + c - a$, and factor each

Big hint:

Both have the factor $t + 1$, which must be positive

Solution:

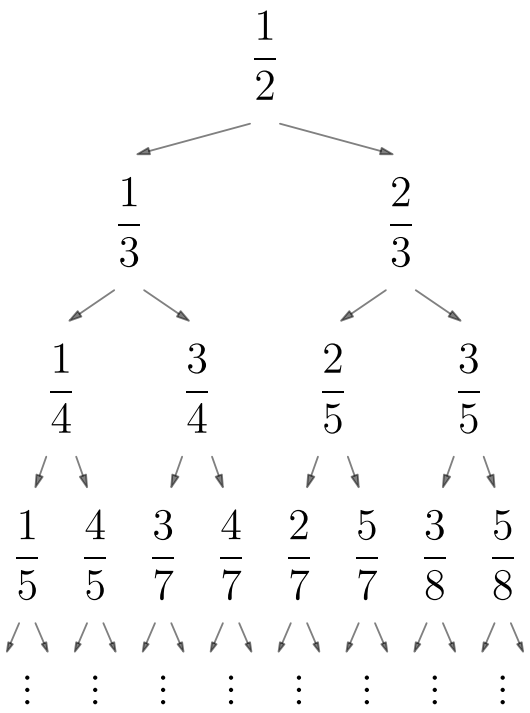
For $c = 4t + 4$ to be positive we need $t > -1$, and then a and b are positive too.

The triangle inequalities factor neatly. First, $a + b - c = t^2 + t + 4$ is always positive. Next, $a + c - b = t^2 - t - 2 = (t - 2)(t + 1)$ and $b + c - a = -t^2 + 9t + 10 = (10 - t)(t + 1)$. Since $t + 1 > 0$, we need $t > 2$ and $t < 10$.

So $t = 3, 4, \ldots, 9$, which is 7 values.

Thus, the correct answer is **B**.

15. The tree diagram below has nodes labeled with rational numbers. The top node, at level 0, is $\frac{1}{2}$. Each node $\frac{a}{b}$ sprouts two nodes in the level below it, $\frac{a}{a+b}$ and $\frac{b}{a+b}$. For example, level 1 has nodes $\frac{1}{1+2} = \frac{1}{3}$ and $\frac{2}{1+2} = \frac{2}{3}$.



At what level does the node $\frac{20}{27}$ appear in the diagram?

A 5

B 8

C 12

D 26

E The node $\frac{20}{27}$ does not appear in the diagram.

Small hint:

Going up a level, $\frac{p}{q}$ becomes a fraction made of p and $q - p$

Big hint:

Starting from 20 and 27, keep subtracting the smaller number from the larger until you reach 1 and 2

Solution:

Every node is a fraction less than 1 in lowest terms. If a node is $\frac{p}{q}$, its parent is made of p and $q-p$, in whichever order gives a fraction less than 1. So each step up replaces the pair of numbers by the smaller one and their difference, as in the Euclidean algorithm.

Starting from $(20, 27)$, the pairs are $(7, 20), (7, 13), (6, 7), (1, 6), (1, 5), (1, 4), (1, 3)$, and $(1, 2)$. That is 8 steps up to $\frac{1}{2}$, so $\frac{20}{27}$ is at level 8.

Thus, the correct answer is **B**.

16. An urn contains 6 red balls, 4 black balls, and 4 white balls. Suppose 7 balls are drawn at random without replacement and a score is computed as follows:

- 1 point is given for each red ball up to a maximum of 3 points.
- 1 point is given for each black ball up to a maximum of 2 points.
- 1 point is given for each white ball up to a maximum of 2 points.

For example, drawing 1 red ball, 4 black balls, and 2 white balls would earn a score of $1 + 2 + 2 = 5$. What is the probability of getting the lowest possible score?

A $\frac{1}{858}$

B $\frac{5}{3432}$

C $\frac{1}{312}$

D $\frac{2}{429}$

E $\frac{5}{264}$

Small hint:

The score is 7 minus the balls beyond their color's cap. Waste as many balls as possible.

Big hint:

At most 3 balls can be wasted: 6 reds and 1 other, or 4 of one other color and 3 of the last

Solution:

Call a ball wasted if it is beyond its color's cap. Then the score is 7 minus the number of wasted balls, so we want to waste as many as possible.

