

# 2017 AMC 10B

## Solutions

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1. Mary thought of a positive two-digit number. She multiplied it by 3 and added 11. Then she switched the digits of the result, obtaining a number between 71 and 75, inclusive. What was Mary's number?

A 11

**B 12**

C 13

D 14

E 15

### Solution(s):

We know that her number was between 71 and 75, so the units digit is between 1 and 5, and the tens digit is 7.

Now, we have to reverse the order of the operations. After reversing, we get the tens digit to be between 1 and 5 and the tens digit to be 7.

Then, subtracting 11 subtracts 1 from the units digit and the tens digit, giving us a number where the tens digit is between 0 and 4 and the units digit is 6. This must be a multiple of 3, so we can only have 36. Dividing this by 3 yields 12.

Thus, the correct answer is **B**.

2. Sofia ran 5 laps around the 400-meter track at her school. For each lap, she ran the first 100 meters at an average speed of 4 meters per second and the remaining 300 meters at an average speed of 5 meters per second. How much time did Sofia take running the 5 laps?

A 5 minutes and 35 seconds

B 6 minutes and 40 seconds

C 7 minutes and 5 seconds

D 7 minutes and 25 seconds

E 8 minutes and 10 seconds

**Solution(s):**

She ran a total of  $5 \cdot 100 = 500$  meters at 4 meters per second and  $5 \cdot 300 = 1500$  meters at 5 meters per second.

Therefore, her time is

$$\frac{500}{4} + \frac{1500}{5} = 425$$

seconds.

This is equal to a total of 7 minutes and 5 seconds.

Thus, the correct answer is **C**.

3. Real numbers  $x$ ,  $y$ , and  $z$  satisfy the inequalities  $0 < x < 1$ ,  $-1 < y < 0$ , and  $1 < z < 2$ .

Which of the following numbers is necessarily positive?

- A  $y + x^2$
- B  $y + xz$
- C  $y + y^2$
- D  $y + 2y^2$
- E  $y + z$

**Solution(s):**

Since  $-1 < y$  and  $1 < z$ , we can add the inequalities to see that  $0 < y + z$ . This naturally proves choice **E** correct.

Furthermore, we can eliminate every other choice with the following values:

$$x = 0.1,$$

$$y = -0.25,$$

$$z = 1.25.$$

Thus, the correct answer is **E**.

4. Supposed that  $x$  and  $y$  are nonzero real numbers such that

$$\frac{3x + y}{x - 3y} = -2.$$

What is the value of

$$\frac{x + 3y}{3x - y}?$$

A  -3

B  -1

C  1

D  2

E  3

**Solution(s):**

Given that

$$\frac{3x + y}{x - 3y} = -2,$$

we can multiply by the denominator to get

$$3x + y = 6y - 2x.$$

Solving, we can see that  $x = y$ .

Therefore,

$$\frac{x + 3y}{3x - y} = \frac{x + 3x}{3x - x} = 2.$$

Thus, the correct answer is **D**.

5. Camilla had twice as many blueberry jelly beans as cherry jelly beans. After eating 10 pieces of each kind, she now has three times as many blueberry jelly beans as cherry jelly beans. How many blueberry jelly beans did she originally have?

A 10

B 20

C 30

D 40

E 50

**Solution(s):**

Let the number of cherry jelly beans be  $c$  and let the number of blueberry jelly beans be  $b$

Then, we know

$$b = 2c$$

$$b - 10 = 3(c - 10)$$

from the first and second statements respectively.

Therefore,

$$2c - 10 = 3c - 30$$

$$c = 20.$$

This means that

$$b = 2 \cdot 20 = 40.$$

Thus, the correct answer is **D**.

6. What is the largest number of solid  $2\text{in} \times 2\text{in} \times 1\text{in}$  blocks that can fit in a  $3\text{in} \times 2\text{in} \times 3\text{in}$  box?

