# 2025 AMC 10A Solutions

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1. Andy and Betsy both live in Mathville. Andy leaves Mathville on his bicycle at 1:30, traveling due north at a steady $8$ miles per hour. Betsy leaves on her bicycle from the same point at 2:30, traveling due east at a steady $12$ miles per hour. At what time will they be exactly the same distance from their common starting point?

   A 3:30

   B 3:45

   C 4:00

   D 4:15

   E 4:30

   ### Solution:

   Let $t$ be the hours since 1:30. Andy has gone $8t$ miles north. Betsy starts an hour later, so she's gone $12(t-1)$ miles east. We want these equal: $8t = 12(t-1)$. That gives $4t = 12$, so $t = 3$. Three hours past 1:30 is 4:30. Thus, **E** is the correct answer.

**2.** A box contains $10$ pounds of a nut mix that is $50$ percent peanuts, $20$ percent cashews, and $30$ percent almonds. A second nut mix containing $20$ percent peanuts, $40$ percent cashews, and $40$ percent almonds is added to the box, resulting in a new nut mix that is $40$ percent peanuts. How many pounds of cashews are now in the box?

A $3.5$

B $4$

C $4.5$

D $5$

E $6$

## Solution:

The starting $10$-pound mix holds $5$ pounds of peanuts and $2$ pounds of cashews. Add $x$ pounds of the second mix, which is $20\%$ peanuts. We want the new peanut fraction to be $40\%$, so $\frac{5+0.2x}{10+x} = 0.4$. This means $5 + 0.2x = 4 + 0.4x$, giving $x = 5$. Those $5$ pounds bring $0.4 \cdot 5 = 2$ more pounds of cashews, so the box now has $2 + 2 = 4$. Therefore, the answer is **B**.

**3.** How many isosceles triangles are there with positive area whose side lengths are all positive integers and whose longest side has length $2025$?

- A $2025$
- B $2026$
- C $3012$
- D $3037$
- E $4050$

## Solution:

Split into two cases. Say two sides both equal $2025.$ Then the third side can be any integer from $1$ to $2025,$ which is $2025$ triangles. Now suppose $2025$ is the unique longest side. The two equal legs $s$ must satisfy $2s > 2025$ by the triangle inequality, and $s \leq 2024.$ So $s$ runs from $1013$ to $2024,$ giving $1012$ triangles. Adding up, $2025 + 1012 = 3037.$ Thus, **D** is the correct answer.

**4.** A team of students is going to compete against a team of teachers in a trivia contest. The total number of students and teachers is $15.$ Ash, a cousin of one of the students, wants to join the contest. If Ash plays with the students, the average age on that team will increase from $12$ to $14.$ If Ash plays with the teachers, the average age on that team will decrease from $55$ to $52.$ How old is Ash?

- A $28$
- B $29$
- C $30$
- D $32$
- E $33$

## Solution:

Let $s$ be the number of students. If Ash joins them, his age is $14(s+1) - 12s = 2s + 14.$ If he joins the teachers instead (there are $15 - s$ of them), his age is $52(16 - s) - 55(15 - s) = 3s + 7.$ Both describe the same Ash, so $2s + 14 = 3s + 7.$ That gives $s = 7,$ and Ash is $2 \cdot 7 + 14 = 28.$ Therefore, the answer is **A**.

**5.** Consider the sequence of positive integers

$$1, 2, 1, 2, 3, 2, 1, 2, 3, 4, 3, 2, 1, 2, 3, 4, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, \ldots$$

What is the $2025$th term in this sequence?

A $5$

B $15$

C $16$

D $44$

E $45$

## Solution:

Group the sequence into blocks. Block $k$ reads $k, k-1, \ldots, 2, 1, 2, \ldots, k-1, k$, which is $2k-1$ terms and ends on $k$. So after block $k$ we've used $1+3+\cdots+(2k-1) = k^2$ terms. Notice $2025 = 45^2$. That's exactly the end of block $45$, whose last term is $45$. Thus, **E** is the correct answer.

**6.** In an equilateral triangle each interior angle is trisected by a pair of rays. The intersection of the interiors of the middle $20^\circ$-angle at each vertex is the interior of a convex hexagon. What is the degree measure of the smallest angle of this hexagon?

A $80$

B $90$

C $100$

D $110$

E $120$

## Solution:

Label the equilateral triangle $ABC$. Each $60^\circ$ angle splits into three $20^\circ$ pieces. Take the outermost trisectors from $A$ and $B$: they meet at base angles $\frac{2}{3} \cdot 60^\circ = 40^\circ$, so the hexagon vertex there has angle $180^\circ - 2 \cdot 40^\circ = 100^\circ$. The innermost trisectors from $A$ and $B$ meet at base angles $20^\circ$, giving apex $180^\circ - 2 \cdot 20^\circ = 140^\circ$, and by vertical angles that's the opposite hexagon angle. So the six angles alternate $100^\circ$ and $140^\circ$. The smallest is $100^\circ$. Therefore, the answer is **C**.