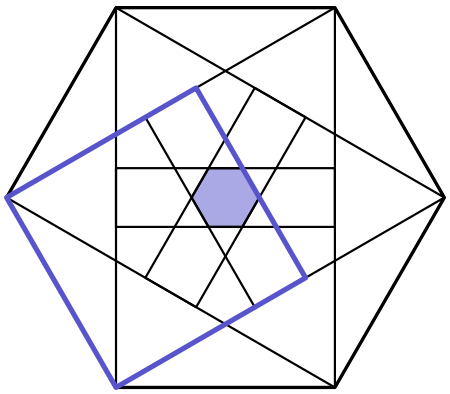
Wasting 4 is impossible. With 6 reds, 3 reds are wasted and the 1 other ball is not. With 5 reds, 2 reds are wasted and the 2 other balls waste nothing. With 4 reds, 1 red is

wasted and the 3 other balls waste at most 1. With at most 3 reds, only black and white can be wasted, and wasting 2 of each would take 8 balls. So at most 3 are wasted, and the lowest score is 4.

Exactly 3 are wasted with 6 reds and 1 of the 8 other balls (8 ways), with 4 black and 3 white (4 ways), or with 3 black and 4 white (4 ways). That is 16 of the $\binom{14}{7} = 3432$ equally likely draws, for a probability of $\frac{16}{3432} = \frac{2}{429}$.

Thus, the correct answer is **D**.

17. For each edge of a regular hexagon with side length 1, a square is constructed inside the hexagon with that edge as one of its sides. The region interior to all six squares is the interior of a smaller hexagon, shown shaded in the figure below.



The area of the smaller hexagon can be expressed in the form $\frac{a\sqrt{b}}{c} - d$, where $a, b, c,$ and d are positive integers, a and c are relatively prime, and b is not divisible by the square of any prime. What is $a + b + c + d$?

A 16

B 17

C 18

D 19

E 20

Small hint:

The center is $\frac{\sqrt{3}}{2}$ from each edge, and each square reaches 1 inward

Big hint:

The small hexagon is regular with the same center, and its sides are $1 - \frac{\sqrt{3}}{2}$ from it

Solution:

Let O be the center. Each edge is $\frac{\sqrt{3}}{2}$ from O, and the square on that edge reaches 1 inward, so its inner side lies just past O, at distance $h = 1 - \frac{\sqrt{3}}{2}$ on the far side.

Together with the square on the opposite edge, it cuts out a strip of width $2h$ centered on O.

The three strips meet in a regular hexagon with the same center and orientation as the big one, whose sides are h from O. (Its corners are only about 0.15 from O, so the squares' other sides, $\frac{1}{2}$ from O, do not cut it.) It is the big hexagon scaled by $\frac{2h}{\sqrt{3}} = \frac{2-\sqrt{3}}{\sqrt{3}}$.

The big hexagon has area $\frac{3\sqrt{3}}{2}$, so the small one has area

$$\begin{aligned} &\frac{3\sqrt{3}}{2} \times \frac{(2-\sqrt{3})^2}{3} \\ &= \frac{\sqrt{3}}{2}\left(7-4\sqrt{3}\right) \\ &= \frac{7\sqrt{3}}{2} - 6. \end{aligned}$$

So $a+b+c+d = 7+3+2+6 = 18$.

Thus, the correct answer is **C**.

18. The strictly decreasing sequence of 11 positive integers $(a_0, a_1, a_2, \ldots, a_{10})$ is defined by the recursive rule $a_k = \frac{1}{2}a_{k-1} + 2$ for $k \geq 1$. Let N be the minimum possible value of a_0. What is the sum of the digits of N?

A 7

B 9

C 10

D 11

E 14

Small hint:

Subtract 4: $a_k - 4 = \frac{1}{2}(a_{k-1} - 4)$

Big hint:

So $a_{10} - 4 = \frac{a_0 - 4}{2^{10}}$ must be an integer

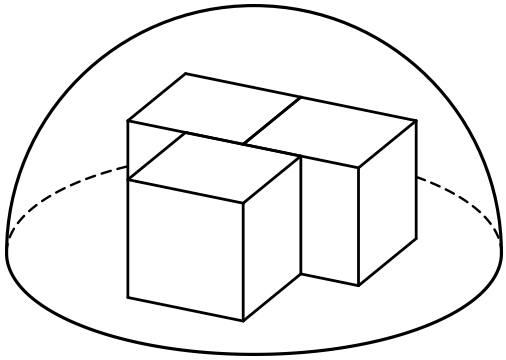
Solution:

Subtracting 4 from both sides of the rule gives $a_k - 4 = \frac{1}{2}(a_{k-1} - 4)$. So the distance from 4 halves at every step, and $a_k - 4 = \frac{a_0 - 4}{2^k}$.