A 3

B 4

C 5

D 6

E 7

**Solution(s):**

The volume of the large solid object is  $3 \cdot 3 \cdot 2 = 18$  and volume of the smaller object is  $2 \cdot 2 \cdot 1 = 4$ . This means we can fit at most 4 of the small objects.

We can make this happen by putting 3 of the small objects in a  $3 \times 2 \times 2$  rectangular prism, and then we have a  $3 \times 2 \times 1$  space left where we can place one small object.

Thus, the correct answer is **B**.

7. Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour.

In all, it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

A 2.0

B 2.2

C 2.8

D 3.4

E 4.4

### Solution(s):

Let the distance she walked be  $d$ . Since this is the same as the amount she biked, represented as  $s$ , we know that  $d = s$ .

Furthermore, let the time she walked (in hours) be  $t$ . Therefore, the amount of time she biked is  $\frac{44}{60} - t$ .

Now, using the definition of speed, we can see that

$$5 = \frac{s}{t}$$
$$17 = \frac{s}{\frac{44}{60} - t}$$

This implies that:

$$s = 5t = 17 \left( \frac{44}{60} - t \right)$$

so

$$22t = \frac{17 \cdot 44}{60}$$

Therefore,

$$t = \frac{17}{30}$$

Since  $s = 5t$ , we have

$$s = 5 \cdot \frac{17}{30} = \frac{17}{6}$$

which approximates to 2.8.

Thus, the correct answer is **C**.



8. Points  $A(11, 9)$  and  $B(2, -3)$  are vertices of  $\triangle ABC$  with  $AB = AC$ . The altitude from  $A$  meets the opposite side at  $D(-1, 3)$ . What are the coordinates of point  $C$ ?

A  $(-8, 9)$

B  $(-4, 8)$

C  $(-4, 9)$

D  $(-2, 3)$

E  $(-1, 0)$

### Solution(s):

Since the triangle  $ABC$  is isosceles, the altitude from  $A$  is the midpoint of the other two sides. Therefore,  $D$  is the midpoint between  $B$  and  $C$ . If  $C = (x, y)$ , then we have

$$\frac{x + 2}{2} = -1,$$

$$\frac{y - 3}{2} = 3.$$

As such,

$$C = (x, y) = (-4, 9).$$

Thus, the correct answer is **C**.

9. A radio program has a quiz consisting of 3 multiple-choice questions, each with 3 choices. A contestant wins if he or she gets 2 or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

A  $\frac{1}{27}$

B  $\frac{1}{9}$

C  $\frac{2}{9}$

D  $\frac{7}{27}$

E  $\frac{1}{2}$

**Solution(s):**

The probability that a contestant gets all 3 correct is

$$\frac{1}{3}^3 = \frac{1}{27}.$$

The probability that a contestant gets exactly 2 is

$$\frac{1}{3}^2 \cdot \frac{2}{3} \cdot \binom{3}{2} = \frac{6}{27}.$$

The combined probability is

$$\frac{6}{27} + \frac{1}{27} = \frac{7}{27}.$$

Thus, the correct answer is **D**.

10. The lines with equations  $ax - 2y = c$  and  $2x + by = -c$  are perpendicular and intersect at  $(1, -5)$ . What is  $c$ ?

A -13

B -8

C 2

D 8

E 13

### Solution(s):

The first equation can be rewritten as

$$y = \frac{a}{2}x - \frac{c}{2}.$$

Similarly, the second equation can be rewritten as

$$y = -\frac{2}{b}x - \frac{c}{b}.$$

Since they are perpendicular, we know the slopes multiply to  $-1$ .

Therefore,

$$\frac{a}{2} \cdot \left(-\frac{2}{b}\right) = -1.$$

This means  $a = b$ , which implies that  $2x + ay = -c$ . We can add this with the first equation to get

$$2x + ay + ax - 2y = 0.$$

Plugging in  $(x, y) = (1, -5)$  yields

$$2 \cdot 1 + a - 5a - 2 \cdot (-5) = 0.$$

This makes

$$2 \cdot (6) = 4a$$

$$a = 3.$$

Therefore,

$$\begin{aligned}c &= 3x - 2y \\ &= 3 \cdot 1 - 2 \cdot (-5) \\ &= 13.\end{aligned}$$

Thus, the correct answer is **E**.