**7.** Suppose $a$ and $b$ are real numbers. When the polynomial $x^3 + x^2 + ax + b$ is divided by $x - 1$, the remainder is $4$. When the polynomial is divided by $x - 2$, the remainder is $6$. What is $b - a$?

A $14$

B $15$

C $16$

D $17$

E $18$

## Solution:

By the Remainder Theorem, just plug in. We get $p(1) = 1 + 1 + a + b = 4$, so $a + b = 2$. And $p(2) = 8 + 4 + 2a + b = 6$, so $2a + b = -6$. Subtract the first from the second: $a = -8$, hence $b = 10$. Then $b - a = 10 - (-8) = 18$. Thus, **E** is the correct answer.

**8.** Agnes writes the following four statements on a blank piece of paper.

- At least one of these statements is true.
- At least two of these statements are true.
- At least two of these statements are false.
- At least one of these statements is false.

Each statement is either true or false. How many false statements did Agnes write on the paper?

A $0$

B $1$

C $2$

D $3$

E $4$

## Solution:

Number them: (1) at least one true, (2) at least two true, (3) at least two false, (4) at least one false. Suppose (3) is true. Then at least two statements are false. But then (1), (2), and (4) all read as true, which leaves at most one false statement. That's a contradiction, so (3) must be false. Now (1), (2), and (4) are all true, and each matches reality with just one false statement. So exactly $1$ statement is false. Therefore, the answer is **B**.

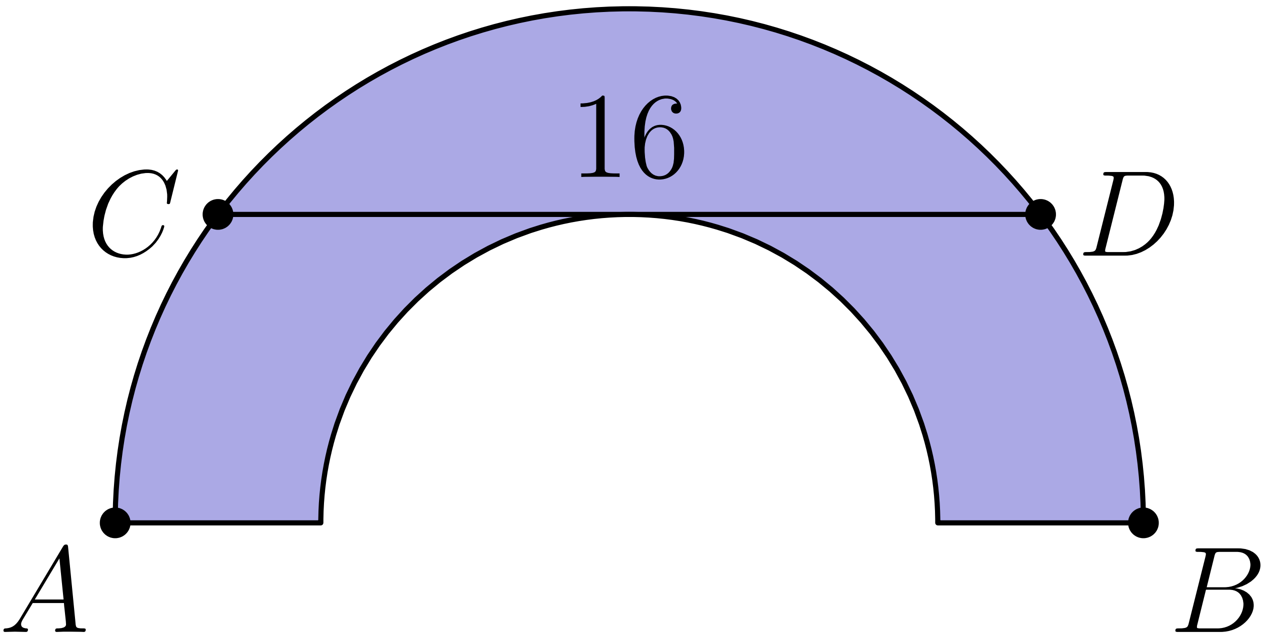
**9.** Let $f(x) = 100x^3 - 300x^2 + 200x$. For how many real numbers $a$ does the graph of $y = f(x-a)$ pass through the point $(1, 25)$?

- A $1$
- B $2$
- C $3$
- D $4$
- E more than $4$

## Solution:

The graph passes through $(1, 25)$ exactly when $f(1-a) = 25$. Let $t = 1 - a$, so we just need the number of solutions to $f(t) = 25$. Factor $f(x) = 100x(x-1)(x-2)$, with roots $0, 1, 2$. Its local maximum on $(0, 1)$ is $f(0.5) = 37.5$, which beats $25$. So the line $y = 25$ cuts the cubic in $3$ points. Each one gives a single $a$, so there are $3$ values. Thus, **C** is the correct answer.

