2018 AMC 10A

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc10/2018A

1. What is the value of

$$\left(\left((2+1)^{-1}+1\right)^{-1}+1\right)^{-1}+1?$$

A $\frac{5}{8}$

B $\frac{11}{7}$

C $\frac{8}{5}$

D $\frac{18}{11}$

E $\frac{15}{8}$

2. Liliane has 50% more soda than Jacqueline, and Alice has 25% more soda than Jacqueline. What is the relationship between the amounts of soda that Liliane and Alice have?

- A Liliane has 20% more soda than Alice.
- B Liliane has 25% more soda than Alice.
- C Liliane has 45% more soda than Alice.
- D Liliane has 75% more soda than Alice.
- E Liliane has 100% more soda than Alice.

3. A unit of blood expires after

$$10! = 10 \cdot 9 \cdot 8 \cdots 1$$

seconds. Yasin donates a unit of blood at noon of January 1. On what day does his unit of blood expire?

- A January 2
- B January 12
- C January 22
- D February 11
- E February 12

4. How many ways can a student schedule 3 mathematics courses — algebra, geometry, and number theory — in a 6-period day if no two mathematics courses can be taken in consecutive periods?

(What courses the student takes during the other 3 periods is of no concern here.)

A 3

B 6

C 12

D 18

E 24

5. Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least 6 miles away," Bob replied, "We are at most 5 miles away." Charlie then remarked, "Actually the nearest town is at most 4 miles away."

It turned out that none of the three statements were true. Let d be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of d?

A $(0,4)$

B $(4,5)$

C $(4,6)$

D $(5,6)$

E $(5,\infty)$

6. Sangho uploaded a video to a website where viewers can vote that they like or dislike a video. Each video begins with a score of 0, and the score increases by 1 for each like vote and decreases by 1 for each dislike vote.

At one point Sangho saw that his video had a score of 90, and that 65% of the votes cast on his video were like votes. How many votes had been cast on Sangho's video at that point?

A 200

B 300

C 400

D 500

E 600

7. For how many (not necessarily positive) integer values of n is the following value an integer?

$$4000 \cdot \left(\frac{2}{5}\right)^n$$

A 3

B 4

C 6

D 8

E 9

8. Joe has a collection of 23 coins, consisting of 5-cent coins, 10-cent coins, and 25-cent coins. He has 3 more 10-cent coins than 5-cent coins, and the total value of his collection is 320 cents. How many more 25-cent coins does Joe have than 5-cent coins?

A 0

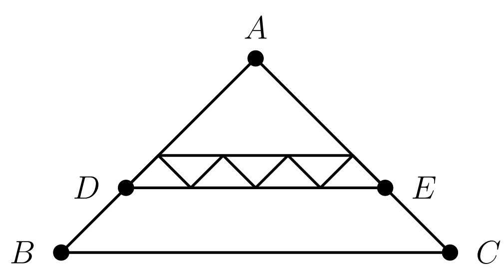
B 1

C 2

D 3

E 4

9. All of the triangles in the diagram below are similar to isosceles triangle ABC, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?



A 16

B 18

C 20

D 22

E 24

10. Suppose that real number x satisfies

$$\sqrt{49-x^2}-\sqrt{25-x^2}=3.$$

What is the value of

$$\sqrt{49-x^2}+\sqrt{25-x^2}?$$

A 8

B $\sqrt{33}+8$

C 9

D $2\sqrt{10}+4$

E 12

11. When 7 fair standard 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 10 can be written as

$$\frac{n}{6^7},$$

where n is a positive integer. What is n?