11. At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

A 10%

B 12%

C 20%

D 25%

E  $33\frac{1}{3}\%$

### Solution(s):

Observe that of the 60% of people that actually like dancing, only 80% say they like dancing. This suggests that 48% of the students say that they like dancing, and as such,  $60\% - 48\% = 12\%$  of the students who like dancing say they don't like it.

Then, we know that 90% of the 40% of people who don't like dancing say they don't like it, which is 36% of the total student population.

This means the total amount of people who say they don't like dancing is  $12\% + 36\% = 48\%$ .

We know then that the fraction of people who say they dislike dancing but actually like it is equal to:

$$\frac{12}{48} = \frac{1}{4} = 25\%.$$

Thus, the correct answer is **D**.

12. Elmer's new car gives 50% percent better fuel efficiency, measured in kilometers per liter, than his old car. However, his new car uses diesel fuel, which is 20% more expensive per liter than the gasoline his old car used. By what percent will Elmer save money if he uses his new car instead of his old car for a long trip?

A 20%

B  $26\frac{2}{3}\%$

C  $27\frac{7}{9}\%$

D  $33\frac{1}{3}\%$

E  $66\frac{2}{3}\%$

### Solution(s):

Every liter can get 1.5 times as many kilometers per liter. This is the same thing as saying she needs  $\frac{2}{3}$  as many liters as per kilometer. However, each liter will cost 1.2 times as many dollars as before.

We need to find the change in dollars over kilometer for the change in cost for the trip. We can see that the change is 1.2 dollars per liter times  $\frac{2}{3}$  liters per kilometer. Solving this, we have  $1.2 \cdot \frac{2}{3} = \frac{4}{5}$  of the cost.

He therefore saves  $\frac{1}{5}$  of the total cost. As such, the savings is 20%.

Thus, the correct answer is **A**.

13. There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three.

There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

- A 1
- B 2
- C 3
- D 4
- E 5

**Solution(s):**

The number of classes taken total is  $10 + 13 + 9 = 32$ .

Let  $x$  represent the number of people who take 1, let  $y$  represent the number of people who take 2 classes, and let  $z$  represent the number of people who take 3 classes.

Then, we know  $x + 2y + 3z = 32$ .

As such, the total number of people is 20, so  $x + y + z = 20$ . This makes  $y + 2z = 12$ .

The number of people who take at least two classes is 9, so  $y + z = 9$ .

Therefore,  $z = 3$ , making that the answer.

Thus, the correct answer is **C**.

14. An integer  $N$  is selected at random in the range  $1 \leq N \leq 2020$ . What is the probability that the remainder when  $N^{16}$  is divided by 5 is 1?

A  $\frac{1}{5}$

B  $\frac{2}{5}$

C  $\frac{3}{5}$

**D  $\frac{4}{5}$**

E 1

### Solution(s):

By Fermat's Little Theorem, we know that  $a^{p-1} \equiv 1 \pmod{p}$  if  $a$  and  $p$  are relatively prime.

Therefore,  $a^4 \equiv 1 \pmod{5}$ , which makes:

$$a^{16} \equiv (a^4)^4 \equiv 1 \pmod{5}$$

if  $a$  and 5 are relatively prime.

Since 5 is a prime, they are relatively prime if  $a$  isn't a multiple of 5. There are  $\frac{2020}{5} = 404$  multiples of 5, so there are 1616 non-multiples of 5.

All multiples of 5, when taken to the 16<sup>th</sup> power, have a remainder of 0 when divided by 5 so they aren't included. Thus, there are exactly 1616 of 2020 numbers that work. This makes the probability  $\frac{1616}{2020} = \frac{4}{5}$ .

Thus, the correct answer is **D**.