All 11 terms are integers exactly when $2^{10} = 1024$ divides $a_0 - 4$. The sequence is strictly decreasing exactly when $a_0 - 4 > 0$, and then every term is above 4, so positive. The smallest choice is $a_0 - 4 = 1024$, so $N = 1028$, and its digit sum is $1 + 0 + 2 + 8 = 11$.

Thus, the correct answer is **D**.

19. Three $2 \times 2 \times 2$ cubes sit on a table so that two of the cubes share a face, and a face of the third cube is centered along a shared edge of the other two cubes, as shown.



A hemispherical cover is placed over the cubes so that its circumference rests on the table and the cubes are under the cover. What is the least possible value of the radius of the cover?

A 3

B $\frac{\sqrt{357}}{6}$

C $\frac{\sqrt{681}}{8}$

D $\frac{\sqrt{178}}{4}$

E $2\sqrt{3}$

Small hint:

A point at height 2 is covered when its horizontal distance from the center is at most $\sqrt{R^2 - 4}$

Big hint:

From above, the cubes form a 4×2 rectangle with a 2×2 square centered on a long side. Find the smallest circle around it.

Solution:

The center of the cover lies on the table. A point at height 2 whose horizontal distance from the center is d is under the cover exactly when $d^2 + 4 \le R^2$. The top corners of the cubes are the hardest points to cover, so we need the smallest circle on the table that contains the cubes' shadow. If its radius is r, then $R^2 = r^2 + 4$.

Put the two side-by-side cubes over $-2 \le x \le 2, 0 \le y \le 2$ and the third cube over $-1 \le x \le 1, -2 \le y \le 0$. The circle must reach the corners $(\pm 2, 2)$ and $(\pm 1, -2)$. By symmetry, try the center $(0, k)$ with all four corners at the same distance:

$$4 + (2 - k)^2 = 1 + (k + 2)^2,$$

which gives $k = \frac{3}{8}$ and $r^2 = 4 + \left(\frac{13}{8}\right)^2 = \frac{425}{64}$. The corners $(\pm 2, 0)$ are inside this circle. The center lies inside the trapezoid with those four corners, so no smaller circle holds them all.

Then $R^2 = \frac{425}{64} + 4 = \frac{681}{64}$, and $R = \frac{\sqrt{681}}{8}$.

Thus, the correct answer is **C**.

20. Boxes with dimensions $77 \times 78 \times 85$ or $2 \times 3 \times 85085$ have volume $510{,}510$. How many different boxes with integer side lengths at least 2, including these two, have volume $510{,}510$? Boxes are considered different if they have different dimensions.

A 63

B 90

C 301

D 377

E 966

Small hint:

$510{,}510 = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17$. Split these primes into three nonempty groups.

Big hint:

Count ordered splits: 3^7 ways, minus those that leave a side equal to 1

Solution:

Since $510{,}510 = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17$, choosing the three side lengths in order means sending each of the seven primes to one of the sides. Every side must be at least 2, so every side must get at least one prime.

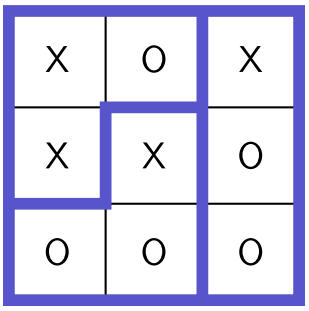
There are $3^7 = 2187$ ways to send the primes. Leaving a particular side empty happens in $2^7 = 128$ ways, and leaving two particular sides empty happens in 1 way. So the ordered triples number $2187 - 3 \times 128 + 3 = 1806$.

The three sides use different primes, so they are three different numbers, and each box is counted $3! = 6$ times. There are $\frac{1806}{6} = 301$ boxes.

Thus, the correct answer is **C**.