**10.** A semicircle has diameter $AB$ and chord $CD$ of length $16$ parallel to $AB$. A smaller semicircle with diameter on $AB$ and tangent to $CD$ is cut from the larger semicircle, as shown below.




What is the area of the resulting figure, shown shaded?

A $16\pi$

B $24\pi$

C $32\pi$

D $48\pi$

E $64\pi$

## Solution:

Let $O$ be the center on $AB$ and $P$ the midpoint of chord $CD$. Set $r = OP$ for the small radius and $R = OD$ for the large one. Since $PD = 8$, the Pythagorean theorem in triangle $OPD$ gives $R^2 - r^2 = 64$. The shaded area is the big semicircle minus the small one: $\frac{1}{2}\pi R^2 - \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(R^2 - r^2) = 32\pi$. Therefore, the answer is **C**.

**11.** The sequence $1, x, y, z$ is arithmetic. The sequence $1, p, q, z$ is geometric. Both sequences are strictly increasing and contain only integers, and $z$ is as small as possible. What is the value of $x + y + z + p + q$?

A $66$

B $91$

C $103$

D $132$

E $149$

## Solution:

From the arithmetic sequence, $z = 1 + 3d$, so $z \equiv 1 \pmod{3}$. From the geometric one, $z = p^3$ for some integer ratio $p \geq 2$. We want the smallest such $z$, so test $p = 2, 3, 4$. Only $p = 4$ works, since $p^3 = 64 \equiv 1 \pmod{3}$. That forces $d = 21$, and the sequences are $1, 22, 43, 64$ and $1, 4, 16, 64$. So $x + y + z + p + q = 22 + 43 + 64 + 4 + 16 = 149$. Thus, **E** is the correct answer.

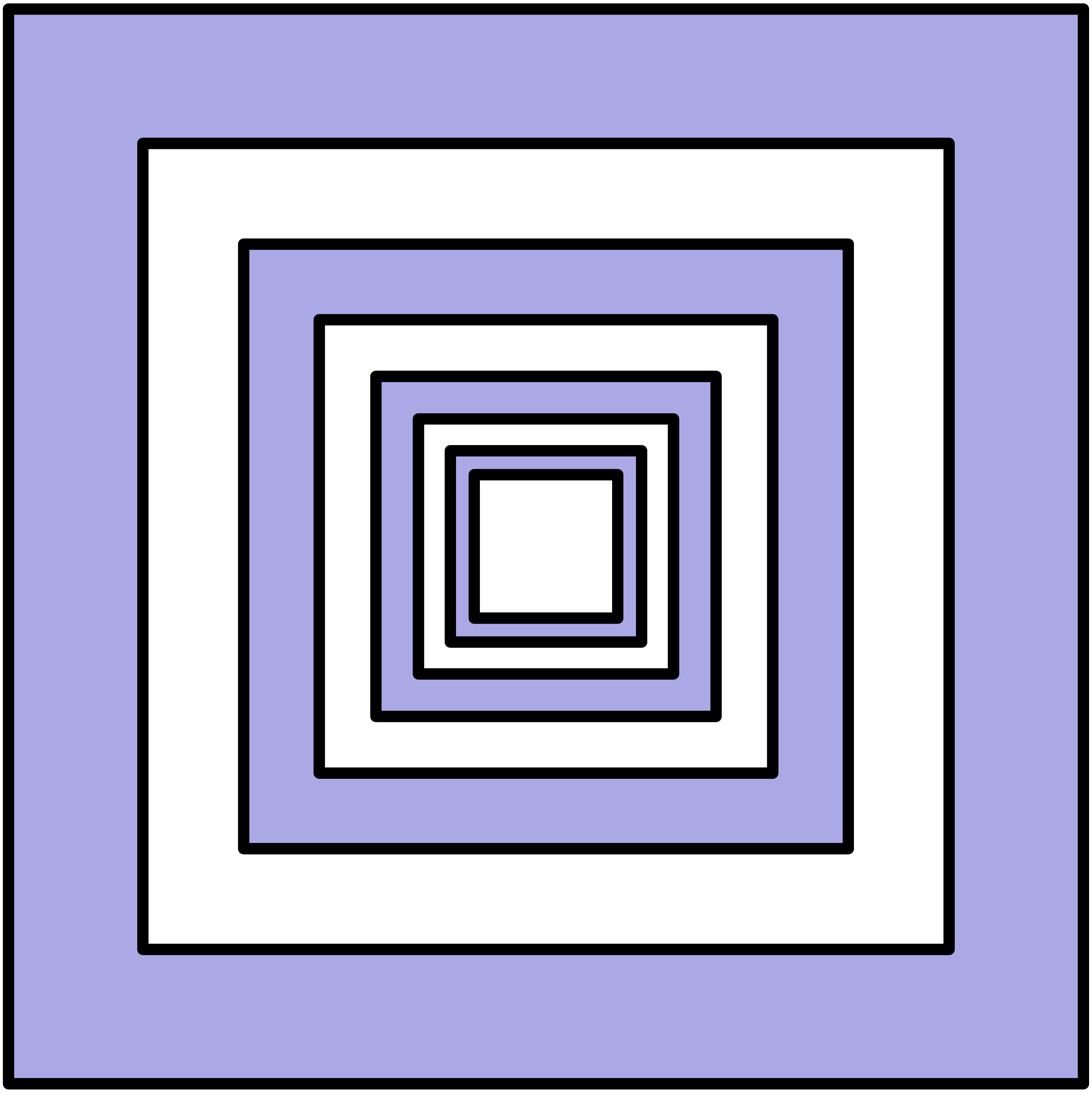
**12.** Carlos uses a $4$-digit passcode to unlock his computer. In his passcode, exactly one digit is even, exactly one (possibly different) digit is prime, and no digit is $0.$ How many $4$-digit passcodes satisfy these conditions?