A 42

B 49

C 56

D 63

E 84

12. How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

$$\begin{cases} x + 3y & = 3 \\ \big||x| - |y|\big| & = 1 \end{cases}$$

A 1

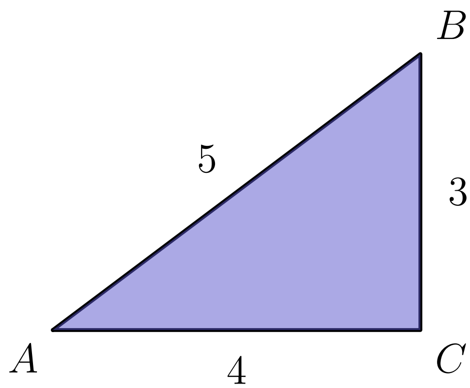
B 2

C 3

D 4

E 8

13. A paper triangle with sides of lengths $3, 4$, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?



A $1+\frac{1}{2}\sqrt{2}$

B $\sqrt{3}$

C $\frac{7}{4}$

D $\frac{15}{8}$

E 2

14. What is the greatest integer less than or equal to

$$\frac{3^{100}+2^{100}}{3^{96}+2^{96}}?$$

A 80

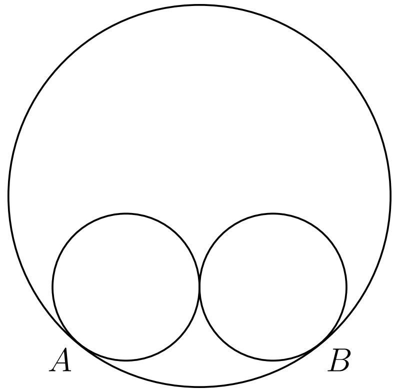
B 81

C 96

D 97

E 625

15. Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points A and B, as shown in the diagram. The distance AB can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?



A 21

B 29

C 58

D 69

E 93

16. Right triangle ABC has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex B to a point on hypotenuse $\overline{AC}$?

A 5

B 8

C 12

D 13

E 15

17. Let S be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a. What is the least possible value of an element in S?

A 2

B 3

C 4

D 5

E 7

18. How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5$$
$$+a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2$$
$$+a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

A 512

B 729

C 1094

D 3281

E $59,048$

19. A number m is randomly selected from the set

$$\{11, 13, 15, 17, 19\},$$

and a number n is randomly selected from

$$\{1999, 2000, 2001, \ldots, 2018\}.$$

What is the probability that m^n has a units digit of 1?

A $\frac{1}{5}$

B $\frac{1}{4}$

C $\frac{3}{10}$

D $\frac{7}{20}$

E $\frac{2}{5}$

20. A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares.

A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of 90° counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides.

What is the total number of possible symmetric scanning codes?

A 510

B 1022

C 8190

D 8192

E $65,534$

21. Which of the following describes the set of values of a for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real xy-plane intersect at exactly 3 points?

A $a = \frac{1}{4}$

B $\frac{1}{4} < a < \frac{1}{2}$

C $a > \frac{1}{4}$

D $a = \frac{1}{2}$

E $a > \frac{1}{2}$

22. Let $a, b, c,$ and d be positive integers such that

$$\gcd(a, b) = 24,$$

$$\gcd(b, c) = 36,$$

$$\gcd(c, d) = 54,$$

and

$$70 < \gcd(d, a < 100.$$

Which of the following must be a divisor of a?

A 5

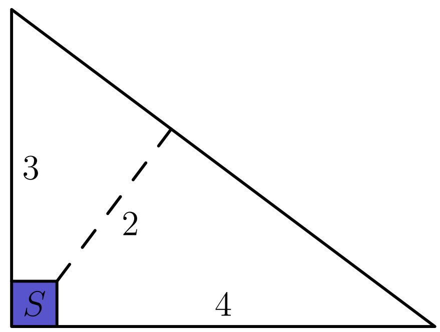
B 7

C 11

D 13

E 17

23. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



A $\frac{25}{27}$

B $\frac{26}{27}$

C $\frac{73}{75}$

D $\frac{145}{147}$

E $\frac{74}{75}$

24. Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

A 60

B 65

C 70

D 75

E 80

25. For a positive integer n and nonzero digits a, b, and c, let A_n be the n-digit integer each of whose digits is equal to a; let B_n be the n-digit integer each of whose digits is equal to b; and let C_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that

$$C_n - B_n = A_n^2?$$

A 12

B 14

C 16

D 18

E 20

Solutions: https://live.poshenloh.com/past-contests/amc10/2018A/solutions