15. Rectangle  $ABCD$  has  $AB = 3$  and  $BC = 4$ . Point  $E$  is the foot of the perpendicular from  $B$  to diagonal  $\overline{AC}$ . What is the area of  $\triangle AED$ ?

A 1

B  $\frac{42}{25}$

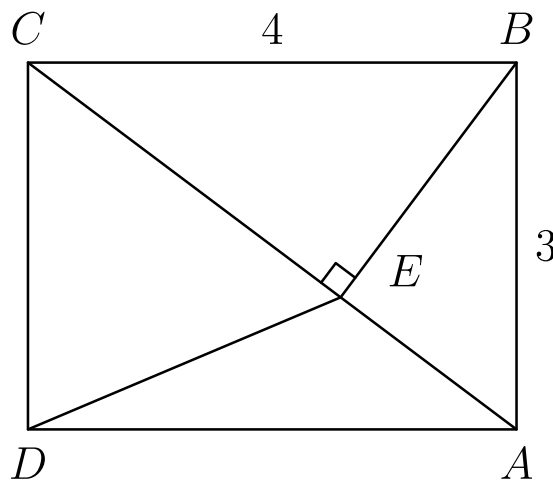
C  $\frac{28}{15}$

D 2

E  $\frac{54}{25}$

**Solution(s):**

Consider the figure:



The area of  $EAD$  is equal to the area of  $CDA$  multiplied by  $\frac{EA}{AC}$  since it has the same altitude and the base has the same line.

The area of  $CDA = \frac{3 \cdot 4}{2} = 6$ . Also, by the Pythagorean Theorem, we get

$$\begin{aligned} CA^2 &= CB^2 + BA^2 \\ &= 4^2 + 3^2 \\ &= 25 \\ CA &= 5 \end{aligned}$$

Next,  $EBA \sim BCA$ , so

$$\frac{EA}{AB} = \frac{BA}{CA} = \frac{5}{3}.$$

Therefore,

$$\frac{AE}{CA} = \frac{3^2}{5} = \frac{9}{25}.$$

As such, the answer is

$$\frac{9}{25} \cdot 6 = \frac{54}{25}.$$

Thus, the correct answer is **E**.

16. How many of the base-ten numerals for the positive integers less than or equal to 2017 contain the digit 0?

A 469

B 471

C 475

D 478

E 481

### Solution(s):

For numbers less than 100, we only have a 0 if its a multiple of 10, of which there are 9.

For numbers between 100 and 999 inclusive, we will use complementary counting. There are 900 total numbers in this range. Also, there are  $9 \cdot 9 \cdot 9 = 729$  numbers in this range with no 0 since there are 9 ways to choose each digit to not be 0. Thus, the total in this range is 171.

For numbers between 1000 and 1999 inclusive, we will use complementary counting again. There are 1000 total numbers in this range. Also, there are  $1 \cdot 9 \cdot 9 \cdot 9 = 729$  numbers in this range with no 0 since there are 9 ways to choose each of the last 3 digits to not be 0 and the first digit must be 1. Thus, the total in this range is 271.

There are 18 numbers between 2000 and 2017 inclusive, each with a 0 in the second digit from the left.

This makes the total

$$9 + 171 + 271 + 18 = 469.$$

Thus, the correct answer is **A**.

17. Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

A 1024

**B 1524**

C 1533

D 1536

E 2048

### Solution(s):

For each unique non-empty subset of

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

we can make a unique monotonous ascending number by taking the numbers in the subset and putting them in ascending order. There are  $2^9 - 1 = 511$  of them.

For each unique non-empty subset of  $S$ , we can make a unique monotonous descending number by taking the numbers in the subset and putting them in descending order. There are  $2^{10} - 1 = 1023$  of them. However, we must remove the subset  $\{0\}$ , which is one case. This yields 1022 cases.