- A $176$
- B $192$
- C $432$
- D $464$
- E $608$

## Solution:

No digit is $0,$ so digits run from $1$ to $9.$ Put the single even digit in the first slot for now and multiply by $4$ at the end to place it. Split on that even digit. If it's the prime $2,$ then the three odd digits all have to be non-prime, so each is $1$ or $9,$ giving $2^3 = 8$ ways. Otherwise the even digit is $4, 6,$ or $8$ ($3$ choices), and exactly one of the odd digits is prime, worth $3, 5,$ or $7$ ($3$ choices) in one of the $3$ odd positions, while the other two odds come from $\{1, 9\}$ ($2^2$ ways). That's $3 \cdot 3 \cdot 3 \cdot 4 = 108.$ Altogether, $4(8 + 108) = 464.$ Therefore, the answer is **D**.

**13.** In the figure below, the outside square contains infinitely many squares, each of them with the same center and sides parallel to the outside square. The ratio of the side length of a square to the side length of the next inner square is $k$, where $0 < k < 1$. The spaces between squares are alternately shaded, as shown in the figure (which is not necessarily drawn to scale).



The area of the shaded portion of the figure is $64\%$ of the area of the original square. What is $k$?

A $\frac{3}{5}$

B $\frac{16}{25}$

C $\frac{2}{3}$

D $\frac{3}{4}$

E $\frac{4}{5}$

### Solution:

Let the outer side be $1$. The squares have sides $1, k, k^2, \ldots,$ and the shaded rings alternate, so the shaded area is $1 - k^2 + k^4 - k^6 + \cdots = \frac{1}{1+k^2}$. We're told this equals $64\% = \frac{16}{25}$, so $1 + k^2 = \frac{25}{16}$. Then $k^2 = \frac{9}{16}$, giving $k = \frac{3}{4}$. Thus, **D** is the correct answer.

**14.** Six chairs are arranged around a round table. Two students and two teachers randomly select four of the chairs to sit in. What is the probability that the two students will sit in two adjacent chairs and the two teachers will also sit in two adjacent chairs?