21. Gerry is playing the Triples Game. Her task is to partition the 3×3 grid of squares shown at the top into three sections, each containing three adjoining squares. (Three squares are adjoining if each shares an edge with at least one of the other two.) One point will be awarded for each section containing more X's than O's. Shown below it is a sample partition that will earn one point in total. A partition that earns two or more points will win the game. What fraction of the possible partitions of this particular grid will result in a win for Gerry?

X	O	X
X	X	O
O	O	O



A $\frac{3}{10}$

B $\frac{1}{3}$

C $\frac{2}{5}$

D $\frac{4}{9}$

E $\frac{1}{2}$

Small hint:

Only 4 squares hold X's. A win needs two sections with 2 X's each, so the third section is all O's.

Big hint:

Only the bottom row and one L-shape consist of three O's. Check the partitions that contain each.

Solution:

Each section is a straight line of three squares or an L-shape. An L-shape fits in a 2×2 block, so it covers at most one corner of the grid, and a middle row or column covers none. The three sections cover all four corners, so one of them is a full row or column along the edge.

That leaves a 2×3 rectangle, which splits in 3 ways: into two straight pieces, or into two L-shapes in 2 ways. Four edges give $4 \times 3 = 12,$ but the three-rows and three-columns partitions are each counted twice, so there are 10 partitions.

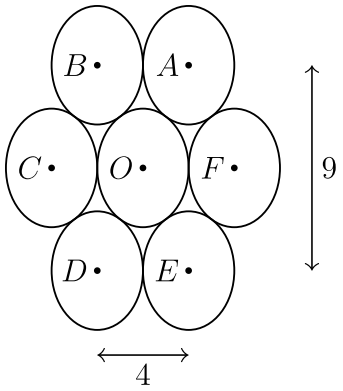
The grid has only 4 X's, and a section earns a point only with at least 2 of them. So a winning partition has two sections with 2 X's each and a third section of three O's. Only two sections consist of three O's: the bottom row, and the L made of the middle-right, bottom-center, and bottom-right squares.

With the bottom row, the top two rows split as two rows (XOX and XXO) or as two L-shapes (XOX and XXO again), both wins, or as XXX and OXO, which earns only 1 point. With the L, the other six squares split only as the top row plus an L (XOX and XXO) or as the left column plus an L (XXO and OXX), both wins.

So 4 of the 10 partitions win, a fraction of $\frac{2}{5}$.

Thus, the correct answer is **C**.

22. An engineer is designing a closely packed field of 7 wind turbines for electricity production. Each turbine, whose center is marked by a labeled dot in the figure, occupies one of 7 congruent elliptical regions. Turbines in the same row have centers that are 4 units apart, and turbines in the same column have centers that are 9 units apart, as shown in the figure below.



The major axis of each ellipse is $m\sqrt{p}$, where m and p are positive integers and p is not divisible by the square of any prime. What is $m+p$?

A 6

B 8

C 9

D 10

E 12

Small hint:

Each ellipse is 4 wide. Squeeze vertically to make them circles of radius 2.

Big hint:

O and A are 2 apart across and 4.5 apart vertically, and their ellipses touch

Solution:

Turbines in the same row touch halfway between their centers, so each ellipse has horizontal semi-axis 2. Call the vertical semi-axis s.

Squeeze the whole picture vertically by the factor $\frac{2}{s}$. Every ellipse becomes a circle of radius 2, and ellipses that touched still touch. The centers O and A were 2 apart across and 4.5 apart up and down, so now they are 2 across and $\frac{9}{s}$ up and down. Touching circles of radius 2 have centers 4 apart, so

$$4 + \frac{81}{s^2} = 16,$$

which gives $s^2 = \frac{27}{4}$ and $s = \frac{3\sqrt{3}}{2}$.

Since $s > 2$, the major axis is vertical, with length $2s = 3\sqrt{3}$. So $m + p = 3 + 3 = 6$.

Thus, the correct answer is **A**.

23. In a list of one or more consecutive integers, there are p multiples of 2 and q multiples of 3. How many ordered pairs (p, q) are possible for this list if p and q are between 0 and 100, inclusive?

A 232

B 233

C 234

D 235

E 236

Small hint:

In n consecutive integers, p is $\frac{n}{2}$ rounded up or down, and q is $\frac{n}{3}$ rounded up or down

Big hint:

Then $2p$ and $3q$ are both close to n. How far apart can they be?