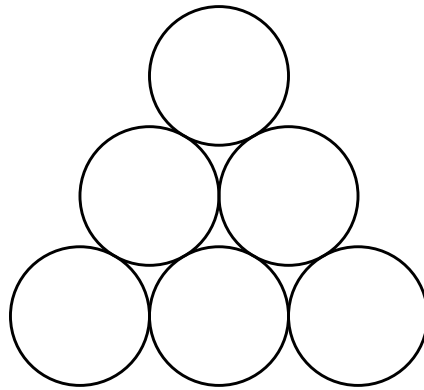
We also must take out the intersection. This would be each of the 9 one digit numbers.

Therefore, the total is

$$1022 + 511 - 9 = 1524.$$

Thus, the correct answer is **B**.

18. In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



- A 6
- B 8
- C 9
- D 12**
- E 15

### Solution(s):

We first will calculate the number of ways when the green is at the top. This is rotationally symmetric with every other corner, so we wouldn't have to count those again. Then, we can multiply our count by 2 since the number of cases when the green is in the inner 3 disks is the same as if we made each corner an edge and each edge piece a corner.

Suppose the green is on the top. Then, there are  $\binom{5}{2} = 10$  places to put the two reds, of which 2 are symmetric. Thus, the number of non symmetric configurations are  $\frac{1}{2} \cdot (10 - 2) = 4$  after dividing by 2 to remove the duplicates, and  $4 + 2 = 6$  when putting those cases back.

This makes the total  $6 \cdot 2 = 12$ .

Thus, the correct answer is **D**.

19. Let  $ABC$  be an equilateral triangle. Extend side  $\overline{AB}$  beyond  $B$  to a point  $B'$  so that  $BB' = 3 \cdot AB$ . Similarly, extend side  $\overline{BC}$  beyond  $C$  to a point  $C'$  so that  $CC' = 3 \cdot BC$ , and extend side  $\overline{CA}$  beyond  $A$  to a point  $A'$  so that  $AA' = 3 \cdot CA$ .

What is the ratio of the area of  $\triangle A'B'C'$  to the area of  $\triangle ABC$ ?

A 9 : 1

B 16 : 1

C 25 : 1

D 36 : 1

E 37 : 1

### Solution(s):

We know that:

$$[A'B'C'] = [ABC] + [A'B'A] \\ + [B'C'B] + [C'A'C].$$

The last three terms on the right hand side of the equation have the same area, so the area:

$$[A'B'C'] = [ABC] + 3[A'B'A].$$

Therefore, to find the ratio in question, we need to find:

$$\frac{[ABC] + 3[A'B'A]}{[ABC]} \\ = 1 + 3 \frac{[A'B'B]}{[ABC]}.$$

Then,

$$[A'B'B] = \frac{1}{2} A'A \cdot A'A \\ \cdot \sin A'AB'$$

and

$$[ABC] = \frac{1}{2} AB \cdot AC \cdot \sin BAC.$$

Since  $\angle A'AB'$  and  $\angle BAC$  are supplements, they have the same sine.

Therefore,

$$\frac{[A'B'B]}{[ABC]} = \frac{A'A \cdot B'A}{AB \cdot AC}.$$

Then,

$$A'B = AB + BB' = 4AB,$$

and

$$AA' = 3AC.$$

This makes

$$\frac{[A'B'B]}{[ABC]} = 4 \cdot 3 = 12.$$

As such, the final ratio is

$$1 + 3 \cdot 12 = 37.$$

Thus, the correct answer is **E**.

20. The number

$$21! = 5.109 \cdot 10^{19}$$

has over 60,000 positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

A  $\frac{1}{21}$

**B**  $\frac{1}{19}$

C  $\frac{1}{18}$

D  $\frac{1}{2}$

E  $\frac{11}{21}$

### Solution(s):

Note that given any integer  $z$ , we can represent it as the product as its even and odd components as  $z = 2^c d$ . This comes from the uniqueness of the prime factorization of integers, as we simply aggregate the odd and even primes (or in other words, 2, and everything else).

With this in mind, using the prime factorization of  $21!$ , the even part of such a representation is  $2^{18}$ . As such, let  $21! = 2^{18}d$ , making  $d$  an odd divisor of  $21!$ .