A $\frac{1}{6}$

B $\frac{1}{5}$

C $\frac{2}{9}$

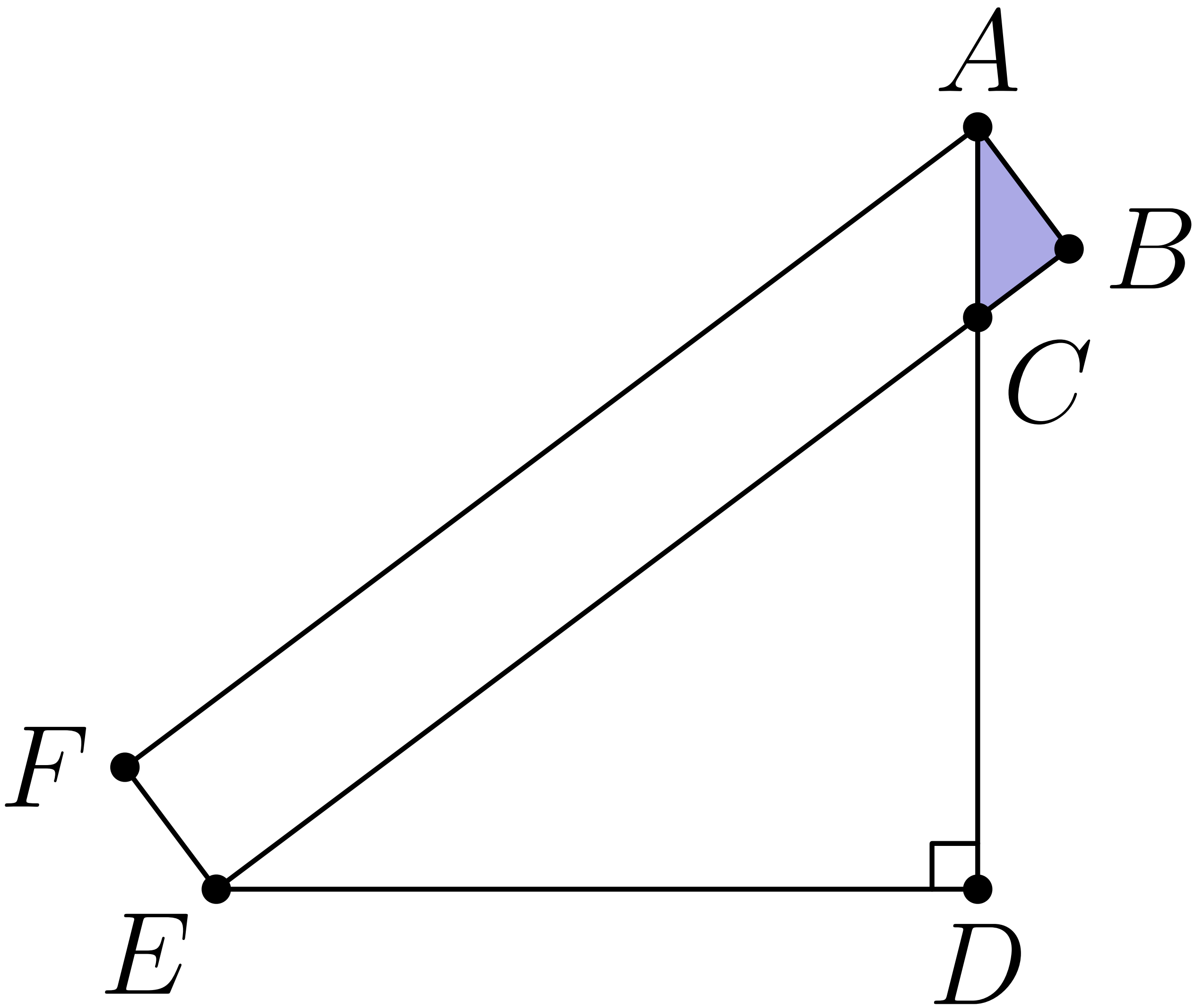
D $\frac{3}{13}$

E $\frac{1}{4}$

### Solution:

Seat the first student anywhere. The second student lands next to them with probability $\frac{2}{5}$, since two of the remaining five chairs are adjacent. Now the teachers fill two of the four leftover chairs. Of the $\binom{4}{2} = 6$ ways to do that, exactly $3$ are adjacent pairs. So the probability is $\frac{2}{5} \cdot \frac{3}{6} = \frac{1}{5}$. Therefore, the answer is **B**.

**15.** In the figure below, $ABEF$ is a rectangle, $AD \perp DE$, $AF = 7$, $AB = 1$, and $AD = 5$. What is the area of $\triangle ABC$?




A $\frac{3}{8}$

B $\frac{4}{9}$

C $\frac{1}{8}\sqrt{13}$

D $\frac{7}{15}$

E $\frac{1}{8}\sqrt{15}$

## Solution:

Let $x = BC$. Since $ABEF$ is a rectangle with $AB = 1$ and $AF = 7$, and $AD = 5$, we get $AC = \sqrt{1+x^2}, CE = 7 - x$, and $CD = 5 - \sqrt{1+x^2}$. The triangles $\triangle ABC$ and $\triangle EDC$ are similar, so $\frac{7-x}{\sqrt{1+x^2}} = \frac{5-\sqrt{1+x^2}}{x}$. Clear denominators and square to get $24x^2 + 14x - 24 = 0$, which factors as $(4x-3)(3x+4) = 0$. The positive root is $x = \frac{3}{4}$. So the area is $\frac{1}{2} \cdot \frac{3}{4} \cdot 1 = \frac{3}{8}$. Thus, **A** is the correct answer.

**16.** There are three jars. Each of three coins is placed in one of the three jars, chosen at random and independently of the placements of the other coins. What is the expected number of coins in a jar with the most coins?

A $\frac{4}{3}$

B $\frac{39}{27}$

C $\frac{5}{3}$

D $\frac{17}{9}$

E $2$

## Solution:

There are $3^3 = 27$ equally likely placements. Of these, $3$ pile all coins into one jar (max $3$), and $6$ put one coin in each jar (max $1$). The other $18$ split $2$-$1$ (max $2$). So the expected maximum is $\frac{3 \cdot 3 + 2 \cdot 18 + 1 \cdot 6}{27} = \frac{51}{27} = \frac{17}{9}$. Therefore, the answer is **D**.