Solution:

Take a list of n consecutive integers. The number of multiples of 2 in it is $\frac{n}{2}$ rounded up or down, so $|n - 2p| \leq 1$. The number of multiples of 3 is $\frac{n}{3}$ rounded up or down, so $|n - 3q| \leq 2$. Every combination of roundings occurs. Which way p rounds depends only on whether the first number is even or odd, and which way q rounds depends only on its remainder when divided by 3. Any six consecutive starting points cover every combination of the two.

So (p, q) is possible exactly when some positive n satisfies both $|n - 2p| \leq 1$ and $|n - 3q| \leq 2$. Such an n exists exactly when $2p$ and $3q$ are at most $1 + 2 = 3$ apart, that is, when $|2p - 3q| \leq 3$. (In that case the smaller of $2p + 1$ and $3q + 2$ works as n, and it is positive.)

Now count by p. The condition says q is within 1 of $\frac{2p}{3}$. If 3 divides p, that allows 3 values of q, and otherwise 2. Among $p = 0, 1, \ldots, 100$, there are 34 multiples of 3 and 67 others. Dropping $q = -1$ at $p = 0$ gives $34 \times 3 + 67 \times 2 - 1 = 235$. (Every q found is at most 67, so $q \leq 100$ holds.)

Thus, the correct answer is **D**.

24. How many polynomials $f(x)$ of degree 3 satisfying

$$f\left(3+\sqrt{5}\right)=f\left(3-\sqrt{5}\right)$$

are there that have leading coefficient equal to 1 and all of whose coefficients are integers between -10 and 10, inclusive?

A 21

B 42

C 63

D 84

E 105

Small hint:

Put $x=3+t$, so that the two inputs are $t=\pm\sqrt{5}$

Big hint:

Replacing t by $-t$ changes only the odd powers of t

Solution:

Write $f(x)=x^3+ax^2+bx+c$. The inputs $3\pm\sqrt{5}$ are symmetric about 3, so put $x=3+t$. Expanding $(3+t)^3$, $a(3+t)^2$, and $b(3+t)$, the coefficient of t^3 is 1 and the coefficient of t is $27+6a+b$.

Replacing t by $-t$ flips the signs of the odd powers of t and leaves the even powers alone, so

$$\begin{aligned} &f(3+t)-f(3-t)\\ &=2t^3+2(27+6a+b)t. \end{aligned}$$

At $t=\sqrt{5}$ this is $2\sqrt{5}\,(5+27+6a+b)$, which must be 0. So $b=-32-6a$.

For $-10\le b\le 10$ we need $-7\le a\le -4$, giving $b=10,4,-2$, and -8, all in range. The constant c can be any of the 21 integers from -10 to 10. So there are $4\times 21=$

84 polynomials.

Thus, the correct answer is **D**.

25. Chris the squirrel has 20 rooms in his cavern for storing food. The first room contains 1 nut, the second room contains 2 nuts, the third room contains 3 nuts, and so on, with the 20th room containing 20 nuts. Starting from today, Chris will collect 18 additional nuts each day and store them one per room in 18 distinct rooms in his cavern. His goal is to have all the rooms contain the same number of nuts once he is finished. What is the minimum number of days required to accomplish this goal?

A 75

B 95

C 105

D 190

E 195

Small hint:

Room 1 gains at most one nut a day, so it ends with at most $d + 1$

Big hint:

After d days the $210 + 18d$ nuts must be 20 times the final count per room

Solution:

Suppose that after d days every room holds N nuts. The rooms start with $1 + 2 + \cdots + 20 = 210$ nuts, so $210 + 18d = 20N$. Room 1 gets at most one nut a day, so $N \leq d + 1$. Then

$$210 + 18d = 20N \leq 20d + 20,$$

so $d \geq 95$.

With $d = 95$ we get $N = \frac{210+1710}{20} = 96$. Each day Chris skips exactly two rooms, and room i needs $96 - i$ nuts, so it must be skipped $95 - (96 - i) = i - 1$ times. These skips add up to $0 + 1 + \cdots + 19 = 190 = 2 \times 95$, as they should.

To schedule them, list the 190 skips in order: room 2 once, then room 3 twice, and so on up to room 20 nineteen times. On day k skip the rooms in positions k and $k + 95$ of

the list. No room appears 95 times, so these are always two different rooms. So 95 days is the minimum.

Thus, the correct answer is **B**.

Problems: https://live.poshenloh.com/past-contests/amc10/2026A