This means every odd divisor of  $21!$  is a divisor of  $d$ . For any odd divisor  $x$  of  $d$ , we know  $2^k x$  is a divisor of  $21!$  for  $0 \leq k \leq 18$ . It is only odd for  $k = 0$ , and as such, there are 19 possible values of  $k$ .

As such, the total probability is

$$\frac{1}{19}.$$

Thus, the correct answer is **B**.



21. In  $\triangle ABC$ ,  $AB = 6$ ,  $AC = 8$ ,  $BC = 10$ , and  $D$  is the midpoint of  $\overline{BC}$ . What is the sum of the radii of the circles inscribed in  $\triangle ADB$  and  $\triangle ADC$ ?

A  $\sqrt{5}$

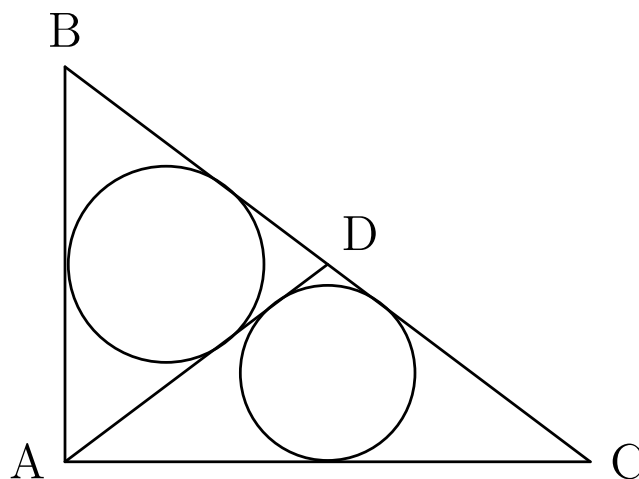
B  $\frac{11}{4}$

C  $2\sqrt{2}$

D  $\frac{17}{6}$

E 3

Solution(s):



The triangle  $ABC$  is a right triangle with a right angle at  $A$ . This makes  $D$  the circumcenter of the triangle since it is the midpoint of the hypotenuse.

Therefore,

$$AD = BD = DC = 5.$$

Also, the area of  $ABC$  is

$$\frac{6 \cdot 8}{2} = 24.$$

Since  $BD$  and  $DC$  have the same altitude and base, the triangles  $ABD$  and  $ACD$  have the same area of 12.

Then, for each triangle, we have  $A = rs$  where  $A$  is the area,  $r$  is the inradius, and  $s$  is the semiperimeter. This means  $12 = \frac{1}{2}rP$  for each triangle, where  $P$  is the

perimeter. Thus, we know that

$$r = \frac{24}{P}.$$

We apply this fact for  $ABD$ , to see that  $r = \frac{24}{5 + 5 + 6} = \frac{3}{2}$ . Similarly, for  $ACD$ , it

$$r = \frac{24}{5 + 5 + 8} = \frac{4}{3}.$$

Their sum is  $\frac{3}{2} + \frac{4}{3} = \frac{17}{6}$ .

Thus, the correct answer is **D**.

22. The diameter  $\overline{AB}$  of a circle of radius 2 is extended to a point  $D$  outside the circle so that  $BD = 3$ . Point  $E$  is chosen so that  $ED = 5$  and line  $ED$  is perpendicular to line  $AD$ . Segment  $\overline{AE}$  intersects the circle at a point  $C$  between  $A$  and  $E$ . What is the area of  $\triangle ABC$ ?

A  $\frac{120}{37}$

B  $\frac{140}{39}$

C  $\frac{145}{39}$

D  $\frac{140}{37}$

E  $\frac{120}{31}$

**Solution(s):**

Since the radius is 2 and  $BD = 3$ , we have  $AD = 7$ . Since  $ED = 5$  and the angle at  $D$  is a right angle, the area of  $ADE$  is  $\frac{5 \cdot 7}{2} = \frac{35}{2}$ .