**17.** Let $N$ be the unique positive integer such that dividing $273436$ by $N$ leaves a remainder of $16$, and dividing $272760$ by $N$ leaves a remainder of $15$. What is the tens digit of $N$?

A $0$

B $1$

C $2$

D $3$

E $4$

## Solution:

Subtract off the remainders. Both $273420 = 273436 - 16$ and $272745 = 272760 - 15$ are multiples of $N$, so their difference $675$ is too. Now $272745 = 404 \cdot 675 + 45$, which makes $45$ a multiple of $N$ as well. The remainder $16$ means $N > 16$, and the only divisor of $45$ bigger than $16$ is $45$ itself. So $N = 45$, and its tens digit is $4$. Thus, **E** is the correct answer.

**18.** The harmonic mean of a collection of numbers is the reciprocal of the arithmetic mean of the reciprocals of the numbers in the collection. For example, the harmonic mean of $4, 4, 5$ is

$$\frac{1}{\frac{1}{3}\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{5}\right)}=\frac{30}{7}.$$

What is the harmonic mean of all the real roots of the $4050$th degree polynomial

$$\prod_{k=1}^{2025}(kx^2-4x-3)=(x^2-4x-3)(2x^2-4x-3)(3x^2-4x-3)\cdots(2025x^2-4x-3)?$$

A $-\frac{5}{3}$

B $-\frac{3}{2}$

C $-\frac{6}{5}$

D $-\frac{5}{6}$

E $-\frac{2}{3}$

## Solution:

Look at one factor $kx^2-4x-3$. Its discriminant $16+12k$ is positive, so it has two distinct real roots. By Vieta, their reciprocals sum to $\frac{x_1+x_2}{x_1x_2}=\frac{4/k}{-3/k}=-\frac{4}{3}$, and notice the $k$ cancels. Summing over all $2025$ factors, the reciprocals total $2025\cdot\left(-\frac{4}{3}\right)=-2700$. There are $4050$ roots in all, so the harmonic mean is $\frac{4050}{-2700}=-\frac{3}{2}$. Therefore, the answer is **B**.

**19.** An array of numbers is constructed beginning with the numbers $-1, 3, 1$ in the top row. Each adjacent pair of numbers is summed to produce a number in the next row. Each row begins and ends with the numbers $-1$ and $1$, respectively. The first three rows are shown below.

$$\begin{array}{ccccccccc} & & -1 & & 3 & & 1 & & \\ & -1 & & 2 & & 4 & & 1 & \\ -1 & & 1 & & 6 & & 5 & & 1 \end{array}$$

If the process continues, one of the rows will sum to $12{,}288$. In that row, what is the third number from the left?

A $-29$

B $-21$

C $-14$

D $-8$

E $-3$

## Solution:

Each interior entry feeds two entries below, and the end values $-1$ and $1$ cancel in the sum. So every row's total doubles the one above. The top row sums to $3$, and $12{,}288 = 3 \cdot 2^{12}$, so this is the $12$th row (counting the top as row $0$). Track the diagonals from the left. The second diagonal is $3, 2, 1, 0, -1, \ldots,$ dropping by $1$ each row. The third diagonal adds these up: for $n \geq 4$ it equals $7 - (0 + 1 + \cdots + (n-4)) = 7 - \frac{(n-4)(n-3)}{2}$. Plug in $n = 12$: $7 - \frac{8 \cdot 9}{2} = -29$. Thus, **A** is the correct answer.

**20.** A silo (right circular cylinder) with diameter $20$ meters stands in a field. MacDonald is located $20$ meters west and $15$ meters south of the center of the silo. McGregor is located $20$ meters east and $g > 0$ meters south of the center of the silo. The line of sight between MacDonald and McGregor is tangent to the silo. The value of $g$ can be written as $\dfrac{a\sqrt{b}-c}{d}$, where $a, b, c,$ and $d$ are positive integers, $b$ is not divisible by the square of any prime, and $d$ is relatively prime to the greatest common divisor of $a$ and $c$. What is $a+b+c+d$?

A $119$

B $120$

C $121$

D $122$

E $123$

## Solution:

Put the silo's center at the origin with radius $10$. Then MacDonald is at $D = (-20, -15)$ and McGregor at $G = (20, -g)$. The tangent length from $D$ is $DT = \sqrt{DS^2 - 10^2} = \sqrt{25^2 - 100} = \sqrt{525}$, and from $G$ it's $TG = \sqrt{g^2 + 20^2 - 10^2}$. The tangent point $T$ sits between the two men, so $DG = DT + TG$. But also $DG = \sqrt{40^2 + (15-g)^2}$. Square twice and simplify: $3g^2 + 150g - 925 = 0$, which gives $g = \frac{20\sqrt{21}-75}{3}$. So $a+b+c+d = 20+21+75+3 = 119$. Therefore, the answer is **A**.

**21.** A set of numbers is called sum-free if whenever $x$ and $y$ are (not necessarily distinct) elements of the set, $x+y$ is not an element of the set. For example, $\{1,4,6\}$ and the empty set are sum-free, but $\{2,4,5\}$ is not. What is the greatest possible number of elements in a sum-free subset of $\{1,2,3,\ldots,20\}$?

A $8$

B $9$

C $10$

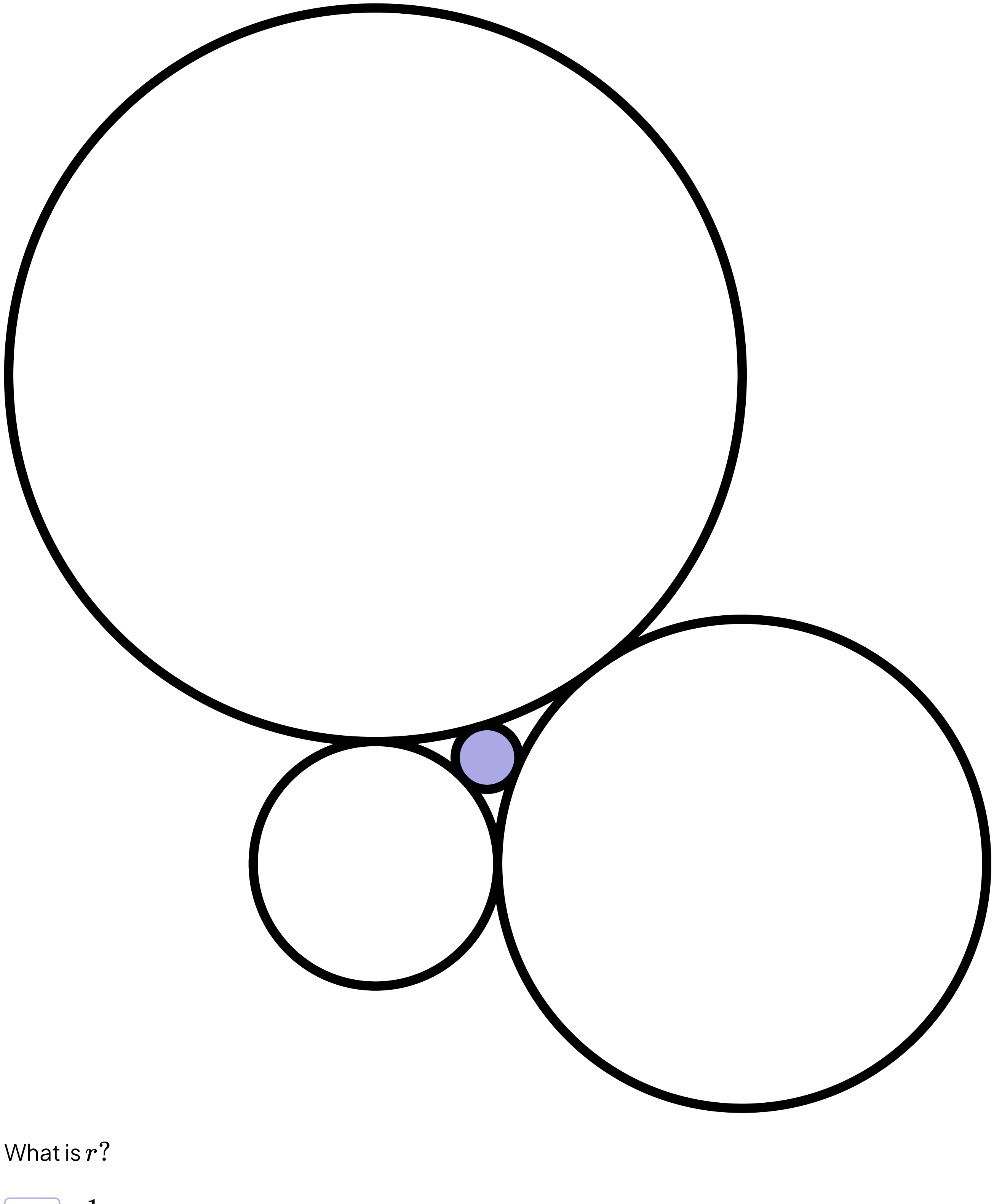
D $11$

E $12$

## Solution:

We can reach $10$. The odds $\{1,3,5,\ldots,19\}$ are sum-free, since two odds sum to an even. So is $\{11,12,\ldots,20\}$, since any two of those sum past $20$. Each has $10$ elements. Now we show you can't beat $10$. Let $m$ be the largest element of a sum-free subset. Pair up $1,2,\ldots,m-1$ as $\{i,m-i\}$. A pair can't contribute both elements, or their sum $m$ would be in the set. There are $\lfloor (m-1)/2 \rfloor$ such pairs, so the subset has at most $\lfloor (m-1)/2 \rfloor + 1 \leq \lfloor 19/2 \rfloor + 1 = 10$ elements. Thus, **C** is the correct answer.