Also, the value of  $AE$  is  $\sqrt{5^2 + 7^2} = \sqrt{74}$  by the Pythagorean Theorem. Also,  $\angle ACB$  is a right angle since  $AB$  is a diameter. Thus, by angle-angle symmetry, we have  $ACB \sim ADE$ .

This means the area of  $ABC$  is equal to the area of  $AED$  times  $\frac{AB^2}{AE^2} =$

$\left(\frac{4}{\sqrt{74}}\right)^2 = \frac{8}{37}$ . Then, we have an area of

$$\frac{8}{37} \cdot \frac{35}{2} = \frac{140}{37}.$$

Thus, the correct answer is **D**.

23. Let

$$N = 123456789101112 \dots 4344$$

be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when  $N$  is divided by 45?

- A 1
- B 4
- C 9
- D 18
- E 44

### Solution(s):

To find the remainder when divided by 45, we must find the remainder when divided by 5 and 9. The remainder when divided by 5 is the remainder when the units digit is divided by 5, making it 4.

To find the remainder when divided by 9, we usually find the sum of the digits. However, each double digit number has the same remainder when divided by 9 as its digit sum, so we can just take the sum of each of the numbers from 1 to 44 as they would have the same remainder. The sum of the first 44 digits is  $\frac{45 \cdot 44}{2}$  which is a multiple of 9. Thus,  $N$  is a multiple of 9.

Since it is a multiple of 9 and has a remainder of 4 when divided by 45, the remainder when divided by 45 is 9.

Thus, the correct answer is **C**.

24. The vertices of an equilateral triangle lie on the hyperbola  $xy = 1$ , and a vertex of this hyperbola is the centroid of the triangle. What is the square of the area of the triangle?

A 48

B 60

C 108

D 120

E 169

### Solution(s):

Since the hyperbola is symmetric, without the loss of generality, we can have  $(1, 1)$  as our vertex. Then, since we have the centroid of an equilateral triangle, the angle at the centroid with any two points is  $120^\circ$ . The branch of the hyperbola with negative coordinates can make an angle of at most  $90^\circ$ . This means that we can't have two points on the negative branch.

Since the hyperbola is symmetric over  $y = x$  and it always decreases, the two points are reflected over  $y = x$ . Also, the altitude is on  $y = x$ , making the other point also on  $y = x$ . This makes the other point  $(-1, -1)$ . Thus, the circumradius is  $2\sqrt{2}$  since it is the distance between the two points. This means we have 3 isosceles triangles with side lengths  $2\sqrt{2}$  and angle  $120^\circ$ .

Therefore, the combined area is

$$\begin{aligned} & 3 \cdot \frac{(2\sqrt{2})^2 \cdot \sin(120^\circ)}{2} \\ &= 12 \sin(120^\circ) \\ &= 12 \cdot \frac{\sqrt{3}}{2} \\ &= \sqrt{108}. \end{aligned}$$

This makes the square 108.

Thus, the correct answer is **C**.

25. Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

A 92

B 94

C 96

D 98

E 100

### Solution(s):

The smallest possible average of the first 6 of them is

$$\frac{91 + \cdots + 96}{7} = 93.5.$$

The largest possible average of the first 6 of them is

$$\frac{95 + \cdots + 100}{7} = 97.5.$$

This makes the bounds of the average of the first 6 of them 94 and 97 inclusive.

Then, let the average of the first 6 of them be  $x$ . Then, the average of all of them is  $\frac{6x + 95}{7}$ , making  $6x + 95$  a multiple of 7.

Therefore,  $6x + 95 \equiv 0 \pmod{7}$ . This means  $x \equiv 4 \pmod{7}$ . The only possible value is  $x = 95$ , making the sum of the first 6 of them 570.

Then, the sum of the first 5 is a multiple of 5, so the 6th score must also be a multiple of 5 since it is their difference. The only not used multiple of 5 is 100, making it the answer.

Thus, the correct answer is **E**.

Problems: <https://live.poshenloh.com/past-contests/amc10/2017B>