**22.** A circle of radius $r$ is surrounded by three circles, whose radii are $1, 2,$ and $3,$ all externally tangent to the inner circle and to each other, as shown.



What is $r$?

A $\frac{1}{4}$

B $\frac{6}{23}$

C $\frac{3}{11}$

D $\frac{5}{17}$

E $\frac{3}{10}$

## Solution:

The three outer centers $A, B, C$ are pairwise $AB = 1 + 2 = 3$, $AC = 1 + 3 = 4$, and $BC = 2 + 3 = 5$ apart, a $3$-$4$-$5$ right triangle. Now apply Descartes' Circle Theorem with curvatures $1, \frac{1}{2}, \frac{1}{3}$, and $\frac{1}{r}$, all mutually tangent: $\frac{1}{r} = 1 + \frac{1}{2} + \frac{1}{3} + 2\sqrt{\frac{1}{2} + \frac{1}{6} + \frac{1}{3}} = \frac{11}{6} + 2\sqrt{1} = \frac{23}{6}$. Inverting, $r = \frac{6}{23}$. Therefore, the answer is **B**.

**23.** Triangle $ABC$ has side lengths $AB = 80$, $BC = 45$, and $AC = 75$. The bisector of $\angle B$ and the altitude to side $AB$ intersect at point $P$. What is $BP$?

A $18$

B $19$

C $20$

D $21$

E $22$

## Solution:

Let the bisector of $\angle B$ hit $AC$ at $D$. By the Angle Bisector Theorem, $\frac{AD}{DC} = \frac{AB}{BC} = \frac{80}{45}$, and since $AC = 75$, we get $AD = 48$ and $CD = 27$. Triangles $BCD$ and $ACB$ share $\angle C$, with sides in ratio $\frac{45}{75} = \frac{3}{5}$, so they're similar. That gives $BD = \frac{3}{5} \cdot 80 = 48$, which equals $AD$. So $\triangle ADB$ is isosceles. Writing $\angle DAB = \angle DBA = \theta$, a short angle chase shows $\angle DPC = \angle DCP$, so $\triangle CDP$ is isosceles too, with $PD = CD = 27$. Since $P$ is on segment $BD$, $BP = BD - PD = 48 - 27 = 21$. Thus, **D** is the correct answer.

**24.** Call a positive integer fair if no digit is used more than once, it has no $0$s, and no digit is adjacent to two greater digits. For example, $23, 196,$ and $12463$ are fair, but $1546, 320,$ and $34321$ are not. How many fair positive integers are there?

- A $511$
- B $2{,}584$
- C $9{,}841$
- D $17{,}711$
- E $19{,}682$

## Solution:

In a fair number the digits climb up to the largest digit $m$ and then fall; otherwise some digit would be trapped between two bigger ones. So count by size. For $k$ digits, pick the digit set from $1$ to $9$ in $\binom{9}{k}$ ways. The largest is $m$, and each of the remaining $k-1$ digits chooses to sit left or right of $m$ ($2^{k-1}$ ways), which pins down the number. Sum over $k$: $\sum_{k=1}^{9}\binom{9}{k}2^{k-1} = \frac{1}{2}\left((1+2)^9-1\right) = \frac{3^9-1}{2} = 9841.$ Therefore, the answer is **C**.

**25.** A point $P$ is chosen at random inside square $ABCD$. The probability that $AP$ is neither the shortest nor the longest side of $\triangle APB$ can be written as $\dfrac{a+b\pi-c\sqrt{d}}{e}$, where $a, b, c, d,$ and $e$ are positive integers, $\gcd(a,b,c,e)=1$, and $d$ is not divisible by the square of a prime. What is $a+b+c+d+e$?

A $25$

B $26$

C $27$

D $28$

E $29$

## Solution:

Place $A=(0,0)$ and $B=(1,0)$ on the unit square. $AP$ is the middle length when $BP < AP < AB$ or $AB < AP < BP$. These two conditions carve out regions bounded by the circle of radius $1$ centered at $A$ (where $AP = AB$) and the perpendicular bisector $x=\frac{1}{2}$ (where $AP = BP$). Let $S$ be where that circle meets $x=\frac{1}{2}$. Then $\triangle ABS$ is equilateral, so $\angle QAS = 60^\circ$. Working out the pieces, the larger region has area $\frac{4\pi-3\sqrt{3}}{24}$ and the smaller $\frac{12-2\pi-3\sqrt{3}}{24}$. They add to $\frac{6+\pi-3\sqrt{3}}{12}$. So $a+b+c+d+e=6+1+3+3+12=25$. Thus, **A** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc10/2025A